Theoretical Studies on the Self Stability and Internal Feedbacks Transfer Functions of Boiling Water Reactor

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The analytical expressions for the internal feedbacks transfer functions of boiling reactor, taking the local effects of both the void and the fuel temperature feedbacks on reactivity into consideration, has been established in the range of linear mathematical treatments for a model with the power densities being distributed spatially and the inlet velocities of feedwater being given as a function of radial locations.

However, this paper does not relate to the mechanisms of heat transfer itself, but, by employing the appropriate heat transfer coefficients, deals especially with the transient fluctuations of the local void fractions which depend upon the generation and movement of the steam bubbles generated by the heat once transmitted to the coolant.

The effects of the thermodynamic properties of the moderator on the self regulative characteristics of boiling reactor has also been obtained in a form of inequality dominting the limitation on the absolute value of the void coefficient of reactivity.

Moreover, the power transfer function of EBWR under the operational conditions of 50 MW has been computed numerically, by using the reactor parameters concerned and the formulae obtained by the author, the results of which show a relatively good agreement with the experimental results obtained at Argonne National Laboratory and reported in such as ANL-5849.

INTRODUCTION

One of the most essential factors for the economic operation of a nuclear reactor is to increase the specific power generation within the core as much as possible.

Generally speaking, the dominant factors restricting such specific power in a nuclear reactor are the maximum temperature of fuel elements and the capability of heat transfer from the surface of fuel elements to the coolant.

However, in the case of boiling reactor since it generates steam within the core the steam void fraction contained in the core will become much more important as the factor for such specific power limitation from the view point of instability phenomena.

The operational experience of various BWR to date shows that when the reactivity absorbed by steam void within the core exceeds 2% or 3%, the stable operation of the reactor may be disturbed and led into resonance instability

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state.

With reference to the essential characteristics of the stability of boiling reactor the hydraulic instability of two phase flow within the channel is of course one of the important factors that have to be taken into consideration of.

However, it will be recognized that in the boiling reactor the existence of void feedbacks transfer functions will be the most fundamental mechanism for manifesting such resonance instability.

In the design and operation of boiling reactor the prediction of the stability limit under given operational conditions is an extremely vital problem, for which it is essential that the power transfer function of the reactor be made known previously.

This paper deals with the establishment of the interrelation between the self regulative characteristics of boiling reactor and the thermodynamic properties of the moderator together with the analytical expressions for the void feedbacks transfer function of such reactor. Moreover, the power transfer function of EBWR under the operational conditions of 50 MW has been computed numerically and compared with the experimental results obtained at Argonne National Laboratory.

EFFECTS OF THE THERMODYNAMIC PROPERTIES OF THE MODERATOR

The boiling reactor unlike the pressurized water reactor it generates steam within the core. When steam displaces a certain amount of water from the core the reactivity will inevitably be changed. Then, it is obvious that the void feedbacks reactivity change must at least be in the negative direction in order to let the boiling reactor possess the self regulative characteristics.

However, as stated in the introduction, experiments performed so far have shown that the reactor is stable only when the total amount of the reactivity change absorbed by steam void contained within the core represents a negative value of not more than 2% or 3% of the effective reactivity[1]. This fact implies that in order to make boiling reactor manifest the stable behavior the void coefficient of reactivity should be negative and moreover its absolute value be restricted within the narrow range for a given amount of steam void fraction within the core, which is deemed to be practicable for the economic operation of an ordinary boiling water power reactor.

Such restriction on the void coefficient of reactivity will first of all originate with the interrelation between the nuclear characteristics of the reactor and the thermodynamic properties of the moderator as shown in the following analysis.

Assuming that the fuel temperature feedbacks can be ignored, the effective reactivity of the boiling reactor with no subcooled region will be given as a function of the average density and temperature of the moderator available within the core and written as

$$K=f(\rho_{av},\ t_m) \tag{1}$$

$$\rho_{av}=\rho_g f_v+\rho_f(1-f_v) \tag{2}$$

where

ρ_{av} = Average density of the moderator within the core,

t_m = saturation temperature of the moderator,

ρ_g = density of the saturated steam,

ρ_f = density of the saturated water,

f_v = average steam void fraction within the core.

Supposing that both the flow rate of the moderator and the velocity ratio of steam to water within the channel are invariable respectively, the average steam void fraction f_v may be determined by both the steam generation rate and the saturation temperature of the moderator and written as

$$f_v = \varphi(W_g, t_m) \tag{3}$$

where

W_g = steam generation rate within the core.

The transient changes of the effective reactivity from the equilibrium value will be obtained by expanding equations (1), (2) and (3) into Taylor's series near equilibrium state.

Neglecting the higher order terms of these expansions, the following three variational equations in the forms of Lapalce transform may by obtained.

$$\delta K(s) = \left(\frac{\partial K}{\partial \rho_{av}}\right)_{t_m} \delta\rho_{av}(s) + \left(\frac{\partial K}{\partial t_m}\right)_{\rho_{av}} \delta t_m(s) \tag{4}$$

$$\delta f_v(s) = \left(\frac{\partial f_v}{\partial W_g}\right)_{t_m} \delta W_g(s) + \left(\frac{\partial f_v}{\partial t_m}\right)_{W_g} \delta t_m(s) \tag{5}$$

$$\delta\rho_{av}(s) = -\Delta\rho\,\delta f_v(s) + \delta t_m(s) \cdot \left[\frac{\partial \rho_f}{\partial t_m} - f_v \frac{\partial \Delta\rho}{\partial t_m}\right] \tag{6}$$

Where $\Delta\rho = \rho_f - \rho_g$ and δ denotes the transient variation of the reactor variables from eqilibrium values.

Elliminating $\delta\rho_{av}(s)$ and $\delta f_v(s)$ from equations (4), (5) and (6), it follows that

$$\delta K(s) = -\Delta\rho \left(\frac{\partial K}{\partial \rho_{av}}\right)_{t_m} \left(\frac{\partial f_v}{\partial W_g}\right)_{t_m} \delta W_g(s) + \Phi\,\delta t_m(s) \tag{7}$$

where $\Phi = B - A$

$$-A = \left(\frac{\partial K}{\partial t_m}\right)_{\rho_{av}} + \left(\frac{\partial K}{\partial \rho_{av}}\right)_{t_m} \frac{\partial \rho_f}{\partial t_m}$$

$$-B = f_v \left(\frac{\partial K}{\partial \rho_{av}}\right)_{t_m} \frac{\partial \Delta\rho}{\partial t_m} + \Delta\rho \left(\frac{\partial K}{\partial \rho_{av}}\right)_{t_m} \left(\frac{\partial f_v}{\partial t_m}\right)_{W_g}.$$

Observing equation (7), it will be seen that the reactor may naturally possess the self regulative characteristics against the disturbance of the transient changes in the steam generation rate since the coefficient of $\delta W_g(s)$ in eqation (7) is inherently negative provided that the void coefficient of reactivity is negative.

However, against the disturbance of the transient changes in the saturation temperature of the moderator, the reactor can be self-regulative only when the coefficient Φ is negative. Here the value of the coefficient Φ will be determined by both the nuclear characteristics of the reactor and the thermodynamic properties of the moderator near equilibrium state as shown in equation (7).

Consequently, the following inequality (8) restricting the value of the negative void coefficient of reactivity under the positive denominator, is yielded by the necessary condition for acquiring the self regulative characteristics in the operation of the reactor without external feedbacks against the transeint variations near equilibrium state in both the steam generation rate and the saturation temperature of the moderator.

$$\left|\left(\frac{\partial K}{\partial f_v}\right)_{t_m}\right| \leq \frac{\Delta\rho\left|\left(\frac{\partial K}{\partial t_m}\right)_{\rho_{av},\, t_f}\right|}{f_v\left|\frac{\partial\Delta\rho}{\partial t_m}\right| + \Delta\rho\left|\left(\frac{\partial f_v}{\partial t_m}\right)_{W_g}\right| - \left|\frac{\partial \rho_f}{\partial t_m}\right|} \qquad (8)$$

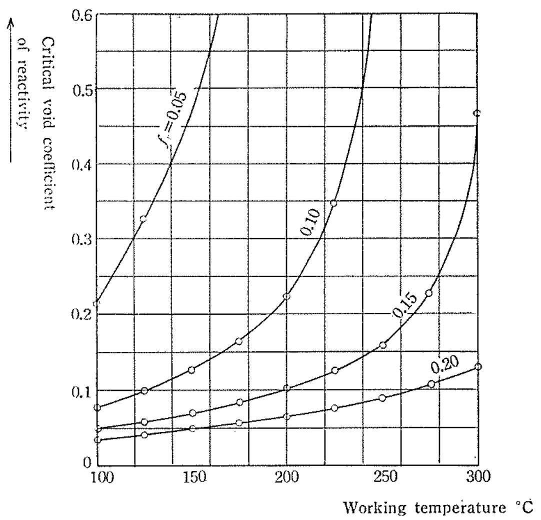

Fig 1. Critical void coefficient of reactivity as a function of saturated temperature of the moderator, $\frac{\partial K}{\partial t_m} = -2.16\times10^{-4}/°C$.

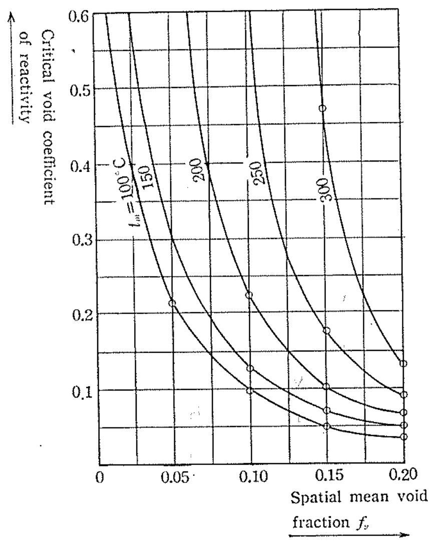

Fig. 2 Critical void coefficient of reactivity as a function of sapatial mean void fraction, $\frac{\partial K}{\partial t_m} = -2.16\times10^{-4}/°C$.

However, when the denominator of the inequality has nagative value under the operational conditions of the reactor, no such limitation occurs on the void coefficient of reactivity. The numerator of inequality (8) is given by the product of the density difference between the saturated liquid and steam and the moderator temperature coefficient of reactivity. On the other hand, the denominator will be determined only by the thermodynamic properties of the moderator under the operational conditions of the reactor.

Assuming the moderator temperature coefficient of reactivity of -2.16×10^{-4} $\frac{\Delta K}{°C}$, some examples of numerical computations on inequality (8) were performed, the results of which are shown in Fig. 1 and Fig. 2, respectively.

ANALYTICAL EXPRESSIONS FOR THE VOID FEEDBACKS TRANSFER FUNCTIONS

Assuming that the core is homogenized, let $w(\tau)q(r,\varphi,z)$ denotes generally the time and space dependent heat transfer rate from fuel surface to the coolant per unit volume of the core in an appropriate cylindrical co-ordinates. The function $q(r,\varphi,z)$ representing the spatial mode of thermal flux distribution, may usually be presumable to be independent of amplitude φ so long as the control rods are all withdrawn by means of applying such as chemical poison to suppress the excess reactivity of the reactor as was often employed to make the thermal flux distribution independent of amplitude φ in the experiments to measure the feedbacks transfer functions of BWR. Moreover, the function $q(r,z)$ is normarized as

$$\int_{V_0}\int q(r,z)2\pi r\,dr\,dz = V_0 \tag{9}$$

Then, by integrating the function $w(o)q(r,z)$ with respect to r and z extending over the entire space of the core, the interrelation between the equilibrium values of $w(\tau)$ and $p(\tau)$ will be established as

$$w(o)=\frac{0.23889P_0}{V_0} \tag{10}$$

where P_0 represents the total power generation within the core, KW. By equation (10), it is seen that $w(o)$ represents the average heat generation rate per unit volume of the core in the equilibrium conditions.

1. Boiling Boundary in the Equilibrium State

Suppose the reactor is operating in the balanced conditions with the inlet velocities of feedwater being given as a function of radial locations. Then, by considering the heat balance within the subcooled region of an elementary channel whose radii are r and $r+dr$, respectivery, the following equation will be established.

$$\int_0^{z_0(r)} q(r,z)dz=\frac{v_0(r)\rho_f\Delta H_s}{w(o)} \tag{11}$$

where

$v_0(r)$ = radial distribution of the inlet velocities of feedwater

ρ_f = specific weight of the feedwater
ΔH_s = enthalpy difference between the feedwater and the saturated water
$z_0(r)$ = radial distribution of the equilibrium locations of boiling boundary.

Solving equation (11) with respect to $z_0(r)$ under the given conditions, we shall obtain the axial locations of boiling boundary as a function of radius r.

2. Local Void Fractions in the Equilibrium Conditions

Here, the local void fraction is defined by the ratio of the volume occupied by steam void to the total volume of mixed fluid in an elementary domain at an arbitrary location within the core.

Since the heat transferred to the coolant within the boiling region of the core will entirely be consumed by steam generation, considering the equilibrium heat balance within the boiling region of the elementary annular channel whose radii are r and $r+dr$, respectively, we shall obtain the expression for the local void fractions as

$$f_v(r,z)=\frac{w(o)}{r_0\rho_g\sigma}\,\frac{1}{v_g(r,z)}\int_{z_0(r)}^{z}q(r,\xi)d\xi \tag{12}$$

where

$f_v(r,z)$ = local void fraction at an arbitrary location (r,z) within the core
r_0 = latent heat of vaporization
$v_g(r,z)$ = local steam flow velocity at an arbitrary location (r,z) within the core
σ = ratio of the total channel area to the total cross sectional area of the entire core.

By the definition of local void fraction, it follows that

$$v_g(r,z)=\frac{v_0(r)\,\varepsilon(r)}{1-f_v(r,z)} \tag{13}$$

in which $\varepsilon(r)$ denotes the local velocity ratios of steam to water within the core. As for the velocity ratios within a channel, it has already been found experimentally by P. A. Lottes[2], ANL, that the velocity ratios are independent approximately of the axial locations. Consequently, we presume that the velocity ratios within the core can be determined only by the radial locations of the channel considered.

Substituting equations (13) into equation (12), we shall obtain the expression for the local steam flow velocities as

$$v_g(r,z)=v_0(r)\,\varepsilon(r)+\frac{w(o)}{r_0\rho_g\sigma}\int_{z_0(r)}^{z}q(r,\xi)d\xi \tag{14}$$

Integrating equation (14) with respect to z extending over the entire boiling region of the channel, the axial mean steam flow velocity may easily be computed and ultimately be reduced to

$$\langle v_g(r,z)\rangle=v_0(r)\left[\varepsilon(r)-\frac{\rho_f\Delta H_s}{r_0\rho_g\sigma}\right]+\frac{w(o)}{r_0\rho_g\sigma}\cdot[\Omega(r,H)-\Omega(r,z_0(r))] \tag{15}$$

where

$$\Omega(r,z)=\int_0^z\int_0^\xi q(r,\eta)d\eta d\xi$$

H = length of the active core.

3. Transient Shifts of Boiling Boudary

When the reactor had been operated in equilibrium conditions, suppose the transient changes have occured in the power abruptly at the instant $\tau=0$, then the heat to be transferred to the coolant within the core may also be changed transiently from the balanced conditions. The transient deviations of this heat rate transferred to the coolant per unit volume of the core are represented by $\delta w(\tau)q(r,z)$ as has already been defined in previous section. In like manner, the transient shifts of boiling boundary will be denoted by $\delta z_0(r,\tau)$.

Let us imagine an elementary annular channel whose radii are r and $r+dr$, respectively. Considering the transient heat passing with the feedwater through the section of the instantaneous boiling boundary in the elementary channel, the transient heat balance equation may easily be obtained as

$$\int_0^{z_0(r)+\delta z_0(r,\tau)}\left[w(o)+\delta w\left(\tau-\frac{z_0(r)-\xi}{v_0(r)}\right)\right]q(r,\xi)d\xi$$
$$+v_0(r)\rho_f\left[H_i(o)+\delta H_i\left(\tau-\frac{z_0(r)}{v_0(r)}\right)\right]=v_0(r)\rho_f[H_s(o)+\delta H_s(\tau)] \quad (16)$$

where

$H_i(o)$ =enthalpy of the feedwater in the equilibrium conditions

$\delta H_i(\tau)$ =transient deviations of the enthalpy of feedwater

$H_s(o)$ =enthalpy of the saturated water in the equilibrium conditions

$\delta H_s(\tau)$ =transient changes of the enthalpy of saturated water.

Equation (16) implies that the sum of the total heat introduced with the feedwater and the heat transferred to the coolant within the subcooled region will be equal to the total heat of the saturated water passing through the section of the instantaneous boiling boundary.

Expanding the integral of equation (16), neglecting the higher order of the terms and subtracting the equilibrium values from both sides of the equation, it may easily be reduced to

$$w(o)q(r,z_0(r)\delta z_0(r,\tau)+\int_0^{z_0(r)}\delta w\left(\tau-\frac{z_0(r)-\xi}{v_0(r)}\right)q(r,\xi)d\xi$$
$$=v_0(r)\rho_f\left[\delta H_s(\tau)-\delta H_i\left(\tau-\frac{z_0(r)}{v_0(r)}\right)\right]. \quad (17)$$

The Laplace transform of equation (17) will be obtained as

$$w(o)q(r,z_0(r))\delta z_0(r,s)+\delta w(s)\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\times$$
$$\int_0^{z_0(r)}\exp\left(\frac{\xi s}{v_0(r)}\right)q(r,\xi)d\xi=v_0(r)\rho_f\left[\delta H_s(s)-\delta H_i(s)\cdot\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\right]. \quad (18)$$

By equation (18), the generalized expression for the Laplace transform of the transient shifts of boiling boundary can be obtained as

$$\delta z_0(r,s)=\frac{1}{w(o)q(r,z_0(r))}\left[v_0(r)\rho_f\left\{\delta H_s(s)-\delta H_i\cdot\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\right\}\right.$$
$$\left.-\delta w(s)\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\times\int_0^{z_0(r)}\exp\left(\frac{-\xi}{v_0(r)}s\right)q(r,\xi)d\xi.\right. \quad (19)$$

For the simplified model without accompanying any transient change of the enthalpy of feedwater or working pressure, equation (19) will be shortened to

$$\delta z_0(r,s)=\frac{-\delta w(s)}{w(o)q(r,z_0(r))}\cdot\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\times \int_0^{z_0(r)}\exp\left(-\frac{\xi}{v_0(r)}s\right)q(r,\xi)d\xi \tag{20}$$

which will give the transient shifts of boiling boundary caused only by the power fluctuations.

Now, it will be convenient to derive, by employing the appropriate method of integration, the another expression for the transient shifts of boiling boundary in the forms of Laplace transforms. Putting $\eta=\tau-\frac{z_0(r)-\xi}{v_0(r)}$ in the integral contained in equation (17) and rewritting ξ for η in the resultant integral, it will ultimately be reduced to

$$\Phi(\tau)=v_0(r)\left[\int_0^{\tau}q\{r,z_0(r)-v_0(r)(\tau-\xi)\}\delta w(\xi)d\xi - \int_0^{\tau-\frac{z_0(r)}{v_0(r)}}q\left\{r,-v_0(r)\left(\tau-\frac{z_0(r)}{v_0(r)}-\xi\right)\right\}\delta w(\xi)d\xi\right]. \tag{21}$$

Taking Laplace transform of equation (21), by using appropriate convolution theorem, we shall obtain

$$\Phi(s)\equiv L\Phi(\tau)=v_0(r)\delta w(s)\left[Lq\{r,z_0(r)-v_0(r)\tau\} -\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)Lq\{r,-v_0(r)\tau\}\right]. \tag{22}$$

Substituting equation (22) into equation (19), we shall obtain the another expression for the transient shifts of boiling boundary as

$$\delta z_0(r,s)=\frac{1}{w(o)q(r,z_0(r))}\left[v_0(r)\rho_f\left\{\delta H_s(s)-\delta H_i(s)\cdot\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\right\} -v_0(r)\delta w(s)\left\{Lq(r,z_0(r)-v_0(r)\tau) -\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\cdot Lq(r,-v_0(r)\tau)\right\}\right]. \tag{23}$$

When the enthalpy of feedwater and the working pressure are invariable transiently, equation (23) is reduced to

$$\delta z_0(r,s)=\frac{-v_0(r)\delta w(s)}{w(o)q(r,z_0(r))}\left[Lq(r,z_0(r)-v_0(r)\tau) -\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)Lq(r,-v_0(r)\tau)\right]. \tag{24}$$

It is interesting to note that the expression for the transient shifts of boiling boundary contains some Lapalce transforms of the function which has been introduced to express the spatial mode of neutron flux distribution and therefore may originally by independent of time.

Comparing equation (20) with equation (24), we shall obtain the following formula representing a certain correlation among some Laplace transforms of an arbitrary function:

$$\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\int_0^{z_0(r)}\exp\left(-\frac{\xi}{v_0(r)}s\right)q(r,\xi)d\xi$$
$$=v_0(r)\left[Lq(r,z_0(r)\tau)-\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)Lq(r,-v_0(r)\tau)\right]. \qquad (25)$$

It will easily be verified that the formula is valid so long as the functions are transformable.

We shall next consider the simplified fuel model with no temperature graduent in the lateral directions and no heat conduction in the axial directions in order to obtain the expression for the transient heat balance in the simplest form

$$\delta w(s)=0.23889\frac{a}{s+a}\delta p(s) \qquad (26)$$

where a denotes the reciprocal time constant of heat transfer, sec.

However, it should be noted that the time constants of heat transfer in both the boiling and the subcooled regions within the heat transfer channels may slightly be different from each other chiefly because of the difference between the heat transfer coefficients in both regions[3]. Consequently, two kinds of reciprocal time constants of heat transfer should be employed in equation (26), namely

$$\delta w_1(s)=0.23889\frac{a_1}{s+a_1}\delta p(s) \qquad (27)$$

$$\delta w_2(s)=0.23889\frac{a_2}{s+a_2}\delta p(s) \qquad (28)$$

where a_1 and a_2 denote the reciprocal time constants of heat transfer in the boiling and subcooled regions, respectively.

Remembering that equation (24) was obtained by considering the transient heat balance within the subcooled region of the heat transfer channel, hereafter we shall write $\delta w_2(s)$ for $\delta w(s)$ in the same equation.

4. Transient Fluctuations of the Local Void Fractions

Generally speaking, the fluctuations of the local void fraction at an arbitrary location within the heat transfer channels will depend upon the transient changes of both the steam generation rate and the locations of boiling boundary. However, for the simplified model with the enthalpy and flow rate of the feedwater as well as the working pressure being invariable transiently, the fluctuations of both the steam generation rate and the locations of boiling boundary will respectively be caused only by the power fluctuations. Hence, for such simplified model, the transient variations of the local void fractions will be given by the sum of the effects of both fluctuations of steam generation rate and the transient shifts of boiling boundary on the local void fractions, each of which depends only upon the power fluctuations.

4.1. Effects of the fluctuations of steam generation rate on the local void

fractions. Hereafter, some appropriate assumptions enumerated below were introduced in the analyses in order to permit the linear mathematical treatments for the establishments of the internal feedbacks transfer functions, namely

a) transient deviations of the reactor's variables from the equilibrium values are small, with linear mathematical treatments being adequate,

b) transient fluctuations of the coolant velocities, steam velocities and working pressure can be ignored,

c) axial distribution of the steam flow velocities in an heat transfer channel can be ignored, the steam flow velocities are therefore presumed to be determined only by the radial location of the channels.

Consider an elementary annular channel whose radii are r and $r+dr$, respectively, in an appropriate cylindrical co-ordinates instituted within the core. Since the heat transferred to the coolant within the boiling region of the channels may entirely be consumed by steam generation, the heat flow rate at an instant τ carried with the steam through the section whose axial locations is z in the boiling region of the elementary channel will be given by

$$\int_{z_0(r)}^{z} w\left(\tau-\frac{z-\xi}{v_g(r)}\right) q(r, \xi) 2\pi r dr d\xi$$

in which the effect of the transient shifts of boiling boundary on the value of integal is ignored and the equilibrium locations of boiling boundary are therefore employed approximately in the integral for the instantaneous boiling boundary.

Consequently, the corresponding mass flow rate of the steam will be

$$\frac{1}{r_0}\int_{z_0(r)}^{z} w\left(\tau-\frac{z-\xi}{v_g(r)}\right) q(r, \xi) 2\pi r dr d\xi$$

in which $v_g(r)$ are the steam flow velocities as a function of radial locations. Since the cross sectional area of fluid flow pass in an elementary annular channel is given by $2\pi r dr \sigma$, the instantaneous local void fractions at section z will easily be obtained as

$$f_{v1}(r, z, \tau)=\frac{1}{r_0\rho_g\sigma v_g(r)}\int_{z_0(r)}^{z} w\left(\tau-\frac{z-\xi}{v_g(r)}\right) q(r, \xi) d\xi \tag{29}$$

in which

σ = ratio of the cross sectional area of mixed flow pass to the total cross sectional area of the core.

Laplace transform of the transient deviations of equation (29) will be given by

$$\delta f_{v1}(r, z, s)=\frac{\delta w_1(s)}{r_0\rho_g\sigma v_g(r)}\exp\left(-\frac{zs}{v_g(r)}\right)\int_{z_0(r)}^{z}\exp\left(\frac{\xi}{v_g(r)}s\right) q(r, \xi) d\xi \tag{30}$$

where $\delta f_{v1}(r, z, s)$ represents, for the sake of simplicity of the notation, the Laplace transform of $\delta f_{v1}(r, z, \tau)$.

We shall next obtain the another expression for the Laplace transform of the transient deviations of the local void fractions, by employing the identical treatments with section 3. Let $A(\tau)$ represents the integral contained in equation (29), namely

$$A(\tau)=\int_{z_0(r)}^{z} w\left(\tau-\frac{z-\xi}{v_g(r)}\right)q(r,\xi)d\xi. \tag{31}$$

Putting $\tau-\frac{z-\xi}{v_g(r)}=\eta$ in equation (31), it is written as

$$A(\tau)=v_g(r)\left[\int_0^{\tau} w(\xi)q\{r,z-v_g(r)(\tau-\xi)\}d\xi\right.$$

$$\left.-\int_0^{\tau-\frac{z-z_0(r)}{v_g(r)}} w(s)q\left\{r,z_0(r)-v_g(r)\left(\tau-\frac{z-z_0(r)}{v_g(r)}-\xi\right)\right\}d\xi\right] \tag{32}$$

in which the integral variable is rewritten ξ for η.

Using appropriate convolution theorem, the Laplace transform of equation (32) will easily be obtained as

$$A(s)=v_g(r)w(s)\left[Lq(r,z-v_g(r)\tau)\right.$$

$$\left.-\exp\left(-\frac{z-z_0(r)}{v_g(r)}s\right)Lq(r,z_0(r)-v_g(r)\tau)\right]. \tag{33}$$

Substituting equation (33) into equation (29), the another expression in the form of Laplace transform for the transient deviations of the local void fractions caused by the fluctuations of steam generation rate will finally be established as

$$\delta f_{v1}(r,z,s)=\frac{\delta w_1(s)}{r_0\rho_g\sigma}\left[Lq(r,z-v_g(r)\tau)-\exp\left(-\frac{z-z_0(r)}{v_g(r)}s\right)\times\right.$$

$$\left.Lq(r,z_0(r)-v_g(r)\tau)\right]. \tag{34}$$

Integrating equation (34) with respect to r and z extending over the entire space of the core, the Laplace transform of the transient deviations of average steam void fraction will be written as

$$\delta f_{v1}(s)=\frac{\delta w_1(s)}{r_0\rho_g\sigma V_0}\left[\iint_{V_1}Lq(r,z-v_g(r)\tau)dV-\right.$$

$$\left.\iint_{V_1}\exp\left(-\frac{z-z_0(r)}{v_g(r)}s\right)Lq(r,z_0(r)-v_g(r)\tau)dV\right] \tag{35}$$

where V_1 represents the volume of boiling region of the core in equilibrium state.

4.2. Effects of the transient shifts of boiling boundary on the local void fractions. Owing to the assumption on the working pressure of being invariable transiently, it will be presumed that the self evaporation of liquid or the extinction of steam bubbles once generated does not occur in their mixed flow.

The effects of the transient shifts of boiling boundary on the local void fractions will easily be understood gemetrically by the aid of Fig. 3. In Fig. 3, point A represents the axial location of the boiling boundary in the equilibrium conditions, while the point C be the instantaneous boiling boundary. The transient shift of boiling boundary at that instant will therefore be represented by AC.

Let line AB in Fig. 3. represents diagrammatically the xial distribution of the

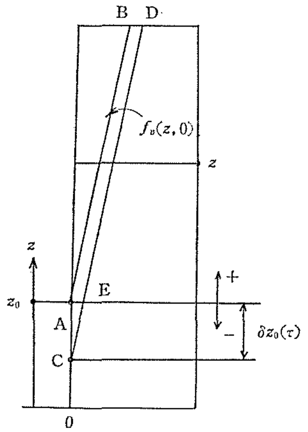

Fig. 3. Effects of transient shifts of boiling boundary on the local void fractions.

local void fractions in the equilibrium conditions, the instantaneous variation of the local void fractions depending upon the transient shifts of boiling boundary will be shown by such a line as CD. Consequently, during transient changes of the reactor the section A located at the equilibrium boiling boundary will become to have some instantaneous void fractions other than zero, as seen in Fig. 3.

The steam bubbles, which are passing through the section A at an instant τ, will continue to move upwards within the channel with the velocity $v_g(r)$ and then will reach an arbitrary section z immediately after the lapse of the time $\frac{z-z_0(r)}{v_g(r)}$. The effect of the transient shifts of boiling boundary on the local void fraction of section z at an instant τ will therefore be determined by the void fraction of section A at the time $\frac{z-z_0(r)}{v_g(r)}$ earlier than τ.

Consequently, we shall now obtain the expression for the transient fluctuations of the local void fractions depending upon the transient shifts of boiling boundary as

$$\delta f_{v_2}(r, z, \tau) = f_v\left\{r, z_0(r), \tau - \frac{z-z_0(r)}{v_g(r)}\right\}. \tag{36}$$

The laplace transform of equation (36) will be

$$\delta f_{v_2}(r, z, s) = f_v(r, z_0(r), s) \cdot \exp\left(-\frac{z-z_0(r)}{v_g(r)} s\right). \tag{37}$$

However, referring to Fig. 3, the instantaneous void fractions of section A during transient changes of the reactor may approximately be given by

$$f_v(r, z_0(r), s) = -\delta z_0(r, s) \cdot \left[\frac{\partial f_v(r, z, o)}{\partial z}\right]_{z=z_0(r)} \tag{38}$$

so long as the transient deviations of the reactor's variables from the equilibrium values are small. Here, $f_v(r, z, o)$ represents the axial distribution of the local void fractions in the equilibrium conditions and is given by equation (12) as

$$f_v(r,z,o)=\frac{w(o)}{r_0\rho_g v_g(r)\sigma}\int_{z_0(r)}^{z} q(r,\xi)d\xi. \tag{12}$$

Therefore

$$\left[\frac{\partial f_v(r,z,o)}{\partial z}\right]_{z=z_0(r)}=\frac{w(o)}{r_0\rho_g v_g(r)\sigma}q(r,z_0(r)). \tag{39}$$

Substituting equation (39) into equation (38), we obtain

$$f_v(r,z_0(r),s)=\frac{-w(o)}{r_0\rho_g v_g(r)\sigma}\delta z_0(r,s)\ q(r,z_0(r)). \tag{40}$$

Again substituting equation (24) into equation (40), the transient fluctuations of the instantaneous void fractions at the location of equilibrium boiling boundary will be obtained as

$$f_v(r,z_0(r),s)=\frac{v_0(r)\delta w_2(s)}{r_0\rho_g v_g(r)\sigma}\Big[Lq(r,z_0(r)-v_0(r)\tau)$$
$$-\exp\Big(-\frac{z_0(r)}{v_0(r)}s\Big)Lq(r,-v_0(r)\tau)\Big]. \tag{41}$$

Combining equation (41) and equation (37), the effects of the transient shifts of boiling boundary of the local void fractions will ultimately be established as

$$\delta f_{v_2}(r,z,s)=\frac{\delta w_2(s)}{r_0\rho_g\sigma}\frac{v_0(r)}{v_g(r)}\exp\Big(-\frac{z-z_0(r)}{v_g(r)}s\Big)\times$$
$$\Big[Lq(r,z_0(r)-v_0(r)\tau)-\exp\Big(-\frac{z_0(r)}{v_0(r)}s\Big)Lq(r,-v_0(r)\tau)\Big]. \tag{42}$$

4.3. Transient fluctuations of the local void fractions. Since the transient fluctuations of the local void fractions from the equilibrium values should be given by the sum of these two kinds of local void fluctuations, they will finally be expressed as

$$\delta f_v(r,z,s)=\delta f_{v_1}(r,z,s)+\delta f_{v_2}(r,z,s)$$
$$=\frac{\delta w_1(s)}{r_0\rho_g\sigma}\Big[Lq(r,z-v_g(r)\tau)-\exp\Big(-\frac{z-z_0(r)}{v_g(r)}s\Big)\times Lq(r,z_0(r)-v_g(r)\tau)\Big]$$
$$+\frac{\delta w_2(s)}{r_0\rho_g\sigma}\cdot\frac{v_0(r)}{v_g(r)}\exp\Big(-\frac{z-z_0(r)}{v_g(r)}s\Big)\times\Big[Lq(r,z_0(r)-v_0(r)\tau)$$
$$-\exp\Big(-\frac{z_0(r)}{v_0(r)}s\Big)Lq(r,-v_0(r)\tau)\Big]. \tag{43}$$

The spatial mean value of the void fluctuations will be given, if necessary, by integrating equation (43) with respect to r and z extending over the entire space of the boiling region of the core and written as

$$\delta f_v(s)=\frac{1}{V_0}\int_{V_1}\int\delta f_v(r,z,s)dV. \tag{44}$$

5. Local Effects of the Void Coefficient of Reactivity

5.1. Perturbation theory on the local effects of the void coefficient of Reactivity. It will generally be presumed that the thermal neutron fluxes of the reactor in the equilibrium conditions will satisfy the diffusion equation

$$D\nabla^2\phi+(k-1)\Sigma_a\phi=0 \tag{45}$$

When the reactor is operating in the balanced conditions, suppose the disturbance has occured in the reactor by the generation of an small void at an arbitrary location within the core. Then the neutron fluxes will be changed from the original values ϕ to such as ϕ' which will satisfy

$$D\nabla^2\phi'+(k-1)\Sigma_a\phi'=\frac{\Delta\omega}{v}\phi' \tag{46}$$

within the domain of no void, and

$$D'\nabla^2\phi'\equiv D\nabla^2\phi'+(k-1)\Sigma_a\phi'+[D'\nabla^2\phi'-D\nabla^2\phi' -(k-1)\Sigma_a\phi']=\frac{\Delta\omega}{v}\phi' \tag{47}$$

within the domain of void, where

v = velocity of the thermal neutrons

ω = reciprocal of the reactor period.

Combining equations (45), (46) and (47), the following integral equation may easily be established.

$$\frac{\Delta\omega}{v}\int_{V_0}\phi\phi' dV=\int_{\delta V_s}\phi[D'\nabla^2\phi'-D\nabla^2\phi'-(k-1)\Sigma_a\phi']dV \tag{48}$$

in which δV_s represents the volume of the small void considered and ϕ' is presumed approximately being equal to ϕ.

It is obvious by equation (47) that $D'\nabla^2\phi'$ in equation (48) is a small quantity of the first order although the diffusion constant D' is infinity within the domain of void. Then, $D\nabla^2\phi'$ is naturally smaller quantity than the first order since $D\ll D'$. Hence, $D'\nabla^2\phi'$ and $D\nabla^2\phi'$ can practically be ignored in comparison with $(k-1)\Sigma_a\phi'$.

Then, equation (48) can approximately be reduced to

$$\frac{\Delta\omega}{v}=\frac{-(k-1)\Sigma_a\phi^2\delta V_s}{\int_V \phi^2 dV}. \tag{49}$$

On the other hand, the interrelation between the small variations of the effective reactivity and corresponding reciprocal reactor period has already been established by perturbation theory and is given by

$$\frac{\Delta K_{eff}}{K_{eff}}=\Delta\omega\frac{\int_V \phi^2 dV}{\int_V k\Sigma_a v\phi^2 dV} \tag{50}$$

Elliminating $\Delta\omega$ from equations (49) and (50), we shall obtain

$$\frac{\Delta K_{eff}}{K_{eff}}=\frac{-(k-1)\Sigma_a v\phi^2\delta V_s}{\int_V k\Sigma_a v\phi^2 dV}. \tag{51}$$

By equation (51), the expression for the local effects of the void coefficient of reactivity will finally be established as

$$\frac{\delta K_{eff}/K_{eff}}{\delta V_s} \equiv \frac{\partial K}{\partial V_s} = -\frac{(k-1)\sum_a v\phi^2}{\int_V k\sum_a v\phi^2 dV}. \tag{52}$$

Consequently, it can be considered by equation (52) that the local effects of the void coefficient of reactivity are weighted by the squre of the neutron fluxes before disturbance.

Equation (52) will brifefly be written as

$$\frac{\partial K}{\partial V_s} = \Psi'_0\phi^2(r,z) = -\Psi_0 q^2(r,z). \tag{53}$$

We shall next consider the correlation between the bulk void coefficient of reactivity and its local effects now investigated.

Let $\left[\frac{\partial K}{\partial f_v}\right]_0$ denotes the bulk void coefficient of reactivity near equilibrium state, the internal feedbacks reactivity changes resulting from the smal deviations of the average void fraction near the equilibrium conditions will be expressed as

$$\delta K = \left[\frac{\partial K}{\partial f_v}\right]_0 \delta\bar{f}_v. \tag{54}$$

On the other hand, by integrating the local effects of the void feedbacks reactivity, we obtain

$$\delta K = \int_{V_s} \frac{\partial K}{\partial V_s}(r,z)\delta f_v(r,z)\sigma dV \tag{55}$$

where V_s represents the total volume of the steam void contained within the core. However, by integrating the local void fractions, the average void fraction within the core will be obtained as

$$\delta\bar{f}_v = \frac{1}{V_0}\int_{V_1} \delta f_v(r,z) dV. \tag{56}$$

Combining equations (54), (55) and (56), we shall obtain

$$\left[\frac{\partial K}{\partial f_v}\right]_0 = \frac{V_0 \iint_{V_1} \frac{\partial K}{\partial V_s}(r,z)\delta f_v(r,z)\sigma dV}{\iint_{V_1} \delta f_v(r,z) dV} \tag{57}$$

which will give the correlation between the bulk void coefficient of reactivity and its local effects near the equilibrium state.

However, the local void fractions in the equilibrium state have already been established and given by

$$\delta f_v(r,z) = \frac{\delta w(o)}{r_0\rho_g\sigma}\frac{1}{v_g(r,z)}\int_{z_0(r)}^{z} q(r,\xi)d\xi. \tag{12}$$

Substituting equation (12) into equation (57), it will finally be reduced to

$$\left[\frac{\partial K}{\partial f_\cdot}\right]_0 = -\Phi_0 \frac{\int_0^{R_0}\int_{z_0(r)}^{H}\int_{z_0(r)}^{z} q^2(r,z)\frac{q^2(r,\xi)}{v_g(r)} r d\xi dz dr}{\int_0^{R_0}\int_{z_0)r)}^{H}\int_{z_0(r)}^{z} \frac{1}{v_g(r)} q(r,\xi) r d\xi dz dr} \tag{58}$$

where $$\Phi_0 = V_0\sigma\Psi_0.$$

When the function $q(r,z)$ and the bulk void coefficient of reactivity are known in the equilibrium state, the value of constant Φ_0 will be evaluated by equation (58), which will allow us to evaluate the local effects of the void coefficient of reactivity by equation (53). Once the local effects of the void coefficient of reactivity were known, the transient fluctuations of the void feedbacks reactivity, taking its local effects into consideration, will be evaluated by

$$\delta K(\tau) = \int_{V_1}\int \frac{\partial K}{\partial V_s}(r,z)\delta f_v(r,z,\tau)dV. \tag{59}$$

The Laplace transform of equation (59) will be written as

$$\delta K(s) = \int_{V_1}\int \frac{\partial K}{\partial V_s}(r,z)\delta f_v(r,z,s)dV. \tag{60}$$

Substituting equation (53) into equation (60), the transient changes of the feedbacks reactivity resulting from the local effects of the void coefficient of reactivity will finally be established as

$$\begin{aligned}\delta K(s) = &-\frac{\Psi_0}{r_0\rho_g}\Big[\delta w_1(s)\Big\{\int_{V_1}\int q^2(r,z)Lq(r,z-v_g(r)\tau dV \\ &-\int_{V_1}\int q^2(r,z)\exp\Big(-\frac{z-z_0(r)}{v_g(r)}s\Big)Lq(r,z_0(r)-v_g(r)\tau)dV\Big\} \\ &-\delta w_2(s)\Big\{\int_{V_1}\int\frac{v_0(r)}{v_g(r)}q^2(r,z)\exp\Big(-\frac{z-z_0(r)}{v_g(r)}s\Big)\times Lq(r,z_0(r)-v_0(r)\tau)dV \\ &-\int_{V_1}\int\frac{v_0(r)}{v_g(r)}q^2(r,z)\exp\Big(-\frac{z_0(r)}{v_0(r)}s\Big)\cdot\exp\Big(-\frac{z-z_0(r)}{v_g(r)}s\Big)\times \\ &\qquad Lq(r,-v_0(r)\tau)dV\Big\}\Big].\end{aligned} \tag{61}$$

Since $q(r,z)=1$ for a simplifed model with uniform power generation, for such case equation (61) will be reduced to

$$\begin{aligned}\delta K(s) = &\frac{-2\pi\Psi_0}{r_0\rho_g}\delta w_1(s)\Big[\frac{R_0^2}{2s}(H-z_0(r))-\int_0^{R_0}\frac{rv_g(r)}{s^2}\Big(1- \\ &\exp\Big(-\frac{H-z_0(r)}{v_g(r)}s\Big)\Big)dr\Big]-\frac{2\pi\Psi_0}{r_0\rho_g}\delta w_2(s)\times\Big[\int_0^{R_0}\frac{rv_0(r)}{s^2}\Big(1- \\ &\exp\Big(-\frac{H-z_0(r)}{v_g(r)}s\Big)\Big)dr-\int_0^{R_0}\frac{rv_0(r)}{s^2}\exp\Big(-\frac{z_0(r)}{v_0(r)}s\Big)\Big(1- \\ &\exp\Big(-\frac{H-z_0(r)}{v_g(r)}s\Big)\Big)dr\Big].\end{aligned} \tag{62}$$

Moreover, since $z_0(r)$ and $v_g(r)$ become constants respectively when the inlet velocity of feedwater is independent of the radial locations, for such case equation (62) will further be reduced to

$$\delta K(s) = -\frac{V_0\Psi_0}{r_0\rho_g s}\Big[y_0\delta w_1(s)\Big\{1-\frac{v_g}{sy_0}\Big(1-\exp\Big(-\frac{y_0}{v_g}s\Big)\Big)\Big\}$$

$$-\delta w_2(s)\frac{v_0}{s}\left(1-\exp\left(-\frac{y_0}{v_g}s\right)\right)\times\left(1-\exp\left(-\frac{z_0}{v_0}s\right)\right). \tag{63}$$

in which $Y_0=H-z_0$

H=active length of the core.

FUEL TEMPERATURS FEEDBACKS REACTIVITY

1. Perturbation Theory on the Local Effects of the Fuel Temperature Coefficient of Reactivity

When the reactor is operating in equilibrium conditions with the fuel temperature of $T_0(r,z)$, suppose a small deviation of fuel temperature δT_f has occured in a small domain at an arbitrary location within the core. Then, the diffusion equation to be satisfied by the thermal neutron fluxes before disturbance is given by

$$D\nabla^2\phi+(k-1)\Sigma_a\phi=0. \tag{64}$$

On the other hand, after the temperature deviation has occured, the thermal neutron fluxes and the diffusion equation dominating them may be altered from those in the balanced conditions as

$$Dv\nabla^2\phi'+(k-1)\Sigma_a v\phi'=\Delta\omega\phi' \tag{65}$$

within the domain of no temperature deviation, and

$$[Dv+\delta(Dv)]\nabla^2\phi'+[(k-1)\Sigma_a v+\delta\{(k-1)\Sigma_a v\}]\phi'=\Delta\omega\phi' \tag{66}$$

within the domain of temperature deviation.

Combining equations (64), (65) and (66) and considering that ϕ' may approximately be equal to ϕ, we shall obtain

$$\Delta\omega\int_{V_0}\phi\phi' dV=\int_{\delta V_f}[\delta(Dv)\phi\nabla^2\phi'+\delta\{(k-1)\Sigma_a v\}\phi\phi']dV \tag{67}$$

in whic δV_f represents the volume of the domain of temperature deviation.

Equation (67) will be, by using the well known reactor equation dominating the thermal neutron fluxes in equilibrium conditions in the form

$$\nabla^2\phi=-B^2\phi \tag{68}$$

written as

$$\Delta\omega=\frac{\int_{\delta V_f}[\delta\{(k-1)\Sigma_a v\}-\delta(Dv)B^2]\phi^2 dV}{\int_{V_0}\phi^2 dV}. \tag{69}$$

Combining equations (50) and (69), we shall obtain

$$\frac{\Delta K_{eff}}{K_{eff}}=\frac{[\delta\{(k-1)\Sigma_a v\}-\delta(Dv)B^2]\phi^2\delta V_f}{\int_{V_0}k\Sigma_a v\phi^2 dV}. \tag{70}$$

By equation (70), it will be seen that the local effects of the fuel temperature coefficient of reactivity are also, similiarly to the case of local void effects, weighted by the square of the thermal neutron fluxes before the disturbance has occured.

Equation (70) can briefly be written as

$$\frac{\partial^2 K}{\partial T_f \partial V_f} = -\Omega_0 q^2(r, z) \tag{71}$$

in which Ω_0 represents a proportional constant to be determined by the reator's parameters.

While, the steady deviation of the fuel temperature feedbacks reactivity resulting from the small shifts of equilibrium conditions is written as

$$\delta K = \sigma_f \int_{V_0}\!\!\int \frac{\partial^2 K}{\partial T_f \partial V_f}(r, z)\delta T_f(r, z) dV \tag{72}$$

in which σ_f indicates the ratio of the entire cross sectional area of the fuel elements to the cross sectional area of the core.

Substituting equation (71) into equation (72), we obtain

$$\delta K = -\Omega_0 \sigma_f \int_{V_0}\!\!\int q^2(r, z)\delta T_f(r, z) dV. \tag{73}$$

We shall next investigate the relationship between the bulk fuel temperature coefficient of reactivity and its local effects. Let $\left[\frac{\partial K}{\partial T_f}\right]_0$ denotes the bulk fuel temperature coefficient of reactivity near balanced conditions, the steady deviation of the reactivity corresponding to the small shifts of average fuel temperature is given by

$$\delta K = \left[\frac{\partial K}{\partial T_f}\right]_0 \delta \overline{T}_f. \tag{74}$$

Combining equations (73) and (74), we obtain

$$\left[\frac{\partial K}{\partial T_f}\right]_0 = -\Omega_0 \sigma_f \int_{V_0}\!\!\int q^2(r, z)\delta T_f(r, z) dV / \delta \overline{T}_f. \tag{75}$$

However, the average fuel temperature deviation near balanced conditions is given by

$$\delta \overline{T}_f = \frac{1}{V_0}\int_{V_0}\!\!\int \delta T_f(r, z) dV. \tag{76}$$

Combining equations (75) and (76), we obtain

$$\left[\frac{\partial K}{\partial T_f}\right]_0 = -\Omega_0 V_0 \sigma_f \cdot \int_{V_0}\!\!\int q^2(r, z)\delta T_f(r, z) dV \Big/ \int_{V_0}\!\!\int \delta T_f(r, z) dV \tag{77}$$

in which $\delta T_f(r, z)$ represents the steady fuel temperature deviations. Equation (77) gives the relation between the bulk fuel temperature coefficient of reactivity and its local effects when the function $q(r, z)$ and $\delta T_f(r, z)$ are known.

2. Transient and Steady Deviations of the Fuel Temperature Near Equilibrium Conditions

We shall now investigate the transient deviations of fuel temperature within the core. Considering the transient heat balance for the fuel per unit volume of the core on the same assumptions in regard of the fuel temperature, that were made in the previous Chapter, the following variational equation may easily be established.

$$\delta T_f(r,z,s)=\frac{a}{s+a}\frac{0.23889}{h\cdot f}\delta p(s)q(r,z) \tag{78}$$

where f indicates the area of heat transfer surface per unit volume of the core. Putting $s=0$ in equation (78), it is seen that the steady deviations of fuel temperature near equilibrium conditions $\delta T_f(0)$ is immediately proportional to the function $q(r,z)$. Hence, we have

$$\delta T_f(r,z,0)=\mu_0 q(r,z). \tag{79}$$

Substituting equation (79) into equation (77), the fuel temperature coefficient of reactivity will ultimately be expressed as

$$\left[\frac{\partial K}{\partial T_f}\right]_0=-\Omega_0\sigma_f V_0\int_{V_0}\int q^3(r,z)dV\Big/\int_{V_0}\int q(r,z)dV. \tag{80}$$

When the bulk fuel temperature coefficient of reactivity and the function $q(r,z)$ are known, the constant Ω_0 can be evaluated by equation (80) and the local effects will therefore be determined by equation (71).

3. Transient Fluctuations of the Fuel Temperature Feedbacks Reactivity

Substituting equation (78) into equation (73), the transient changes of the fuel temperature feedbacks reactivity, taking the local effects into consideration, will finally be written as

$$\delta K(s)=-\Omega_0\sigma_f\frac{0.23889}{h\cdot f}\delta p(s)\frac{a}{s+a}\int_{V_0}\int q^3(r,z)dV. \tag{81}$$

However, as stated previously, two kinds of reciprocal time constants of heat transfer should be employed to express the transfer functions of heat transfer from fuel to the coolant because of the difference between the heat transfer coefficients for the boiling and the subcooled regions, respectively. Consequently, equation (81) will finally be written as

$$\delta K(s)=-\Omega_0\sigma_f\times 0.23889\delta p(s)\times\left[\frac{1}{h_1 f}\frac{a_1}{s+a_1}\times\right.$$
$$\left.\int_{V_1}\int q^3(r,z)dV+\frac{1}{h_2 f}\frac{a_2}{s+a_2}\int_{V_2}\int q^3(r,z)dV\right] \tag{82}$$

where

h_1 = heat transfer coefficient for the boiling region
h_2 = heat transfer coefficient for the subcooled region
a_1 = reciprocal time constant of heat transfer for the boiling region
a_2 = reciprocal time constant of heat transfer for the subcooled region
V_1, V_2 = the volume of boiling and subcooled regions of the core in equilibrium state, respectively.

Since $q(r,z)=1$ for the model with uniform power generation, for such case equation (82) will be reduced to

$$\delta K(s)=-2\pi\Omega_0\sigma_f\times 0.23889\delta p(s)\times\left[\frac{1}{h_1 f}\frac{a_1}{s+a_1}\times\right.$$

$$\int_0^{R_0}(H-z_0(r))rdr+\frac{1}{h_2f}\frac{a_2}{s+a_2}\times\int_0^{R_0}rz_0(r)dr\Big]. \tag{83}$$

Moreover, since the equilibrium location of boiling boundary becomes constant provided the inlet velocity of feedwater is independent of the radial locations, for such case equation (83) will further be reduced to

$$\delta K(s)=-\Omega_0\sigma_f\times 0.23889\delta p(s)$$
$$\times\Big[\frac{V_1}{h_1f}\frac{a_1}{s+a_1}+\frac{V_2}{h_2f}\frac{a_2}{s+a_2}\Big]. \tag{84}$$

TRANSIENT CHANGES OF THE INTERNAL FEEDBACKS REACTIVITY

Assuming that the moderator temperature feedbacks reactivity can be ignored, we can now write the ulitimate expression for the transient changes of the internal feedbacks reactivity resulting from the effects of both the local void fluctuations and the fuel temperature deviations as

$$\delta K(s)=-\frac{\Psi_0}{r_0\rho_g}\times 0.23889\delta p(s)\Big[\frac{a_1}{s+a_1}\int_{V_1}\int q^2(r,z)\times$$
$$Lq(r,z-v_g(r)\tau)dV-\frac{a_1}{s+a_1}\int_{V_1}\int q^2(r,z)\cdot\exp\Big(-\frac{z-z_0(r)}{v_g(r)}s\Big)\times$$
$$Lq(r,z_0(r)-v_g(r)\tau)dV\Big]-\frac{\Psi_0}{r_0\rho_g}\times 0.23889\delta p(s)\times$$
$$\Big[\frac{a_2}{s+a_2}\int_{V_1}\int\frac{v_0(r)}{v_g(r)}q^2(r,z)\cdot\exp\Big(-\frac{z-z_0(r)}{v_g(r)}s\Big)\times Lq(r,z_0(r)-v_0(r)\tau)dV$$
$$-\frac{a_2}{s+a_2}\int_{V_1}\int\frac{v_0(r)}{v_g(r)}q^2(r,z)\cdot\exp\Big(-\frac{z_0(r)}{v_0(r)}s-\frac{z-z_0(r)}{v_g(r)}s\Big)\times$$
$$Lq(r,-v_0(r)\tau)dV\Big]-\Omega_0\sigma_f\times 0.23889\delta p(s)\times$$
$$\Big[\frac{1}{h_1f}\frac{a_1}{s+a_1}\int_{V_1}\int q^3(r,z)dV+\frac{1}{h_2f}\frac{a_2}{s+a_2}\int_{V_2}\int q^3(r,z)dV\Big]. \tag{85}$$

Equation (85) indicates the generalized expression for the feedbacks transfer function of boiling reactor required.

For the uniform power generation model with uniform inlet velocity of feedwater, equation (85) will be reduced to

$$\frac{\delta K(s)}{\delta p(s)}=-\frac{\Psi_0}{r_0\rho_g s}V_1\times 0.23889\Big[1-\frac{v_g}{sy_0}\Big(1-\exp\Big(-\frac{y_0}{v_g}s\Big)\Big)\Big]\times$$
$$\frac{1}{1+T_{1B}s}-\frac{\Psi_0\pi R_0^2v_0}{r_0\rho_g s^2}\times 0.23889\times\frac{1}{1+T_{1W}s}\times\Big[1-\exp\Big(-\frac{y_0}{v_g}s\Big)\Big]\times$$
$$\Big[1-\exp\Big(-\frac{z_0}{v_0}s\Big)\Big]-\Omega_0\sigma_f\times 0.23889\times$$
$$\Big[\frac{V_1}{h_1f}\frac{1}{1+T_{1B}s}+\frac{V_2}{h_2f}\frac{1}{1+T_{1W}s}\Big] \tag{86}$$

in which $Y_0=H-Z_0$.

It will easily be verified that the equation (86) is in coincidence with the result obtained by J. A. Thie, ANL, such a report as ANL-5849, given in the form

$$\frac{\delta K(s)}{p(s)/P_0}=\Big[\frac{\partial K}{\partial\theta_m}\Big]_0\cdot\Big[\frac{z_0}{H}\frac{\theta_{W_0}}{1+T'_{1W}s}+\frac{y_0}{H}\frac{\theta_{B_0}}{1+T'_{1B}s}\Big]$$
$$+\Big[\frac{\partial K}{\partial V_s}\Big]_0\Big[\frac{y_0}{\rho l s}\times\frac{\phi_0}{1+T_{1B}s}\times\Big\{1-\frac{u}{sy_0}\Big(1-\exp\Big(-\frac{y_0}{u}s\Big)\Big)\Big\}$$
$$+\frac{W}{\rho l s^2}\Big(1-\exp\Big(-\frac{y_0}{u}s\Big)\Big)\Big(1-\exp\Big(-\frac{z_0}{W}s\Big)\times\frac{\Psi_0}{1+T_{1W}s}\Big]. \qquad (87)$$

However, it should be noted that the meanings of the constant Ψ_0 in both equations are quite different from each other.

TRANSFER FUNCTION OF THE SINUSOIDAL POWER GENERATION MODEL

We shall now obtain the feedbacks transfer functions of the model whose spatial mode of power density distribution is given by

$$q(r,z)=B_0\sin\frac{\pi(z+a_0)}{H+a_0+a_0'}J_0\Big(\frac{2.405r}{R_0+\delta R_0}\Big) \qquad (88)$$

where

H =active length of the core
R_0 =radius of the active core
a_0 =reflector saving at the bottom surface of the core
a_0' =reflector saving at the top surface of the core
δR_0=reflector saving in the radial direction.

Putting $m=\frac{\pi}{(H+a_0+a_0'}$ and $b_0=\frac{2.405}{(R_0+\delta R_0)}$ equation (88) is written as

$$q(r,z)=B_0\sin m(z+a_0)J_0(b_0r). \qquad (89)$$

Further, equation (89) is normalized as

$$\int_{V_0}\int B_0\sin m(z+a_0)J_0(b_0r)dV=V_0. \qquad (90)$$

Performing the integral of equation (90), we obtain

$$B_0=\frac{b_0mV_0}{2\pi R_0J_1(b_0r)[\cos ma_0-\cos m(H+a_0)]}. \qquad (91)$$

Hence, $q(r,z)$ becomes

$$q(r,z)=\frac{b_0mV_0\sin m(z+a_0)J_0(b_0r)}{2\pi R_0J_1(b_0r)[\cos ma_0-\cos m(H+a_0)]}. \qquad (92)$$

The expressions for the feedbacks transfer functions and other formulae concerned, which have so far been established in the previous Chapters, will be determined for this model as the followings.

1. The Axial Locations of the Boiling Boundary in Equilibrium Conditions as a Function of Radial Locations

$$z_0(r)=\frac{1}{m}\mathrm{Cos}^{-1}\Big[\cos(ma_0)-\frac{v_0(r)\rho_f\Delta H_s m}{B_0J_0(b_0r)w(o)}\Big]-a_0 \qquad (93)$$

By equation (93), $z_0(r)$ will be evaluated provided the inlet velocities and subcooling of feedwater are known.

2. Spatial Distribution of the Local Void Fractions in Equilibrium Conditions

$$f_v(r,z)=\frac{w(o)B_0}{r_0\rho_g\sigma}\frac{J_0(b_0r)}{v_g(r,z)}\cdot\frac{2}{m}\cdot\sin\left[m\ a_0-\frac{z+z_0(r)}{2}\right)\Big]\times$$
$$\sin\left[\frac{m(z-z_0(r))}{2}\right] \qquad (94)$$

3. Spatial Distribution of the steam Flow Velocities

$$v_g(r,z)=v_0(r)\varepsilon(r)+\frac{w(o)}{r_0\rho_g\sigma}\int_{z_0(r)}^{z}q(r,\xi)d\xi \qquad (95)$$

4. The Axial Mean Steam Flow Velocity as a Function of Radial Locations

Integrating equation (95) with respect to z extending over the entire boiling region, the axial mean value of steam flow velocities will be obtained as

$$<v_g(r,z)>\equiv v_g(r)=v_0(r)\varepsilon(r)+\frac{w(o)}{r_0\rho_g\sigma}\cdot B_0J_0(b_0r)\times$$
$$\left[\frac{1}{m}\cos m(z_0(r)+a_0)-\frac{1}{m^2(H-z_0(r))}\{\sin m(H+a_0)\right.$$
$$\left.-\sin m(z_0(r)+a_0)\}\right] \qquad (96)$$

which will be used for the evaluation of the void feedbacks transfer functions.

5. Transient Shifts of Boiling Boundary

$$\delta z_0(r,s)=-\frac{v_0(r)\delta w_2(s)}{w(o)q(r,z_0(r))}B_0J_0(b_0r)\cdot\Big[\sin m(z_0(r)+a_0)\times$$
$$\frac{s}{s^2+m^2v_0^2(r)}-\frac{mv_0(r)}{s^2+m^2v_0^2(r)}\cos m(z_0(r)+a_0)$$
$$-\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\cdot\left\{\frac{s}{s^2+m^2v_0^2(r)}\sin(ma_0)-\frac{mv_0(r)}{s^2+m^2v_0^2(r)}\cos(ma_0)\right\}\Big] \qquad (97)$$

6. Relationship Between the Bulk Void Coefficient of Reativity and its Local Effects

$$\left[\frac{\partial K}{\partial f_v}\right]_0=-mB_0^2\Phi_0\cdot\frac{\gamma_0}{\xi_0} \qquad (98)$$

where

$$\xi_0=\int_0^{R_0}\frac{rJ_0(b_0r)}{v_g(r)}[m(H-z_0(r))\cdot\cos m(z_0(r)+a_0)+$$
$$\sin m(z_0(r)+a_0)-\sin m(H+a_0)]dr \qquad (99)$$

$$\gamma_0=\int_0^{R_0}\frac{r}{v_g(r)}J_0^3(b_0r)\Big[\frac{1}{2}\cos m(z_0(r)+a_0)\cdot\Big\{(H-z_0(r))$$
$$-\frac{1}{2m}\sin 2m(H+a_0)+\frac{1}{2m}\sin 2m(z_0(r)+a_0)\Big\}-$$

$$\frac{1}{3m}\sin^3 m(z_0(r)+a_0)\Big]dr. \tag{100}$$

Equation (98) allows us to evaluate the constant Φ_0 provided the bulk void coefficient of reactivity is known. Once Φ_0 was evaluated, the local effects of the void coefficient of reactivity are given by the relationship

$$\frac{\partial K}{\partial V_s}(r,z) = -\frac{\Phi_0}{V_{0\sigma}}q^2(r,z) \tag{101}$$

in which $q(r,z)$ is given by equation (92).

7. Relationship Between the Bulk Fuel Temperature Coefficient of Reactivity and its Local Effects

$$\left[\frac{\partial K}{\partial T_f}\right]_0 = -\Omega_{0\sigma f}V_0\frac{B_0^2 b_0}{R_0 J_1(b_0 r)}\cdot\frac{\gamma_1}{\xi_1} \tag{102}$$

where

$$\gamma_1 = \int_0^{R_0} rJ_0^3(b_0 r)\Big[\frac{1}{3}\{\cos ma_0 \sin^2 ma_0 - \cos m(H+a_0)\times$$

$$\sin^2 m(H+a_0)\} - \frac{2}{3}\{\cos ma_0 - \cos m(H+a_0)\}\Big]dr \tag{103}$$

$$\xi_1 = [\cos ma_0 - \cos m(H+a_0)]. \tag{104}$$

8. Void Feedbacks Transfer Function

$$\frac{\delta K^V(s)}{\delta p(s)} = -\frac{2\pi\Psi_0}{r_0\rho_g}\times 0.23889\frac{a_1}{s+a_1}\Big[B_0^3\int_0^{R_0} rJ_0^3(b_0 r)\times$$

$$\frac{s\alpha_1(r,s)}{3m(s^2+m^2v_g^2(r))}dr - B_0^3\int_0^{R_0} rJ_0^3(b_0 r)\times\frac{msv_g^2(r)}{s^2+4m^2v_g^2(r)}\times$$

$$\frac{\alpha_2(r,s)\beta_2(r,s)}{s^2+m^2v_g^2(r)}dr\Big] - \frac{2\pi\Psi_0}{r_0\rho_g}\times 0.23889\frac{a_2}{s+a_2}\times$$

$$\Big[B_0^3\int_0^{R_0} r\cdot\frac{v_0(r)}{v_g(r)}J_0^3(b_0 r)\times\frac{mv_g^2(r)\alpha_2(r,s)}{s^2+4m^2v_g^2(r)}\times$$

$$\frac{s\beta_3(r,s)}{s^2+m^2v_0^2(r)}dr - B_0^3\int_0^{R_0} r\cdot\frac{v_0(r)}{v_g(r)}J_0^3(b_0 r)\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\times$$

$$\frac{mv_g^2(r),s}{s^2+4m^2v_g^2(r)}\times\frac{\alpha_2(r,s)\beta_4(r,s)}{s^2+m^2v_0^2(r)}dr\Big] \tag{105}$$

where

$$\alpha_1(r,s) = \Big[\cos m(z_0(r)+a_0)\cdot\sin^2 m(z_0(r)+a_0)$$

$$-\cos m(H+a_0)\cdot\sin^2 m(H+a_0)+$$

$$2\cos m(z_0(r)+a_0) - 2\cos m(H+a_0) -$$

$$\frac{mv_g(r)}{s}\times\{\sin^3 m(H+a_0) - \sin^3 m(z_0(r)+a_0)\}\Big] \tag{106}$$

$$\alpha_2(r,s) = \Big[\frac{s}{mv_g(r)}\sin^2 m(z_0(r)+a_0) + \sin 2m(z_0(r)+a_0)$$

$$+\frac{2mv_g(r)}{s} - \exp\left(-\frac{H-z_0(r)}{v_g(r)}s\right)\times\Big\{\frac{s}{mv_g(r)}\times$$

$$\sin^2 m(H+a_0)+\sin 2m(H+a_0)+\frac{2mv_g(r)}{s}\}\Big] \tag{107}$$

$$\beta_2(r,s)=\left[\sin m(z_0(r)+a_0)-\frac{mv_g(r)}{s}\cdot\cos m(z_0(r)+a_0)\right] \tag{108}$$

$$\beta_3(r,s)=\left[\sin m(z_0(r)+a_0)-\frac{mv_0(r)}{s}\cdot\cos m(z_0(r)+a_0)\right] \tag{109}$$

$$\beta_4(r,s)=\left[\sin (ma_0)-\frac{mv_0(r)}{s}\cos ma_0\right]. \tag{110}$$

9. Fuel Temperature Feedbacks Transfer Function

$$\frac{\delta K^T(s)}{\delta p(s)}=-2\pi\Omega_0\sigma_f\times 0.23889\cdot\Big[\frac{B_0{}^3}{3mh_1f}\frac{a_1}{s+a_1}\times \int_0^{R_0} rJ_0{}^3(b_0r)\gamma_1(r)dr+\frac{B_0{}^3}{3mh_2f}\frac{a_2}{s+a_2}\times \int_0^{R_0} rJ_0{}^3(b_0r)\gamma_2(r)dr\Big] \tag{111}$$

where

$$\gamma_1(r)=[\cos m(z_0(r)+a_0)\cdot\{\sin^2 m(z_0(r)+a_0)+2\} -\cos m(H+a_0)\cdot\{\sin^2 m(H+a_0)+2\}] \tag{112}$$

$$\gamma_2(r)=[\cos ma_0\{\sin^2 ma_0+2\}-\cos m(z_0(r)+a_0) \{\sin^2 m(z_0(r)+a_0)+2\}]. \tag{113}$$

10. Internal Feedbacks Transfer Function

$$\frac{\delta K(s)}{\delta p(s)}=\frac{\delta K^V(s)}{\delta p(s)}+\frac{\delta K^T(s)}{\delta p(s)}$$

$$=-\frac{2\pi\Psi_0}{r_0\rho_g}\times 0.23889\frac{a_1}{s+a_1}\Big[B_0{}^3\int_0^{R_0}\frac{rJ_0{}^3(b_0r)}{3m(s^2+m^2v_g{}^2(r))}\times sa_1(r,s)dr-B_0{}^3\int_0^{R_0} rJ_0{}^3(b_0r)\cdot\frac{msv_g{}^2(r)}{s^2+4m^2v_g{}^2(r)}\times \frac{a_2(r,s)\beta_2(r,s)}{s^2+m^2v_g{}^2(r)}dr\Big]-\frac{2\pi\Psi_0}{r_0\rho_g}\times 0.23889\frac{a_2}{s+a_2}\times$$

$$\Big[B_0{}^3\int_0^{R_0} r\cdot\frac{v_0(r)}{v_g(r)}\cdot J_0{}^3(b_0r)\cdot\frac{mv_g{}^2(r)a_2(r,s)}{s^2+4m^2v_g{}^2(r)}\times \frac{s\beta_3(r,s)}{s^2+m^2v_0{}^2(r)}dr-B_0{}^3\int_0^{R_0} r\cdot\frac{v_0(r)}{v_g(r)}\cdot J_0{}^3(b_0r)\exp\left(-\frac{z_0(r)}{v_0(r)}s\right)\times \frac{mv_g{}^2(r)s}{s^2+4m^2v_g{}^2(r)}\times\frac{a_2(r,s),\beta_4(r,s)}{s^2+m^2v_0{}^2(r)}dr\Big]-$$

$$2\pi\Omega_0\sigma_f\times 0.23889\Big[\frac{B_0{}^3}{3mh_1f}\frac{a_1}{s+a_1}\int_0^{R_0} rJ_0{}^3(b_0r)\gamma_1(r)dr +\frac{B_0{}^3}{3mh_2f}\frac{a_2}{s+a_2}\int_0^{R_0} rJ_0{}^3(b_0r)\gamma_2(r)dr\Big] \tag{114}$$

11. Average Values of Void Fractions and Moderator Densities Within the Core in Equilibrium Conditions as the Functions of Radial Locations

Let $\bar{f}_v$ and ρ_{av} denote the average values of void fractions and moderator densities within the core, respectively, these are calculated as the followings.

1) in the case $|C(r)|>|F(r)|$

$$\bar{f}_v=1-\frac{2[C(r)-D(r)]}{m(H-z_0(r))\sqrt{C^2(r)-F^2(r)}}\times\Big[\mathrm{Tan}^{-1}\sqrt{\frac{C(r)+F(r)}{C(r)-F(r)}}\times \tan\frac{m(H+a_0)}{2}-\mathrm{Tan}^{-1}\sqrt{\frac{C(r)+F(r)}{C(r)-F(r)}}\times \tan\frac{z_0(r)+a_0}{2}\cdot m\Big] \tag{115}$$

$$\frac{\rho_{av}}{\rho_f}=1-\Big(1-\frac{\rho_g}{\rho_f}\Big)\Big[1-\frac{2\{C(r)-D(r)\}}{m(H-z_0(r))\sqrt{C^2(r)-F^2(r)}}\times \Big\{\mathrm{Tan}^{-1}\sqrt{\frac{C(r)+F(r)}{C(r)-F(r)}}\times\tan\frac{m(H+a_0)}{2}-\mathrm{Tan}^{-1}\sqrt{\frac{C(r)+F(r)}{C(r)-F(r)}}\times\tan\frac{m(z_0(r)+a_0)}{2}\Big\}\Big] \tag{116}$$

2) in the case $|C(r)|<|F(r)|$

$$\bar{f}_v=1-\Big[\frac{2\{C(r)-D(r)\}}{m(H-z_0(r))\sqrt{F^2(r)-C^2(r)}}\mathrm{Tan\,h}^{-1}\sqrt{\frac{F(r)+C(r)}{F(r)-C(r)}}\times \tan\frac{m(H+a_0)}{2}-\mathrm{Tan\,h}^{-1}\sqrt{\frac{F(r)+C(r)}{F(r)-C(r)}}\times \tan\frac{m(z_0(r)+a_0)}{2}\Big] \tag{117}$$

$$\frac{\rho_{av}}{\rho_f}=1-\Big(1-\frac{\rho_g}{\rho_f}\Big)\Big[1-\frac{2\{C(r)-D(r)\}}{m(H-z_0(r))\sqrt{F^2(r)-C^2(r)}}\times \Big\{\mathrm{Tan\,h}^{-1}\sqrt{\frac{F(r)+C(r)}{F(r)-C(r)}}\times\tan\frac{m(H+a_0)}{2}\times \mathrm{Tan\,h}^{-1}\sqrt{\frac{F(r)+C(r)}{F(r)-C(r)}}\times\tan\frac{m(z_0(r)+a_0)}{2}\Big\}\Big] \tag{118}$$

where

$$C(r)=v_0(r)\varepsilon(r)+F(r)\cos m(z_0(r)+a_0)$$
$$D(r)=F(r)\cos m(z_0(r)+a_0)$$
$$F(r)=\frac{w(o)B_0J_0(b_0r)}{r_0\rho_g\sigma m}.$$

NUMERICAL CALCULATIONS ON THE POWER TRANSFER FUNCTIONS OF EBWR UNDER THE OPERATIONAL CONDITIONS OF 50 MW, 600 PSIG

1. Reactor Parameters Necessary for the Calculations

The reactor parameters necessaɹy for the numerical calculations on the feedbacks transfer functions of EBWR under the operational conditions of 50 MW, 600 psig are as tabulated below.

Table 1. Reactor Parameter[4)]

Items	Notations	Values	Unit
Reactor power in equilibrium	P_0	50	MW
Working pressure		600	psig
Radius of active core	R_0	0.64	m
Length of active core	H	1.2192	m
Area ratio, channel to core	σ	0.6085	
Area ratio, fuel to core	σ_f	0.28396	
Volume of core	V_0	1.56886	m^3
Heat transfer area per unit volume of the core	f	91.135	m^{-1}
Saturation temperature	t	488.88 253.82	°F °C
Specific weight of saturated water	ρ_f	793.587	Kg/m^3
Specific weight of saturated steam	ρ_g	21.322	Kg/m^3
Latent heat of vaporization	r_0	405	Kcal/kg
Enthalpy of saturated water	i'	260.44	Kcal/kg
Subcooled temperature	Δt_s	5.49 3.05	°C °F
Enthalpy difference between feedwater and saturated water	ΔH_s	3.16	Kcal/kg
Overall volume flow rate of feedwater	W_q	1.07172	m^3/sec.
Overall mass flow rate of feedwater	W_t	855.447	Kg/sec.
Specific weight of feedwater	ρ_f'	798.2	Kg/m^3
Steam generation rate in equilibrium conditions	W_g	82.144	T/hr
Circulation ratio in equilibrium conditions	W_t/W_g	37.49	
Average velocity ratio	ε	1.85	
Average heat generation rate	$w(0)$	7613.5	$Kcal/m^3$ sec.
Total heat transfer area	F	142.978	m^2
Average heat flux	q	300,745	$Kcal/m^2$ Hr.
Heat transfer coefficient in boiling region	h_1	5.6131	$Kcal/m^2$ sec. °C
Heat transfer coefficient in subcooled region	h_2	4.7330	$Kcal/m^2$ sec. °C
Reciprocal time constant of heat transfer in boiling region	α_1	3.049	$sec.^{-1}$
Reciprocal time constant of heat transfer in subcooled region	α_2	2.5707	$sec.^{-1}$
Average power density in equilibrium conditions		3.187×10^4	KW/m^3
Average inlet velocity of feedwater		4.5	fps
Prompt neutron life time	l^*	6.25×10^{-5}	sec.
Bulk void coefficient of reactivity	$\left(\frac{\partial K}{\partial f_v}\right)_0$	-0.11	
Bulk fuel temperature coefficient of reactivity	$\left(\frac{\partial K}{\partial T_f}\right)_0$	-1.20×10^{-5}	$\frac{\Delta K}{K}$/°C
Reflector saving, bottom	a_0	0.352	m
Reflector saving, top	a_0'	0.0895	m
Reflector saving, radial	R_0	0.11	m
$2.405/(R_0+\delta R_0)$	b_0	3.2067	m^{-1}
$\pi/(H+a_0+a_0')$	m	1.8917	m^{-1}
Constant given by equation (91)	B_0	2.3316	—

POWER TRANSFER FUNCTION OF THE REACTOR WITH INTERNAL FEEDBACKS

The power transfer function of the reactor with internal feedbacks is defined, referring to Fig. 4, by

$$G(s)=\frac{\delta p(s)/P_0}{\delta K_{in}(s)} \text{ [3]}. \tag{119}$$

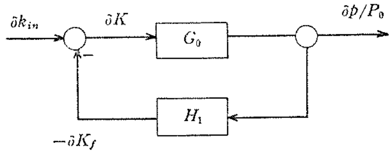

Fig. 4. Block diagram of the reactor with internal feedbacks.

The zero power transfer function $G_0(s)$ and the feedbacks transfer function $H_1(s)$ will respectively be given by

$$G_0(s)=\frac{\delta p(s)/P_0}{\delta K(s)}=\frac{1}{l^*}\frac{1}{s\left[1\times\sum\frac{\beta_i}{l^*(s+\lambda_i)}\right]} \tag{120}$$

$$H_1(s)=\frac{P_0}{V_0}\left[\frac{\delta K^V(s)}{\delta p(s)}+\frac{\delta K^T(s)}{\delta p(s)}\right]. \tag{121}$$

From elementary feedback theory, the power transfer function is obtained as

$$\frac{\delta p(s)/P_0}{\delta K_{in}(s)}=\frac{G_0(s)}{1+G_0(s)H_1(s)}. \tag{122}$$

RESULTS OF THE NUMERICAL COMPUTATIONS

Using the expressions obtained by the author for the sinusoidal power genera-

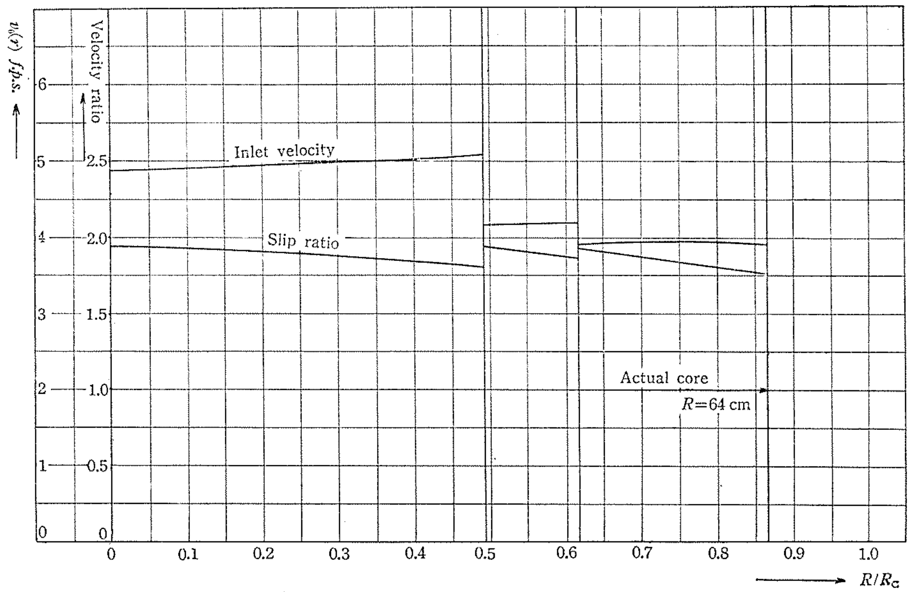

Fig. 5. Inlet velocity and velocity ratio as the functions of radical location, EBWR 50 MW.

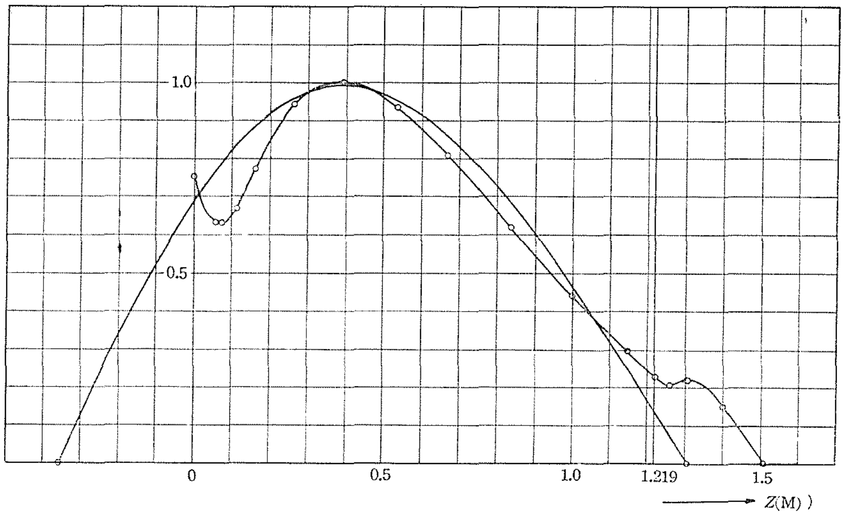

Fig. 6. Axial distribution of power density and approximate chopped sine, EBWR 50 MW.

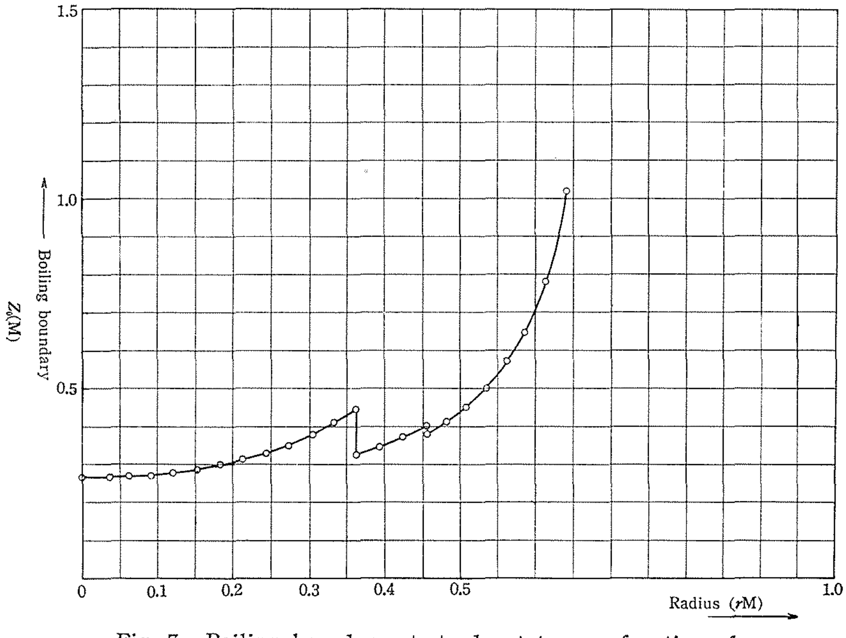

Fig. 7. Boiling boundary at steady state as a function of radial location, EBWR 50 MW.

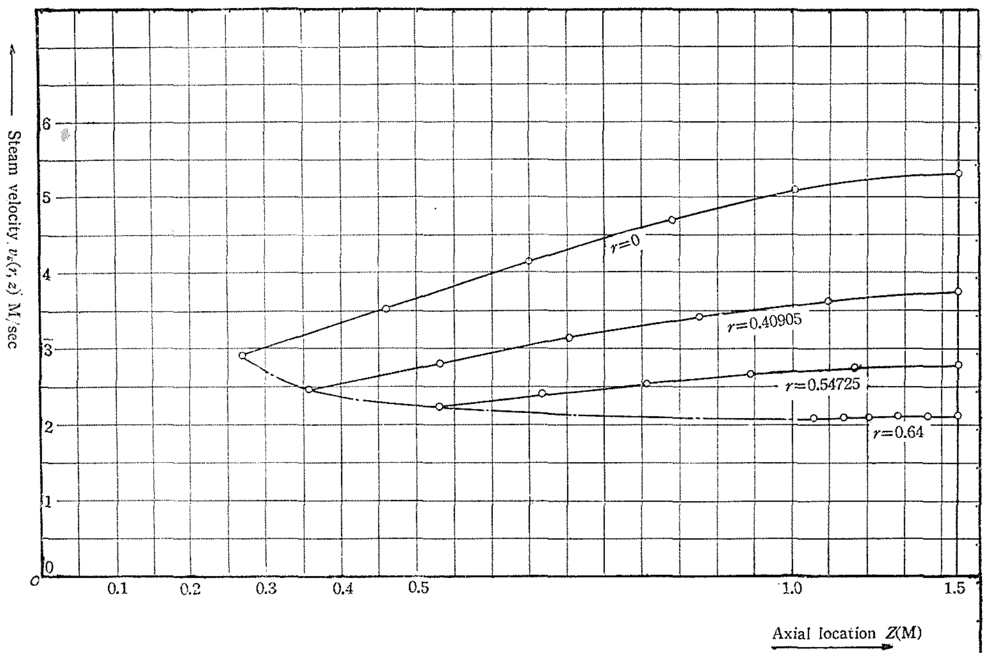

Fig. 8. Spatial distribution of steam flow velocities, EBWR 50 MW.

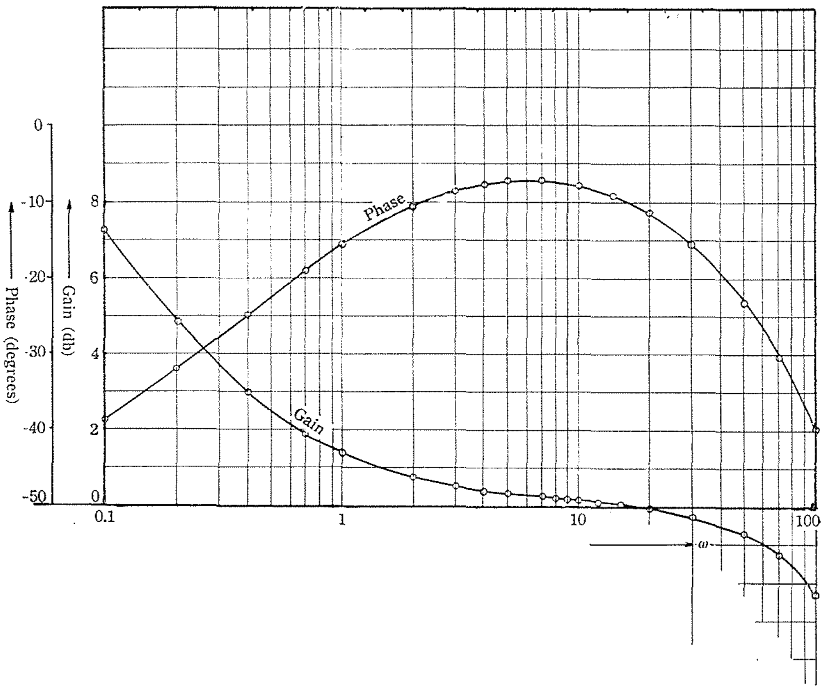

Fig. 9. Zero power reactor transfer function $G_0(j\omega)$, EBWR.

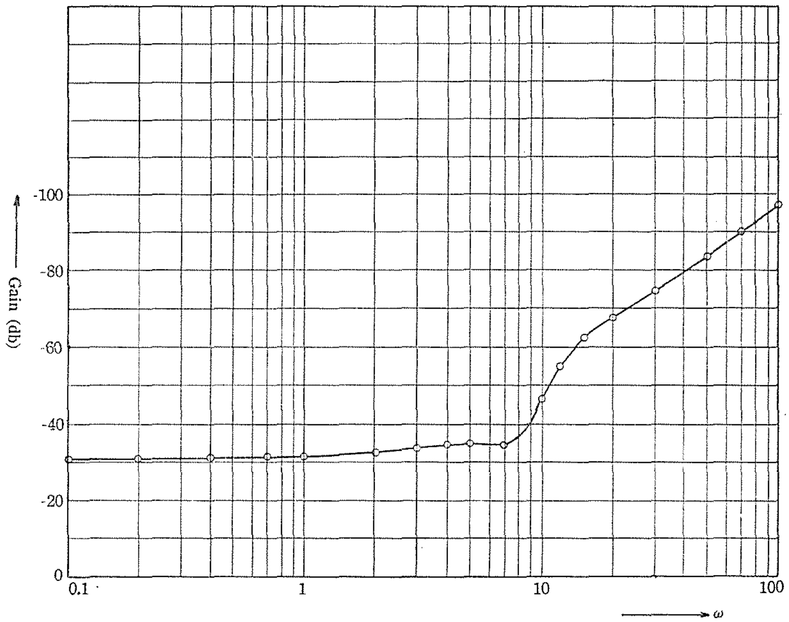

Fig. 10. Feedbacks transfer function $H_1(j\omega)$ EBWR 50 MW.

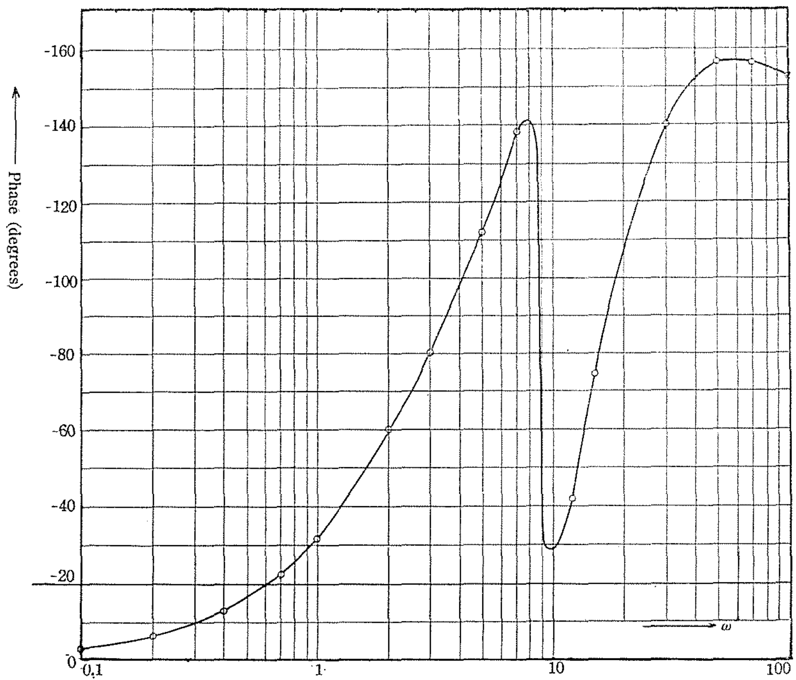

Fig. 11. Feedbacks transfer function, EBWR 50 MW.

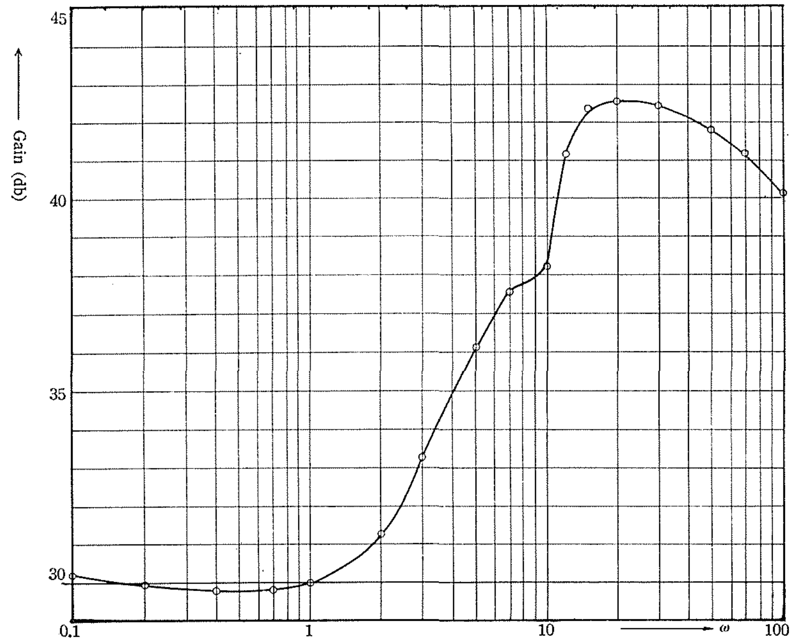

Fig. 12. Power transfer function, EBWR 50 MW.

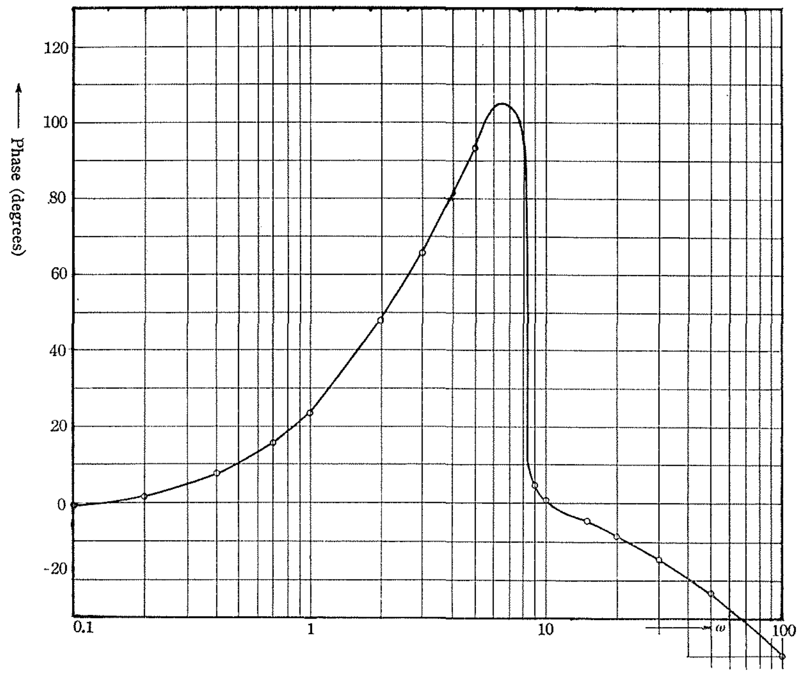

Fig. 13. Power transfer function, EBWR 50 MW.

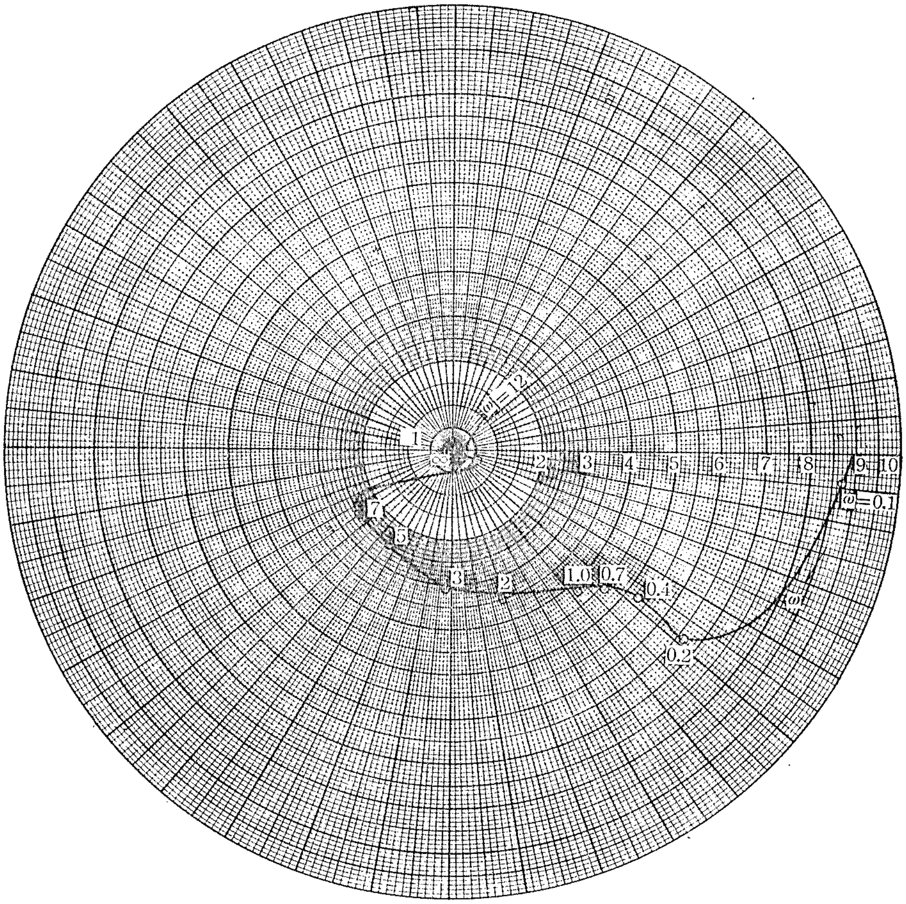

Fig. 14. Vector diagram of $H_1(j\omega)\ G_0(j\omega)$.

tion model, the numerical computations were performed on the feedbacks transfer functions as well as the power transfer functions of EBER under the operational conditions of 50 MW, 600 psig, the results of which are shown in Fig. 5 to Fig. 14.

When the results of these computations were compared with the experimental results obtained by ANL, such a report as ANL-5849, the agreement between both the results was satisfactory with the maximum difference being not more than 10 db in gain curves.

The calculated phase angle curve shows a remarkable agreement with the experimental result.

CONCLUSIONS

The good agreement between the theories and experiments on the power transfer functions of BWR shows the usefulness of such theories in the range of linear mathematical treatments, that have been established by the author in this paper, for the prediction of the stability limit of BWR. Some differences between the theories and experiments are presumed to be due to the following items:

1) Non-linearity of the system.

2) Error resulting from the evaluations of the local void and fuel temperature coefficients of reactivity.
3) Error resulting from the evaluations of the bulk void and fuel temperature coefficients of reactivity.
4) Error resulting from the assumption regarding spatial distribution of the equilibrium power densities.
5) Transient fluctuations of the working pressure.
6) Axial distribution of the steam flow velocities within the heat transfer channels.

However, the agreement between the theories and experiments is expected to be improved still more provided that more detailed informations regarding the items enumerated above may become to be available, the theories established by the author in this paper on the power transfer functions of boiling reactor are presumed to be very useful for the prediction of the stability limit of such reactor.

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