# Cartesian Displacements of Normal Vibrations of $1,2,4,5$－Tetracyanobenzene and $1,2,4,5$－ Tetracyanobenzene－ $\mathrm{d}_{2}$ Molecules 

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#### Abstract

Displacements of the intramolecular normal vibrations of the TCNB and TCNB－${ }_{2}$ molecules were calculated in terms of the Cartesian coordinates，and graphically represented in a diagram． The results were compared with the numerical representation of the normal vibrations by means of the potential energy distribution．


## INTRODUCTION

It is well known that fully conjugated cyano－compounds such as tetracyanoethylene （TCNE），7，7，8，8－tetracyanoquinodimethane（TCNQ），and 1，2，4，5－tetracyano－ benzene（TCNB）are strong electron acceptors in charge－transfer complexes．In a series of studies on molecular vibrations of these compounds，we have recently carried out the normal coordinate analysis of the TCNB and TCNB－ $\mathrm{d}_{2}$ molecules， using a modified Urey－Bradley force field for the in－plane vibrations and a valence force field for the out－of－plane vibrations．${ }^{1)}$ In that study，the assignments of the fundamental vibrations have been made on the basis of the potential energy dis－ tribution among the internal symmetry coordinates．It was found，however，that descriptions of the precise vibrational modes of some fundamentals were very difficult because of the wide distribution of their potential energies among many internal symmetry coordinates．The same difficulties have also been found for the fundamental vibrations of planar ring molecules，such as benzene ${ }^{2)}$ and halogenated benzenes．${ }^{3)}$

In the present paper，the atomic displacements of the fundamental vibrations of the TCNB and TCNB－ $\mathrm{d}_{2}$ molecules were calculated in terms of the Cartesian displacement coordinates and the precise vibrational modes were schematically drawn in diagrams．The representation of the fundamental vibrations by this method was of satisfactory for the purpose of visualization of the precise vibrational modes．

## PROCEDURE OF CALCULATION

In the normal coordinate analysis by Wilson＇s $\boldsymbol{G F}$ matrix method，${ }^{4}$ ）it is possible to obtain useful informations about the fundamental vibrational modes from the $\boldsymbol{L}$ matrix whose columns are the characteristic vectors of the $\boldsymbol{G F}$ matrix．The $\boldsymbol{L}$

[^0]matrix is given by the internal symmetry coordinate matrix $\boldsymbol{R}$ and the normal coordinate matrix $\boldsymbol{Q}$ as
\[

$$
\begin{equation*}
R=L Q \tag{1}
\end{equation*}
$$

\]

However, the potential energy distribution $P_{i j}$ defined by

$$
\begin{equation*}
P_{i j}=L_{j i}{ }^{2} F_{j j} / \lambda_{i} \tag{2}
\end{equation*}
$$

is usually utilized for the quantitative evaluation of the potential energy distribution of the $i$-th fundamental vibration into the $j$-th internal symmetry coordinate $R_{j}$. Here $\lambda_{i}$ is the frequency parameter of the $i$-th fundamental vibration, i.e. the $i$-th characteristic value of the $\boldsymbol{G F}$ matrix, $L_{j i}$ is the $j$-th element of the $i$-th characteristic vector, and $F_{j j}$ the $j$-th diagonal element of the potential energy matrix $\boldsymbol{F}$. Among all the terms of $P_{i j}$ 's with a fixed value of $i$, usually only one or two terms become so large that the fundamental vibration can easily be assigned to the corresponding internal symmetry coordinates. When, however, many terms of $P_{i j}$ 's have fairly large values, in other words, when the potential energy is widely distributed among many internal symmetry coordinates, the assignment is difficult in terms of the internal symmetry coordinate. In this case, the calculation of the $\boldsymbol{L}_{X}$ matrix defined by

$$
\begin{equation*}
\boldsymbol{X}=\boldsymbol{L}_{X} \boldsymbol{Q} \tag{3}
\end{equation*}
$$

is useful, because the $\boldsymbol{L}_{X}$ matrix permits us a graphical representation of the normal vibrations. Here, $\boldsymbol{X}$ is the column matrix whose elements are the Cartesian coordinates in unit of $\AA$. Equation (3) means that the $\boldsymbol{L}_{X}$ matrix represents the atomic displacements in terms of the Cartesian coordinate when the normal coordinate makes a unit change. The $\boldsymbol{L}_{X}$ matrix is also given by ${ }^{5,6)}$

$$
\begin{equation*}
\boldsymbol{L}_{\boldsymbol{X}}=\boldsymbol{M}^{-1} \tilde{\boldsymbol{B}} \boldsymbol{G}^{-1} \boldsymbol{L} \tag{4}
\end{equation*}
$$

where $\boldsymbol{M}^{-1}$ is the diagonal matrix whose elements are the inverses of the atomic masses, $\tilde{\boldsymbol{B}}$ the transpose matrix of the $\boldsymbol{B}$ matrix which is defined by $\boldsymbol{R}=\boldsymbol{B} \boldsymbol{X}$, and $\boldsymbol{G}^{-1}$ the inverse matrix of the kinetic energy matrix $\boldsymbol{G}$ presented in terms of the internal symmetry coordinate. Equation (4) can be modified to ${ }^{7}$ )

$$
\begin{equation*}
\boldsymbol{L}_{\boldsymbol{X}}=\boldsymbol{M}^{-1} \tilde{\boldsymbol{B}} \boldsymbol{L}_{\boldsymbol{G}}{ }^{\circ} \boldsymbol{\Lambda}_{G^{-1}} \boldsymbol{L}_{\boldsymbol{c}} \tag{5}
\end{equation*}
$$

which is the useful form for calculation of the $\boldsymbol{L}_{X}$ matrix. Here, $\boldsymbol{\Lambda}_{G}{ }^{-1}$ is the inverse matrix of the $\boldsymbol{\Lambda}_{\boldsymbol{G}}$ matrix whose elements are characteristic values of the $\boldsymbol{G}$ matrix, $\boldsymbol{L}_{G}{ }^{\circ}$ the product of $\boldsymbol{\Lambda}_{\boldsymbol{G}}{ }^{1 / 2}$ and the $\boldsymbol{L}_{\boldsymbol{G}}$ matrix whose columns are characteristic vectors of the $\boldsymbol{G}$ matrix, and $\boldsymbol{L}_{\boldsymbol{c}}$ the matrix whose columns are characteristic vectors of $\tilde{\boldsymbol{L}}_{G}{ }^{\circ} \boldsymbol{F} \boldsymbol{L}_{G}{ }^{\circ}$.

In the present study, the $\boldsymbol{L}_{X}$ matrix was calculated by the use of Eq. (5) with the aid of the previous results of the normal coordinate analysis of the TCNB and TCNB$\mathrm{d}_{2}$ molecules. ${ }^{1)}$ All computations were carried out with a Facom 230-60 computer at Data Processing Center, Kyoto University, and the displacements obtained were directly drawn in a diagram by a Calcomp model 770/763 off-line XY plotter. The molecular-fixed axes of TCNB were chosen as shown in Fig. 1.


Fig. 1. Molecular-fixed axes of TCNB.

## RESULTS AND DISCUSSIONS

The Cartesian displacements of the twenty-nine in-plane normal vibrations of the TCNB molecule (the point group is $D_{2 h} \equiv V_{h}$ ), eight of which belong to the $A_{g}$ species, seven to the $B_{1 g}$ species, seven to the $B_{2 u}$ species, and seven to the $B_{3 u}$ species, are shown by arrows in Fig. 2A, together with their calculated frequencies ${ }^{1)}$ in $\mathrm{cm}^{-1}$. The displacements of the thirteen out-of-plane normal vibrations of the TCNB molecule, two of which belong to the $B_{2 g}$ species, four to the $B_{3 g}$ species, three to the $A_{u}$ species, and four to the $B_{1 u}$ species, are also shown in Fig. 2B, where the positive displacements with respect to the molecular-fixed $z$-axis are given by upward arrows and the negative ones by downward arrows. The arrows were drawn with a magnification of 3.5 times as compared with the molecular scale.

It is apparent from Fig. 2 that the graphical representation of the normal vibrations has some advantages over the numerical representation by the potential energy distribution, especially when the potential energy are widely distributed among many internal symmetry coordinates. The previous paper ${ }^{1)}$ has reported that the potential energy of the $\nu_{5}$ vibration ( $730 \mathrm{~cm}^{-1}$ ) belonging to the $A_{g}$ species is distributed among the internal symmetry coordinates $S_{3}$ (the C-CN stretching) by $12 \%, \mathrm{~S}_{4}$ (the CCN-CCN ring stretching) by $30 \%, S_{5}$ (the CCN-CH ring stretching) by $7 \%, S_{6}$ (the C-CH-C ring bending) by $36 \%$, and $S_{7}$ (the C-C-CH ring bending) by $12 \%$. Furthermore, it has been reported ${ }^{1)}$ that the potential energy of the $\nu_{7}$ vibration ( $406 \mathrm{~cm}^{-1}$ ) belonging to the $A_{\mathrm{g}}$ species is distributed among $S_{3}$ by $29 \%, S_{5}$ by $20 \%$, $S_{6}$ by $34 \%$, and $S_{7}$ by $11 \%$. In these cases, the precise descriptions of these vibrations are very difficult as far as only the results of the potential energy distribution are used. But the Cartesian displacements of these vibrations apparently show that the $\nu_{5}$ and $\nu_{7}$ vibrations are the elongation modes of the benzene ring along the $y$ and $x$-axes, respectively.

From Fig. 2A, the $\nu_{4}$ vibration ( $1261 \mathrm{~cm}^{-1}$ ) of the $A_{g}$ species is easily found to have a character of the so-called "breathing" mode which generally gives a large intensity in Raman spectra. In fact, the Raman spectrum of this molecule gives the strong peak at $1262 \mathrm{~cm}^{-1}$. ${ }^{1)}$

The results of the normal coordinate analysisis ${ }^{1}$ that the Kekulé interaction force constant contributes mainly to the $\nu_{25}$ vibration ( $1276 \mathrm{~cm}^{-1}$ ) of the $B_{3 u}$ species is well understood by a glance at the displacement of this vibration drawn in Fig. 2A, because this is a pure "Kekulé deformation" mode of the benzene ring.

It has been indicated by the normal coordinate analysis that the $\mathrm{C}-\mathrm{C} \equiv \mathrm{N}$ bending mode and the C-C-CN bending mode are often coupled with each other. ${ }^{1)}$ This is clearly seen in the Cartesian displacements of many vibrations such as the $\nu_{6}$ vibration ( $557 \mathrm{~cm}^{-1}$ ) of the $A_{g}$ species, the $\nu_{13}$ vibration ( $701 \mathrm{~cm}^{-1}$ ) of the $B_{1 g}$ species, the $\nu_{21}$ vibration ( $466 \mathrm{~cm}^{-1}$ ) of the $B_{2 u}$ species, and the $\nu_{28}$ vibration ( $499 \mathrm{~cm}^{-1}$ ) of the $B_{3 u}$ species. The similar coupling between the $\mathrm{C} \equiv \mathrm{N}$ and $\mathrm{C}-\mathrm{C}$ stretching modes are found in the $\nu_{2}$ vibration ( $2251 \mathrm{~cm}^{-1}$ ) of the $A_{g}$ species, the $\nu_{9}$ vibration ( $2249 \mathrm{~cm}^{-1}$ ) of the $B_{1 g}$ species, the $\nu_{17}$ vibration ( $2249 \mathrm{~cm}^{-1}$ ) of the $B_{2 u}$ species, and the $\nu_{23}$ vibration ( $2250 \mathrm{~cm}^{-1}$ ) of the $B_{3 u}$ species.

It is also apparent from Fig. 2 that the displacements of the hydrogen atoms are very large as compared with those of the carbon and nitrogen atoms, especially in the C-H stretching vibrations (the $\nu_{1}$ vibration of the $A_{g}$ species and the $\nu_{16}$ vibration of the $B_{2 u}$ species). This fact suggests that the anharmonisity must be taken into account in the theoretical analysis of such vibrations.

In the case of the out-of-plane skeletal deformation vibrations, the advantages of the graphical representation of the normal vibrations over the description by the potential energy distribution are much more remarkable, because of the difficulty of a visual grasp of the atomic movements represented by the tortional symmetry coordinates and of the wide distribution of the potential energies among many internal symmetry coordinates. Figure 2B apparently shows that the $\nu_{40}$ vibration ( $522 \mathrm{~cm}^{-1}$ ) of the $B_{1 u}$ species and the $\nu_{32}$ vibration ( $888 \mathrm{~cm}^{-1}$ ) of the $B_{3 g}$ species, have the characters of the so-called "butterfly" and "chair form deformation" modes, respectively, with respect to the benzene ring, although they can not be imagined from the numerical representation by the potential energy distribution.

The Cartesian displacements of the normal vibrations of the TCNB- $\mathrm{d}_{2}$ molecule are almost the same as those of the corresponding vibrations of the TCNB molecule, except for six in-plane vibrations as well as six out-of-plane vibrations which are given in Figs. 3A and 3B, respectively. In these vibrations, the amplitudes of the displacements of the deuterium atoms in the TCNB- $\mathrm{d}_{2}$ molecule are much smaller than those of the hydrogen atoms in the TCNB molecule, as is expected from the difference of their atomic masses.

## (A) IN-PLANE FUNDAMENTALS




Fig. 2. Cartesian displacements of the normal vibrations of TCNB.
The displacements are magnified by 3.5 times as compared with the molecular size.







Fig. 2. Continued.
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Fig. 2. Continued.







Fig. 2. Continued.
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(B) OUT-OF-PLANE FUNDAMENTALS



Fig. 2. Continued.


Fig. 2. Continued.
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Fig. 2. Continued.
(A) IN-PLANE FUNDAMENTALS


Fig. 3. Cartesian displacements of the normal vibrations of TCNB- $\mathrm{d}_{2}$. The displacements are scaled as in Fig. 2.
(B) OUT-OF-PLANE FUNDAMENTALS



Fig. 3. Continued.

## Cartesian Displacements of Normal Vibrations of TCNB and TCNB-d ${ }_{2}$

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