

Computer Program for Three Body Break Up Reaction

Kiyoji FUKUNAGA* and Shigeru KAKIGI*

Received October 11, 1978

A FORTRAN program is developed to calculate the energy spectrum for three body break up reaction at intermediate energy. Energy spectra are calculated on the basis of AGS (Alt, Grassberger and Sandhas) formalism assuming quasi free scattering with final state interaction. The calculated curves reproduced well energy spectra of ^3He in coincidence with protons for $^3\text{He}(\tau, \tau p)$ reaction and deuterons for $^3\text{He}(\tau, \tau d)$ reaction at 120 MeV.

KEY WORDS Nuclear Reaction $^3\text{He}(\tau, \tau p)$ at 120 MeV /
Program for QF Scattering with FSI/

I INTRODUCTION

Three body problems could be treated exactly on the basis of Faddeev's equations with given two body interaction potentials and the energy spectrum of nucleons from break up reaction of deuterons by nucleons could be successfully explained using a separable potential for nucleon-nucleon interaction¹⁾. For the case where components of high angular momentum contribute to the two body interaction in the break up reaction, the problem is not so easy because that realistic separable potential has not yet been obtained and a long computer time needs to solve the integral equations. Furthermore if composite particles are concerned in the final state complicated reaction mechanism is presumed. At intermediate energies, however, continuous energy spectra of emitted particles show rather simple shape with large peaks due to quasi free scattering and some times with peaks for final state interaction appearing at low relative energies of the interacting two particles²⁾. Then it is reasonable to assume some simplifications in calculation for the break up reaction. Alt *et al.*³⁾ have derived one dimensional integral equations for the three body scattering problem and derived a scattering amplitude at the special case where, in the first place, a quasi free process produces three particles and in the final state any two particles interact. We have obtained a computer program to calculate this amplitude and using it evaluated energy spectra for the break up reaction of ^3He nucleus by ^3He beam at 120 MeV.

II EQUATIONS

Break up amplitude for three body break up reaction can be calculated using Eq. (5. 2) in Ref. 3 as follows,

* 福永清二, 柿木 茂: Nuclear Science Research Facility, Institute for Chemical Research, Kyoto University.

Program for Three Body Break Up Reaction

$$X_{p_1, 1m}(z) = -\sum_{r \neq 1} \langle \mathbf{p}_1 | T'_r(z) G_0(z) | 1, m; z \rangle + \sum_{r \neq 1} \langle \mathbf{p}_1 | T_1(z) G_0(z) T_r(z) G_0(z) | 1, m; z \rangle, \quad (1)$$

where T'_r are non separable "amplitude" and small compared with the two body amplitude T_1 . In the calculation the following conditions are assumed. 1) two body amplitude T_1 and T'_r have not any singularity on the energy shell, 2) the Green's function G_0 at the last position in the second term does not given any singularity because the cancellation of amplitudes at two integration angles \mathbf{p}_1'' and $-\mathbf{p}_1''$, and 3) a pole approximation is applied to integrate the other Green's function in the second term. Then the cross section is given as follows,

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = \frac{2\pi}{\hbar^2} \cdot \frac{\mu_{in}}{k_{in}} \rho |M_+ + M_-|^2, \quad (2)$$

$$\begin{aligned} M_{\pm} &= \langle \mathbf{q}_1 | X_{\pm} | \mathbf{q}_0 \rangle \\ &= -\sum_{r \neq 1} \left(\frac{1}{S_{r1}} \right)^3 \langle \mathbf{p}'_r | T'_r(z - q_1^2) | \mathbf{p}_r \rangle \frac{1}{\left\{ \mathbf{p}'_1 - \frac{C_{r1}}{S_{r1}} (\mathbf{q}_1 \mp \mathbf{q}_0) \right\}^2 - Q} \\ &\quad \times \langle \mathbf{p}'_1 - \frac{C_{r1}}{S_{r1}} (\mathbf{q}_1 - \mathbf{q}_0) | 1, m; z \rangle \\ &\quad + \sum_{r \neq 1} \left(\frac{1}{S_{r1}} \right)^3 \int d\Omega'' \langle \mathbf{p}_1 | T_1(z - q_1^2) | \mathbf{p}_1'' \rangle \frac{\pi}{2} i \mathbf{p}_1'' \langle \mathbf{p}'_r | T'_r(z - q_1^2) | \mathbf{p}_r \rangle \\ &\quad \times \frac{1}{\left\{ \mathbf{p}'_1 - \frac{C_{r1}}{S_{r1}} (\mathbf{q}_1 - \mathbf{q}_0) \right\}^2 - Q} \langle \mathbf{p}'_1 - \frac{C_{r1}}{S_{r1}} (\mathbf{q}_1 \mp \mathbf{q}_0) | 1, m; z \rangle, \end{aligned} \quad (3)$$

$$p_1''^2 = z - q_1^2,$$

where ρ is the phase space for factor three body system in the final state. M_+ and M_- are matrix elements for the target break up process and projectile break up process, respectively. The variable p_1^2 and q_1^2 have a dimension of energy and relate to the usual wave vector \mathbf{k} and mass m ,

$$\left. \begin{aligned} \mathbf{p}_1 &= [2m_2 m_3 (m_2 + m_3)]^{-1/2} (m_3 \mathbf{k}_2 - m_2 \mathbf{k}_3), \\ \mathbf{q}_1 &= [2m_1 (m_2 + m_3) (m_1 + m_2 + m_3)]^{-1/2} [m_1 (\mathbf{k}_2 + \mathbf{k}_3) - (m_2 + m_3) \mathbf{k}_1], \end{aligned} \right\} \quad (4)$$

and the variables in the i - jk system are related to the variables in the j - ki system through the following relations,

$$\left. \begin{aligned} \mathbf{p}_2 &= C_{21} \mathbf{p}_1 + S_{21} \mathbf{q}_1, \\ \mathbf{q}_2 &= -S_{21} \mathbf{p}_1 + C_{21} \mathbf{q}_1. \end{aligned} \right\} \quad (5)$$

where

$$\left. \begin{aligned} C_{21} &= - \left[\frac{m_1 m_2}{(m_1 + m_3) (m_2 + m_3)} \right]^{1/2} = C_{12}, \\ S_{21} &= \sqrt{1 - C_{21}^2}, \end{aligned} \right\} \quad (6)$$

Four types of potential are assumed in the calculation of the initial interaction, that is Gaussian type, Yukawa type, Woods Saxson type and Woods Saxson for real potential with the first derivative of Woods Saxson for imaginary potential,

$$\langle \mathbf{p}_r | T_r(z - q_0^2) | \mathbf{p}'_r \rangle = T_r(\mathbf{p}'_r - \mathbf{p}_r), \quad (7)$$

$$= -\pi^{3/2} \frac{V}{\alpha^3} \exp\left[-\frac{(\mathbf{p}'_r - \mathbf{p}_r)^2}{4\alpha^2}\right], \quad \text{for Gaussian} \quad (8)$$

$$= -4\pi V \left[\frac{1}{\alpha_1^2 + (\mathbf{p}'_r - \mathbf{p}_r)^2} + \frac{\eta}{\alpha_2^2 + (\mathbf{p}'_r - \mathbf{p}_r)^2} \right], \quad \text{for Yukawa} \quad (9)$$

$$= \int d\mathbf{r}^3 V(\mathbf{r}) e^{i(\mathbf{p}'_r - \mathbf{p}_r) \cdot \mathbf{r}}, \quad \text{for Woods Saxon } V(\mathbf{r}). \quad (10)$$

The form factor of the target nucleus is assumed as,

$$\begin{aligned} \langle \mathbf{p}_1 | 1, m; z \rangle &= (z - q_0^2) \phi_{1m}(\mathbf{p}_1), \\ \phi_{1m}(\mathbf{p}_1) &= \frac{\sqrt{\alpha}}{\pi} \frac{1}{\alpha^2 + \mathbf{p}_1^2}. \end{aligned} \quad (11)$$

S wave final state interaction between particle 2 and particle 3 relates to the effective range parameters at zero energy limit in the two body scattering,

$$|\langle \mathbf{p}_1 | T_1(z - q_1^2) | \mathbf{p}'_1 \rangle|^2 = \frac{C^2(\eta)}{4\pi^2} \cdot \frac{1}{C^2(\eta)q^2 + \left(-\frac{1}{a} + \frac{1}{2}r q^2\right)}, \quad (12)$$

$$\begin{aligned} q^2 &= (\mathbf{p}'_1 - \mathbf{p}_1)^2, \\ C^2(\eta) &= 2\pi\eta / [\exp(2\pi\eta) - 1], \\ \eta &= z_2 z_3 e^2 / \hbar v, \end{aligned}$$

where $C(\eta)$ is the Coulomb penetration factor. For the l -th partial wave interaction in the final state Brait-Wigner type resonance form is assumed as follows,

$$|\langle \mathbf{p}_1 | T_1(z - q_1^2) | \mathbf{p}'_1 \rangle|^2 = A \frac{\Gamma_1^2}{(z - q_1^2 - E_1)^2 + \frac{1}{4}\Gamma_1^2} P_1(\cos\omega). \quad (13)$$

The angular integral of Eq. (3) is done analytically where the angular momentum of l is less than or equal to 2. The values of parameters in Eq. (8), (9), and (11) are transformed from the radial parameters in fm using following relation,

$$\alpha^2_{\text{MeV}} = \frac{\gamma}{\mu a_{\text{fm}}}, \quad \gamma = \frac{41.6}{2},$$

μ : reduced mass

III COMPUTER PROGRAM

The computer program "AGSIA" calculates the three body break up cross section on the basis of AGS formalism. Seven types of card have to read in the format of free field as the input data. Three body kinematics is calculated using a subprogram "KINEMA" which is the same as the program developed by Ohlsen⁴), and the numerical integration of Eq. (10) is made using a sub-program "SIMPSON" which is a usual program of Simpson Integral. The matrix element in Eq. (3) is calculated using a sub-program "XMAT". Four types of initial interaction can be selected by **KCODE4** and five print out format can be selected by **KCODE3**. All calculated energy spectra are print out in units of $10^{-26} \text{cm}^2/\text{sr}^2 \cdot \text{MeV}$. as a function of the laboratory energy of particle 1. The FORTRAN programs are given in an Appendix.

The energy spectrum of ${}^3\text{He}$ in coincidence with protons for ${}^3\text{He}(\tau, \tau p)d$ reaction

Program for Three Body Break Up Reaction

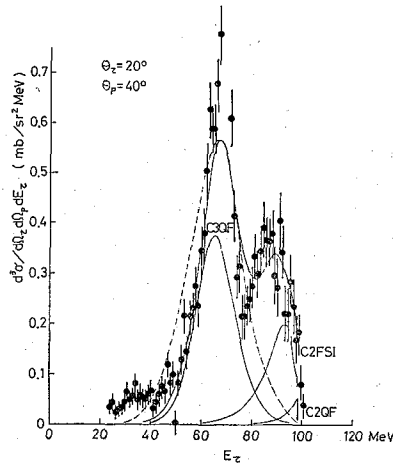


Fig. 1. Energy spectrum projected for the ${}^3\text{He}(\tau, \tau p)d$ reactions at $\theta_\tau=20^\circ$ and $\theta_p=40^\circ$. All solid curves are calculated on the basis of the AGS formalism. The dashed curve shows the calculated one of the simple spectator model.

at 120 MeV was calculated using the program of "AGSIA". Figure 1 shows the calculated curves at the angular set $\theta_\tau=20^\circ$, $\theta_p=40^\circ$ and $\varphi_p=0^\circ$ (opposite side of, and in coplanar with, the beam) and shows the experimental energy spectrum which was obtained from the data in Ref. 2. The curves denoted as **C2QF** and **C2FSI** are calculated using only the first term at $\gamma=2$ and the second term at $\gamma=2$ in Eq. (1). The former corresponds to the cross section assuming the quasi free scattering of particle 1 (${}^3\text{He}$) and particle 3 (unobserved particle) and the latter corresponds to that for quasi scattering with the final state interaction of particle 2 and particle 3. The dashed curve is calculated using a simple impulsive spectator model. Nevertheless a normalization constant was needed to reproduce the experimental energy spectrum, the calculated energy spectrum is reproduced well using a reasonable parameter set.

ACKNOWLEDGMENT

Authors would like to thanks to Mr. T. Ohsawa, Dr. S. Tanaka, and Mr. A. Okihana for their usefull discussions and cooperations to obtain the data of ${}^3\text{He}(\tau, \tau p)d$ reaction. Numerical calculation was made by FACOM 230 computer at the Institute for Chemical Research of Kyoto University.

REFERENCES

- (1) W. Ebenhöb, *Nuclear Phys.*, **A191**, 92 (1972).
- (2) K. Fukunaga, N. Fujiwara, S. Kakigi, T. Ohsawa, H. Nakamura-Yokota, S. Tanaka, A. Okihana, T. Sekioka, T. Higo, and T. Miyayaga, *J. Phys. Soc. Japan*, **45**, 1783 (1978).
- (3) E. O. Alt, P. Grassberger, and W. Sandhas, *Nuclear Phys.*, **B2**, 167 (1967).
- (4) G. G. Ohlsen, *Nuclear Inst. and Meth.*, **37**, 240 (1965).

Appendix

Instructions for use of "AGSIA"

Card 1 (general conditions in the calculation)

:KCODE1, KCODE2, KCODE3, KCODE4, IK, NO, FACT
 KCODE1 = 1 print subtitle "XM3 IS NOT A COMPOSITE PARTICLE"
 2 print subtitle "XM3=XM4+XM5 AND BREAKS TO SINGLET DEUTERON"
 3 print subtitle "XM3=XM4+XM5 AND BREAKS TO TRIPLET p+n"
 SYSTEM"

if any number is chosen, the program does not change to calculate the reaction

KCODE2 = 0 calculate the target breakup reaction ; $M=M_+$
 1 calculate the projectile breakup reaction ; $M=M_-$
 2 calculate the target breakup plus the projectile breakup reaction ; $M=M_+ + M_-$
 KCODE3 = 0 print out the kinematics of three body breakup reaction
 1 print out all reaction amplitudes and phase space factor
 2 print out the final cross sections
 3 print out the cross sections for (1-2) quasi free, the cross sections for (1-3) quasi free and the final cross sections
 4 print out all cross sections of components in Eq. (3) and the final cross sections
 KCODE4 = 0 initial interaction of Gaussian type
 1 initial interaction of double Yukawa type
 2 initial interaction of Woods-Saxson type
 3 initial interaction of Woods-Saxson (real) with the first derivative of the Woods-Saxson (imaginary)
 LK number of resonance in the final state interaction
 NO number of angular set
 FACT factor of the second term by the first term in Eq. (3), and set 1 in the usual calculation.

Card 2 (kinematical parameters)

:XMP, XMT, XM1, XM2, XM3, Q, EPL, DE1L, ZZ
 XMP mass of the projectile in atomic mass unit
 XMT Mass of the target in atomic mass unit
 XM1 Mass of the particle 1 in atomic mass unit
 XM2 mass of the particle 2 in atomic mass unit
 XM3 mass of the particle 3 in atomic mass unit
 Q reaction Q value in MeV
 EPL incident energy in MeV
 DE1L energy increment in MeV
 ZZ product of electric charges Z_2 and Z_3

Card 3 (form factor and effective range parameters)

:ALPHTO, ALPHAO, ERANGT
 ALPHTO width of the form factor in fm
 ALPHAO scattering length in fm
 ERANGT effective range in fm

Card 4 (potential parameters for the first step interaction between particle 1 and particle 2.)

:VTPO1, RHTPO1, DHTPO1, YETAP1, VTPO2, RHTPO2, DHTPO2, YETAP2
 VTPO1 real well depth in MeV
 RHTPO1 real well width in fm
 DHTPO1 real well diffuseness (for Gaussian or Woods-Saxson) or second well width in fm (for double Yukawa)
 YETAP1 ratio of second Yukawa potential to the first Yukawa potential
 VTPO2 imaginary well depth in MeV
 RHTPO2 imaginary well width in fm
 DHTPO2 imaginary well diffuseness (for Gaussian or Woods-Saxson) or the second well

Program for Three Body Break Up Reaction

width in fm (for double Yukawa)

YETAP2 ratio of the imaginary depth of second Yukawa to that of first Yukawa

Card 5 (potential parameters for the first step interaction between particle 1 and particle 3)
:VTDO1, RHTDO1, DHTDO1, YETAD1, VTDO2, RHXDO2, DHTDO2, YETAD2
similar to the Card 4

Card 6 (resonance parameters for the S wave final state interaction)

:COEF(1), ER(1) WIDTH(1)

COEF(1) coefficient of the Brait-Wigner resonance

ER(1) resonance energy in MeV

WIDTH(1) resonance width in MeV

If the resonances for the P wave and the D wave final state interactions are calculated, the Card 7 and Card 8 are used for the resonance parameters. If any resonance is not calculate, the COEF(1) in the Card 6 must be equal to zero. And the Card of angular set follows to the resonance Card.
Card 7 (angular set)

:TH1R, TH2R, PHI2

TH1R polar angle of particle 1 in degrees in the laboratory

TH2R polar angle of particle 2 in degrees

PHI2 azimuthal angle of particle 2 in degrees. Set zero when the detectors for particle 1 and particle 2 are set opposite side of, in coplaner, with, the incident beam.

