Bull. Inst. Chem. Res., Kyoto Univ., Vol. 60, No. 2, 1982

Four Particle Model For Three Body Breakup Reaction

Kiyoji FUKUNAGA*, Sigeru KAKIGI*, Takao OHSAWA**, and Akira OKIHANA***

Received March 31, 1982

A mechanism for three body breakup reaction is studied with a four particle model neglecting the process of multiple scattering. Energy spectra of protons for the d+d→d+p+n reaction show three large enhancements corresponding to the target breakup quasi-free process, the projectile breakup quasi-free process and a new type of enhancement at low energies of protons associated with the forward deuterons. The last enhancement apparts from the kinematical condition for a spectator model and that is mainly produced through the negative energy scattering off the energy shell.

I. INTRODUCTION

Breakup cross sections for three nucleon system (nucleon+deuteron) were calculated using a Faddeev theory and compaired with the experimental data in good fits.[1] Three body theory was also used to calculate the breakup cross section for the $^2H(\alpha, \alpha p)n$ reaction at a low energy and the reasonable fit was obtained.[2] The Faddeev theory is not yet applied to four nucleon system (deuteron+deuteron, nucleon+triton or helium 3), and the experimental energy spectra[3] were usually analysed using a simple theory for quasi-free scatterings. The theory however can not explaine the absolute value of the breakup cross section and also fails to reproduce the angular dependence of the enhancement maximum in the energy spectrum, although the simple theory reproduced the spectral shape of emitted particles. On the other hand a four body model calculation[4] can explaine angular correlation functions for three body breakup reaction. Then it is worthy to examine the validity of the assumptions of three body model in the case of four particle system.

The three body breakup reaction of four particle system is studied on the bases of Sloan's four particle theory[5] neglecting multiple scattering process.

II. THEORY

1. Differential Cross Section

For the nuclear reaction as $P+T\rightarrow A+B+C$, the breakup cross section is given as

$$\frac{d\sigma}{d\Omega_A d\Omega_B dE_A}=\frac{2\pi}{\hbar v_{in}}\frac{1}{(2J_p+1)(2J_T+1)}\sum_J(2J+1)\rho|U_{fi}|^2(10\,mb/sr^2MeV). \quad (1)$$

* 福永清二, 柿木 茂: Nuclear Science Facility, Institute for Chemical Research, Kyoto University, Kyoto

** 大沢孝夫: Laboratory of Nuclear Reaction, Institute for Chemical Research, Kyoto University, Kyoto

*** 沖花 彰: Kyoto University of Education, Kyoto

with

$$\frac{1}{\hbar v_{in}^{*}}=\sqrt{\frac{m_p(m_p+m_T)}{2m_T E_p}}\gamma^{1/2},$$

$$\gamma=\frac{m_0}{\hbar}\sim\frac{1}{41.6\ MeV\cdot fm^2},$$

m_p : mass of the projectile in AMU,
m_T: mass of the target in AMU,
J_p : spin of the projectile in $\hbar$ units,
J_T : spin of the target in $\hbar$ units,
J : total spin of the four particle system in $\hbar$ units.

The quantity ρ is the phase space factor as,

$$\rho=\frac{1}{(2\pi)^6}\gamma^3\rho_{MeV}, \tag{2}$$

$$\rho_{MeV}=\frac{m_A m_B m_C p_A p_B}{m_C+m_B-m_B(\boldsymbol{p}_{in}-\boldsymbol{p}_A)\cdot\boldsymbol{p}_B/p_B^2},$$

with

$p_A=\sqrt{2m_A E_A}$,
$p_B=\sqrt{2m_B E_B}$,
m_A : mass of outgoing particle A in AMU,
m_B : mass of outgoing particle B in AMU,
m_C : mass of outgoing particle C in AMU,
E_A : energy of particle A in MeV,
E_B : energy of particle B in MeV,

where $\boldsymbol{p}_{in}$, $\boldsymbol{p}_A$ and $\boldsymbol{p}_B$ are momenta of the projectile, particle A and particle B, respectively. The phase space factor ρ_{MeV} is measured in MeV. The U is the transition amplitude for the three body breakup reaction and is defined as

$$|U_{fi}|^2=(2\pi)^9\gamma^{-9/2}|T_{fi}|^2, \tag{3}$$

where $|T|^2$ is measured in units of $MeV^{-5/2}$.

2. Momentum Coordinates

For the channel representation of four particle system, there are two types; 1+3 channel (ijk$-l$) and 2+2 channel (ij$-$kl). There are four 1+3 channels and three 2+2 channels. One of the 1+3 channel consists of three subchannels which are specified by one of three internal pairs in the three particle subsystem and one of the 2+2 channel consists of two subchannels specified by one of two internal pairs. The definition of the internal pair is same as used by Sloan.[5]

For a system of four particles with masses m_i, m_j, m_k and m_l, it is convenient to use the following ($\boldsymbol{p}$, $\boldsymbol{q}$, $\boldsymbol{s}$, $\boldsymbol{w}$) coordinates instead of the usual relative momenta ($\boldsymbol{k}_{ij}$, $\boldsymbol{k}_{ij-k}$, $\boldsymbol{k}_{ijk-l}$, $\boldsymbol{w}$), ($\boldsymbol{k}_{ij}$, $\boldsymbol{k}_{kl}$, $\boldsymbol{k}_{ij-kl}$, $\boldsymbol{w}$) or particle momenta ($\boldsymbol{k}_i$, $\boldsymbol{k}_j$, $\boldsymbol{k}_k$, $\boldsymbol{k}_l$) in the laboratory system.

For the 1+3 type,

$$\left.\begin{aligned}
\boldsymbol{p}_{i-j} &= [2m_im_j/(m_i+m_j)]^{-1/2}\boldsymbol{k}_{i-j}\\
&= [2m_im_j(m_i+m_j)]^{-1/2}(m_j\boldsymbol{k}_i-m_i\boldsymbol{k}_j),\\
\boldsymbol{q}_{ij-k} &= [2m_k(m_i+m_j)/(m_i+m_j+m_k)]^{-1/2}\boldsymbol{k}_{ij-k}\\
&= [2m_k(m_i+m_j)(m_i+m_j+m_k)]^{-1/2}[m_k(\boldsymbol{k}_i+\boldsymbol{k}_j)-(m_i+m_j)\boldsymbol{k}_k],\\
\boldsymbol{s}_{ijk-l} &= [2m_l(m_i+m_j+m_k)/(m_i+m_j+m_k+m_l)]^{-1/2}\boldsymbol{k}_{ijk-l}\\
&= [2m_k(m_i+m_j+m_k)(m_i+m_j+m_k+m_l)]^{-1/2}\\
&\qquad\times[m_l(\boldsymbol{k}_i+\boldsymbol{k}_j+\boldsymbol{k}_k)-(m_i+m_j+m_k)\boldsymbol{k}_l],\\
\boldsymbol{w} &= [2(k_i+m_j+m_k+m_l)]^{-1/2}(\boldsymbol{k}_i+\boldsymbol{k}_j+\boldsymbol{k}_k+\boldsymbol{k}_l),
\end{aligned}\right\}\qquad(4)$$

and for the 2+2 type,

$$\left.\begin{aligned}
\boldsymbol{p}_{i-j} &= [2m_im_j/(m_i+m_j)]^{-1/2}\boldsymbol{k}_{i-j}\\
&= [2m_im_j(m_i+m_j)]^{-1/2}(m_j\boldsymbol{k}_i-m_i\boldsymbol{k}_j),\\
\boldsymbol{q}_{k-l} &= [2m_km_l/(m_k+m_l)]^{-1/2}\boldsymbol{k}_{k-l}\\
&= [2m_km_l(m_k+m_l)]^{-1/2}(m_l\boldsymbol{k}_k-m_k\boldsymbol{k}_l),\\
\boldsymbol{s}_{ij-kl} &= [2(m_i+m_j)(m_k+m_l)/(m_i+m_j+m_k+m_l)]^{-1/2}\boldsymbol{k}_{ij-kl}\\
&= [2(m_i+m_j)(m_k+m_l)(m_i+m_j+m_k+m_l)]^{-1/2}\\
&\qquad\times[(m_k+m_l)(\boldsymbol{k}_i+\boldsymbol{k}_j)-(m_i+m_j)(\boldsymbol{k}_k+\boldsymbol{k}_l)],\\
\boldsymbol{w} &= [2(m_i+m_j+m_k+m_l)]^{-1/2}(\boldsymbol{k}_i+\boldsymbol{k}_j+\boldsymbol{k}_k+\boldsymbol{k}_l),
\end{aligned}\right\}\qquad(5)$$

where $\boldsymbol{w}$ is total momentum of the four particle system. The momenta ($\boldsymbol{p}_\beta$, $\boldsymbol{q}_\beta$, $\boldsymbol{s}_\beta$ and $\boldsymbol{w}$) in the subchannel β is connected with the momenta ($\boldsymbol{p}_\gamma$, $\boldsymbol{q}_\gamma$, $\boldsymbol{s}_\gamma$ and $\boldsymbol{w}$) in the subchannel γ through the 4×4 matrix $[C^{\beta\gamma}]$ as,

$$\begin{pmatrix}\boldsymbol{p}_\beta\\ \boldsymbol{q}_\beta\\ \boldsymbol{s}_\beta\\ \boldsymbol{w}\end{pmatrix}=\begin{pmatrix} & & & \\ & C^{\beta\gamma} & & \\ & & & \\ & & & \end{pmatrix}\begin{pmatrix}\boldsymbol{p}_\gamma\\ \boldsymbol{q}_\gamma\\ \boldsymbol{s}_\gamma\\ \boldsymbol{w}\end{pmatrix},$$

with

$$[C^{\beta\gamma}]=[A^\beta]\cdot[D^{\beta\gamma}]\cdot[A^\gamma]^{-1},$$
$$[\boldsymbol{k}_\beta]=[D^{\beta\gamma}][\boldsymbol{k}_\gamma],$$

where $[\boldsymbol{k}_\beta]$ and $[\boldsymbol{k}_\gamma]$ are one column matrices of particle momenta in the subchannel β and γ, respectively. The $[A^\beta]$ is the 4×4 matrix which is composed of the coefficients of particle momenta in equation (4) or (5). The $[A^\beta]^{-1}$ is the inverse matrix of the $[A^\beta]$ and the $[D^{\beta\gamma}]$ presents the relation between the different labelings γ and β for the kinematically equivalent state. Then, the momenta $\boldsymbol{p}_\beta$, $\boldsymbol{q}_\beta$ and $\boldsymbol{s}_\beta$ are explicitly written by the momenta $\boldsymbol{p}_\gamma$, $\boldsymbol{q}_\gamma$ and $\boldsymbol{s}_\gamma$ as,

$$\left.\begin{aligned}
\boldsymbol{p}_\beta &= C^{\beta\gamma}_{pp}\boldsymbol{p}_\gamma+C^{\beta\gamma}_{pq}\boldsymbol{q}_\gamma+C^{\beta\gamma}_{ps}\boldsymbol{s}_\gamma,\\
\boldsymbol{q}_\beta &= C^{\beta\gamma}_{qp}\boldsymbol{p}_\gamma+C^{\beta\gamma}_{qq}\boldsymbol{q}_\gamma+C^{\beta\gamma}_{sq}\boldsymbol{s}_\gamma,\\
\boldsymbol{s}_\beta &= C^{\beta\gamma}_{sp}\boldsymbol{p}_\gamma+C^{\beta\gamma}_{sq}\boldsymbol{q}_\gamma+C^{\beta\gamma}_{ss}\boldsymbol{s}_\gamma,
\end{aligned}\right\}\qquad(6)$$

and

$$Z=p_\beta^2+q_\beta^2+s_\beta^2=p_\gamma^2+q_\gamma^2+s_\gamma^2,$$

where Z is the total energy in the center of mass system. In these relations, it is not necessary that β and γ belong to the same channel.

When the four particles have the same mass of unity, the matrix $[A]$ is given as

$$[A]=\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & -\frac{3}{2\sqrt{6}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{pmatrix} \text{ for the } 1+3 \text{ type channel,}$$

and

$$[A]=\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{pmatrix} \text{ for the } 2+2 \text{ type channel.}$$

Using the new momentum coordinates, the state vectors in the relative momentam coordinates are written by using the vector $|\boldsymbol{p}\rangle$, $|\boldsymbol{q}\rangle$ and $|\boldsymbol{s}\rangle$ which are normalized to the δ-function as

$$|\boldsymbol{k}_{ij}\rangle=\frac{1}{\left|\frac{d\boldsymbol{k}_{ij}}{d\boldsymbol{p}_{ij}}\right|^3}|\boldsymbol{p}_{ij}\rangle,$$

$$|\boldsymbol{k}_{kl}\rangle=\frac{1}{\left|\frac{d\boldsymbol{k}_{kl}}{d\boldsymbol{q}_{kl}}\right|^3}|\boldsymbol{q}_{kl}\rangle,$$

$$|\boldsymbol{k}_{ij-kl}\rangle=\frac{1}{\left|\frac{d\boldsymbol{k}_{ij-kl}}{d\boldsymbol{s}_{ij-kl}}\right|^3}|\boldsymbol{s}_{ij-kl}\rangle,$$

$$\langle\boldsymbol{p}'_{ij}\boldsymbol{q}'_{kl}\boldsymbol{s}'_{ij-kl}|\boldsymbol{p}_{ij}\boldsymbol{q}_{kl}\boldsymbol{s}_{ij-kl}\rangle =\delta^3(\boldsymbol{p}'_{ij}-\boldsymbol{p}_{ij})\delta^3(\boldsymbol{q}'_{kl}-\boldsymbol{q}_{kl})\delta^3(\boldsymbol{s}'_{ij-kl}-\boldsymbol{s}_{ij-kl}),$$

$$\langle\boldsymbol{k}'_{ij}\boldsymbol{k}'_{kl}\boldsymbol{k}'_{ij-kl}|T_{fi}|\boldsymbol{k}_{ij}\boldsymbol{k}_{kl}\boldsymbol{k}_{ij-kl}\rangle =\frac{1}{2\sqrt{2}}\langle\boldsymbol{p}'_{ij}\boldsymbol{q}'_{kl}\boldsymbol{s}'_{ij-kl}|T_{if}|\boldsymbol{p}_{ij}\boldsymbol{q}_{kl}\boldsymbol{s}_{ij-kl}\rangle. \qquad (7)$$

The transition amplituds $\langle\boldsymbol{k}'|T_{fi}|\boldsymbol{k}\rangle$ in the relative momentum coordinates is equal to that in Eq. (3).

3. Spin Coefficients

As shown in the previous sections, the channel of the four particle state is specified by the subchannel and the interacting pair. The state vector of the subchannel is composed of two functions of the momentum state $|\boldsymbol{pqs}\rangle$ and the spin state $|J_\beta J_\mu\rangle$ as,

$$|12,\ 34\rangle = |\boldsymbol{p}_{12},\ \boldsymbol{q}_{34},\ \boldsymbol{s}_{12\text{-}34}\rangle |J_{12\text{-}34},\ J_{12}\rangle$$
$$|12,\ 3,\ 4\rangle = |\boldsymbol{p}_{12},\ \boldsymbol{q}_{123},\ \boldsymbol{s}_{123\text{-}4}\rangle |J_{123\text{-}4},\ J_{12}\rangle.$$

For the $d+d\to d+p+n$ reaction, two protons are labeled as particles 1 and 3 and two neutrons are labeled as particles 2 and 4.

The antisymmetrized amplitude is written by using the amplitudes represented with the labeled four particles as,

$$\begin{aligned} &{}_f^A\langle 12,\ 3,\ 4|T|12,\ 34\rangle_i^A \\ &= \frac{1}{8}\{({}_f\langle 12,\ 3,\ 4| - {}_f\langle 32,\ 1,\ 4| - {}_f\langle 14,\ 3,\ 2| + {}_f\langle 34,\ 1,\ 2|) \\ &\quad T(|12,\ 34\rangle_i + |34,\ 12\rangle_i)\}. \end{aligned} \tag{8}$$

We assume that two particle interaction is not spin dependent, then the transition amplitudes are factorized to two parts as the momentum components and the spin coefficients. The spin coefficient is written for the unpolarized experiment as

$$S = S(J_\beta J_\mu,\ J_\gamma J_\nu,\ J_\alpha J_\lambda) = \langle J_\beta J_\mu | J_\gamma J_\nu\rangle \langle J_\gamma J_\nu | J_\alpha J_\lambda\rangle, \tag{9}$$

where J_β, J_γ and J_α are the spins of the subchannels and J_μ, J_ν and J_λ are the spins of interacting pair in the final state, the intermediate state and the initial state, respectively. The spin of the subchannel is the total spin of the three particle subsystem in the 1+3

Table I Transition amplitude for $d+d\to d+p+n$
(Proton: 1, 3 and neutron: 2, 4)

	α	12-34							
	λ	12				34			
	γ	123-4		124-3		134-2		234-1	
	ν	13	23	14	24	13	14	23	24
$J=0$	$\mu=12$	$\frac{3}{4}t^0_{13}$	$-\frac{3}{4}t^0_{23}-\frac{1}{4}t^1_{23}$	$-\frac{3}{4}t^0_{14}-\frac{1}{4}t^1_{14}$	$\frac{3}{4}t^0_{24}$				
	14					$-\frac{3}{4}t^0_{13}$			
	23	$-\frac{1}{4}t^0_{13}$			$-\frac{3}{4}t^0_{24}$				$-\frac{3}{4}t^0_{24}$
	24					$\frac{3}{4}t^0_{13}$	$\frac{3}{4}t^0_{14}-\frac{1}{4}t^1_{14}$	$-\frac{3}{4}t^0_{23}-\frac{1}{4}t^1_{23}$	$\frac{3}{4}t^0_{24}$
$J=1$	12	$-\frac{1}{2}t^0_{13}$	$-\frac{1}{2}t^0_{23}-\frac{1}{2}t^1_{23}$	$-\frac{1}{2}t^0_{14}-\frac{1}{2}t^1_{14}$	$\frac{1}{2}t^0_{24}$				
	14				$\frac{1}{2}t^0_{24}$	$\frac{1}{2}t^0_{13}$			
	23	$\frac{1}{2}t^0_{13}$							$\frac{1}{2}t^0_{24}$
	24					$-\frac{1}{2}t^0_{13}$	$-\frac{1}{2}t^0_{14}-\frac{1}{2}t^1_{14}$	$-\frac{1}{2}t^0_{23}-\frac{1}{2}t^1_{23}$	$-\frac{1}{2}t^0_{24}$
$J=2$	12		t^1_{23}	t^1_{14}					
	14								
	23								
	24						t^1_{14}	t^1_{23}	

Table II Transition amplitude for $p+{}^3He\rightarrow d+p+p$
(proton: 1, 2, 3 and neutron: 4)

	α			234-1		
	λ			34		
	J_{34}	1			0	
	γ	12-34	134-2		12-34	134-2
	ν	12	13	14	12	13
$J=0$	$\mu=14$		$-\frac{3}{4}t^0_{13}$			$\frac{\sqrt{3}}{4}t^0_{13}$
	24					
	34		$\frac{3}{4}t^0_{13}$	$\frac{3}{4}t^0_{14}+\frac{1}{4}t^1_{14}$		$-\frac{\sqrt{3}}{4}t^0_{13}$
$J=1$	14		$-\frac{1}{4}t^0_{13}$			
	24					
	34	$\frac{1}{\sqrt{3}}t^0_{12}$	$\frac{1}{4}t^0_{13}$	$\frac{1}{4}t^0_{14}+\frac{1}{12}t^1_{14}$		

channel or the spin of the pair coupled with the momentum q in the $2+2$ channel. The overlapping of two spin state is written as, for example,

$$\begin{aligned}&\langle J_\gamma\, J_\nu \mid J_\alpha\, J_\lambda\rangle \\ &=(-)^{J_i+J_k-J_{ik}+J_k+J_{ij}-J_{ijk}}\times\sqrt{(2J_{ik}+1)\,(2J_{ij}+1)\,(2J_{ijk}+1)\,(2J_{kl}+1)} \\ &\times W(J_kJ_iJ_{ijk}J_j;\ J_{ik}J_{ij})\,W(J_{ij}J_kJJ_l;\ J_{ijk}J_{kl}), \\ &\gamma=(ijk-l),\ \nu=(ik),\ \alpha=(ij-kl),\ \lambda=(ij),\end{aligned}$$

where W is Racah Coefficient.

For the initial $d+d$ channel and the final $d+p+n$ channel, the transition amplitudes with the spin coefficients are given in Table I. For the initial $p+{}^3He$ channel and the final $d+p+p$ channel, the amplitudes are given in Table II.

4. Transition Amplitude

For four particle system, the transition amplitude T_{fi} in Eq. (3) is given by the solution of seven coupled integral equations studied by Sloan.[5] To simplify the calculation, the single scattering amplitude takes in the place of the transition amplitude T_{fi} as

$$T_{fi}=\frac{1}{n_i n_f}\sum_{\gamma\beta\alpha}\ \sum_{\mu\in I(\beta)\cap I(\gamma)}\ \sum_{\nu\in I(\gamma)\cap E(\beta)\cap E(\alpha)}\ \sum_{\lambda\in I(\alpha)\cap I(\alpha)}(-)^{\phi}\sum_{J_\rho J_\gamma J_\nu J_\alpha}S\cdot T^{\nu}_{\mu\lambda}, \tag{10}$$

where the S is the spin coefficient defined in Eq. (9) and $(-)^{\phi}$ is the sign in Eq. (8). For the reaction $d+d\rightarrow d+p+n$, the initial channel is the $2+2$ type and the intermediate channel is one of four $1+3$ channels. The final channel is a three body channel including one bound pair. The n_i and n_f are the number of the initial subchannel and the number of the final subchannel, respectively. Then, $n_i\times n_f=8$ as shown in Eq (8). The suffices λ, μ and ν of single scattering amplitude mean the interacting pairs in the initial subchannel α, the final subchannel β and the intermediate subchannel γ, respectively.

The amplitude $T^{\nu}_{\mu\lambda}$ is written as,

$$T^{\nu}_{\mu\lambda}=\frac{1}{|D_f D_i|^3}\phi^-_{\alpha}(\boldsymbol{q}''_{\lambda})\int d\boldsymbol{q}'_{\nu}\phi_{\beta}(\boldsymbol{p}'_{\mu})<\boldsymbol{p}'_{\nu}|T_{\nu}(z-s'^2_{\nu}-q'^2_{\nu})|\boldsymbol{q}''_{\nu}>\phi_{\alpha}(\boldsymbol{q}'') \tag{11}$$

where

$$\left.\begin{aligned}
\boldsymbol{q}''_{\lambda}&=(C^{\alpha\gamma}_{qp}D_f\boldsymbol{s}_{\lambda}-C^{\beta\gamma}_{qp}D_5\boldsymbol{s}_{\mu})/C^{\alpha\gamma}_{sp}D_f,\\
\boldsymbol{p}'_{\mu}&=(-D_7\boldsymbol{s}_{\mu}+D_8\boldsymbol{q}_{\mu}+(C^{\beta\gamma}_{pq}D_f-C^{\beta\gamma}_{pp}D_1)\boldsymbol{q}'_{\nu})/D_f,\\
\boldsymbol{p}'_{\nu}&=(C^{\beta\gamma}_{qs}\boldsymbol{s}_{\mu}+C^{\beta\gamma}_{ss}\boldsymbol{q}_{\mu}-D_1\boldsymbol{q}'_{\nu})/D_f,\\
\boldsymbol{s}'_{\nu}&=C^{\beta\gamma}_{qp}\boldsymbol{s}_{\mu}/D_f,\\
\boldsymbol{p}''_{\nu}&=(D_f\boldsymbol{s}_{\lambda}-C^{\alpha\gamma}_{ss}C^{\beta\gamma}_{qp}\boldsymbol{s}_{\mu}-C^{\alpha\gamma}_{sq}D_f\boldsymbol{q}'_{\nu})/C^{\alpha\gamma}_{sp}D_f,\\
\boldsymbol{p}''_{\lambda}&=(C^{\alpha\gamma}_{pp}D_f\boldsymbol{s}_{\lambda}-C^{\alpha\gamma}_{qp}D_3\boldsymbol{s}_{\mu}-D_4D_f\boldsymbol{q}'_{\nu})/C^{\alpha\nu}_{sp}D_f
\end{aligned}\right\} \tag{12}$$

with

$$\begin{aligned}
D_i&=C^{\alpha\gamma}_{sp},\\
D_f&=C^{\beta\gamma}_{ss}C^{\beta\gamma}_{qq}-C^{\beta\gamma}_{qs}C^{\beta\gamma}_{sp},\\
D_1&=C^{\beta\gamma}_{qq}C^{\beta\gamma}_{ss},\\
D_3&=C^{\beta\gamma}_{pp}C^{\beta\gamma}_{ss},\\
D_4&=C^{\alpha\gamma}_{pp}C^{\alpha\gamma}_{sq}-C^{\alpha\gamma}_{sp}C^{\alpha\gamma}_{pq},\\
D_5&=C^{\alpha\gamma}_{qp}C^{\alpha\gamma}_{ss}-C^{\alpha\gamma}_{sp}C^{\alpha\gamma}_{qs},\\
D_7&=C^{\beta\gamma}_{pp}C^{\beta\gamma}_{qs},\\
D_8&=C^{\beta\gamma}_{pp}C^{\beta\gamma}_{ss}.
\end{aligned}$$

The momentum s_{μ} is chosen to relate with the momentum s'_{ν} as,

$$s^2_{\mu}=s'^2_{\nu}$$

The $<\boldsymbol{p}'|T_{\nu}|\boldsymbol{p}''>$ in Eq. (11) is an off energy amplitude of nucleon-nucleon scattering.

5. Nucleon-Nucleon Scattering Amplitude

For nucleon-nuclcon scattering amplitude, the separable potential is taken as

$$V(\boldsymbol{p}',\ \boldsymbol{p}'')=-\lambda g(\boldsymbol{p}')g(\boldsymbol{p}'')$$

and S-wave amplitude is given as

$$<\boldsymbol{p}'|T(\varepsilon)|\boldsymbol{p}''>=g(\boldsymbol{p}')\tau(\varepsilon)g(\boldsymbol{p}'') \tag{13}$$

with

$$\begin{aligned}
g(\boldsymbol{p})&=\frac{N}{\beta^2+p^2},\\
\tau(\varepsilon)&=\frac{J(\sqrt{\varepsilon})}{\kappa+i\sqrt{\varepsilon}},\\
J(\sqrt{\varepsilon})&=-\frac{(\beta-i\sqrt{\varepsilon})^2}{\kappa(\beta+\kappa)(2\beta+\kappa-i\sqrt{\varepsilon})}.
\end{aligned}$$

These definitions equal to those of Ebedhöh[1] and values of parameters β, κ and N are given in Table III.

Table Ⅲ Parameters for nucleon-nucleon scattering.

	β(MeV$^{1/2}$)	κ(MeV$^{1/2}$)	N(MeV$^{5/4}$)
triplet n-p	9.113	1.4910	40.630
singlet p-p	7.725	−0.7160	13.891 i
n-p	7.576	−0.2581	8.811 i
n-n	7.203	−0.3332	8.879 i

6. Singularity of τ

Assuming the separable potential, the transition amplitude of Eq. (11) can be written as

$$T^{\nu}_{\mu\lambda}=\frac{1}{|D_f D_i|^3}\phi^{-}_{\alpha}(q''_{\lambda})\int q'^{2}_{\nu}dq'_{\nu}\tau_{\nu}(z-s^{2}_{\mu}-q'^{2}_{\nu})$$
$$\times\int d\Omega_{q'_{\nu}}\phi_{\beta}(p'_{\mu})g(p'_{\nu})g(p''_{\nu})\phi_{\alpha}(p''_{\lambda}). \quad (14)$$

The τ has a pole at the energy of the n-p triplet bound state ($\varepsilon=z-s^{2}_{\mu}-q'^{2}_{\nu}=-\kappa^2$, $\kappa>0$). Then the q'_{ν} integral in Eq. (14) can be separated to the residue of the pole and the principal integral with the aid of the formal relation as

$$\frac{1}{q^2-q^2_0-i\varepsilon}=P\left(\frac{1}{q^2-{}^2_0q}\right)+i\pi\delta(q^2-q^2_0). \quad (15)$$

The principal integral can be evaluate numerically using a standard technique dividing the integration area to two regions on both sides of the pole. The angular integral in Eq. (14) has no any singularity, then the integration can be evaluate without any trouble numerically.

III. CALCULATED ENERGY SPECTRA AND DISCUSSIONS

Figure 1 shows the calculated energy spectra of protons emitted from the reaction $^2H(d,\,pd)n$ at 60 MeV. The proton angle is fixed at 20° and the deuteron angle are chosen in the opposit side of the proton direction in the range from 2° to 70° in the step of 4°.

Large enhancements are seen at the region for the low energy protons associated with the forward deuterons and also seen at two quasi-free scattering regions corresponding to the target breakup and the projectile breakup reactions. Figure 2 shows the angular dependence of the proton energy at the maximum enhancement. In this figure, the dashed curve shows the kinematical points corresponding to the minimum energy of the neutron. If a three body impulse approximation calculation is valid for this reaction, the enhancements will be expected at energies on the dashed curve. As shown in the figure, the dashed curve splits to two curves avoiding the area NES corresponding to the large enhancement. The enhancement in the NES region is produced from the nucleon-nucleon scattering at negative energy and then if such an enhancement is observed experimentally, it is an evidence that the negative energy scattering is important.

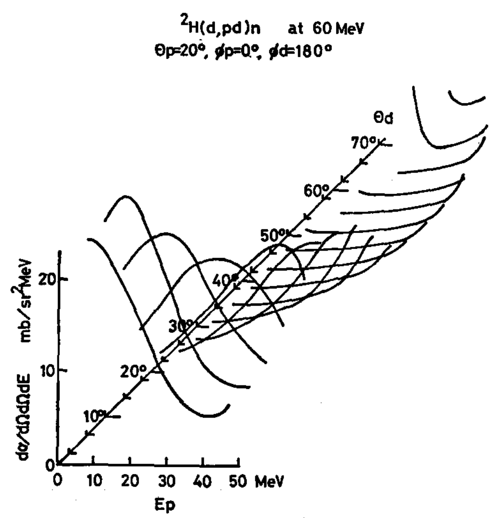

Fig. 1. Breakup cross sections calculated with the four particle model.

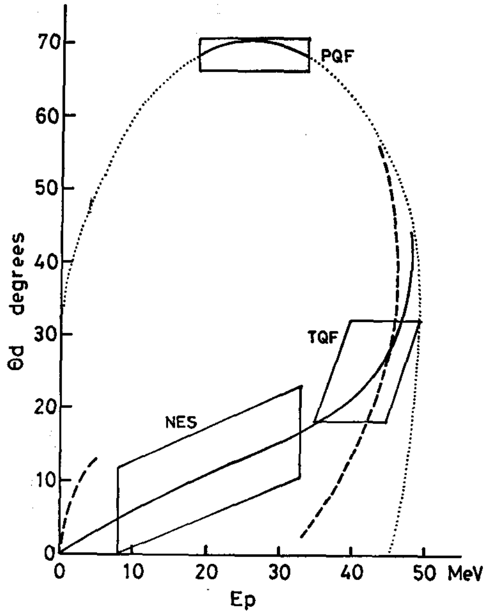

Fig. 2. Proton energies at the cross section maximum as a function of deuteron angle for the $^2H(d, pd)n$ reaction at 60 MeV. The solid curves show calculated maxima and areas PQF, TQF and NES show the regions maingly produced cross sections through the quasi free scattering of the projectile breakup reaction and the target breakup reaction and through the negative energy scattering. The dashed curves show the kinematical points corresponding to the minimum energies of the spectator neutrons.

In conclusion, the large enhancement is predicted at the low energies of protons associated with the forward deuterons. Then it is hoped to measure the breakup cross section in this kinematical region.

Numerical calculations were made using the FACOM 160 AD at the Computer Center of the Institute for Chemical Research.

REFERENCES

(1) W. Ebenhöh, *Nucl. Phys.*, **A191**, 97 (1972).
(2) Y. Koike, *Nucl. Phys.*, **A337**, 23 (1980).
(3) W. Kluge, H. Matthäy, R. Schlüfter, H. Schneider and K. Wick, *Nucl. Phys.*, **A302**, 93 (1978).
(4) K. Fukunaga, S. Kakigi, T. Ohsawa, A. Okihana, S. Tanaka, H. Nakamura-Yokota and S. Kato, *Nucl. Phys.*, **A369**, 289 (1981).
(5) I. H. Sloan, *Phys. Rev.*, **C6**, 1945 (1972).