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Kinematics for Quasi-Free Scattering♯

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Relativistic formulae are given for calculations of the scattering energy and the scattering angle for quasi-free process. The relative energies between the scatteing particles and the pectator are not taken into account in the plane-wave impulse approximation. But they are significant variables in studying multiple scattering effects. Calculation formulae for them are also given.

KEY WORDS: Quasi-free scattering/ Three-particle kinematics/ Relativistic formulae/

I. INTRODUCTION

For reactions in few-nucleon system, the reaction mechanism of quasi-free scattering (QFS) is an important one. The simplest approximation to the theoretical calculation of cross section for the QFS process is the plane-wave impulse approximation (PWIA). Discrepancies are frequently found between differential cross sections observed in the QFS regions and ones calculated in the PWIA. The ratio of the experimental cross section to the PWIA one is referred to as normalization factor and it is a measure of the discrepancy. As a cause of it, one can take a multiple scattering effect, which seems to vary with the kinetic energies of the relative motions between the scattering particles and the spectator in the QFS process. Then it is worthwhile to investigate the systematics about the variation of normalization factor taking account of the kinetic energies mentioned above other than the ordinary variables in the PWIA calculation. From this point of view, kinematic considerations are made for the QFS process.

II. KINEMATICAL CONDITIONS

Consider the reaction $a+b \rightarrow 1+2+3$, where particle a is the projectile and particle b the target. Suppose that the target b consists of two particles 2 and 3 being bounded and the projectile a is scattered on the particle 2. The particle 3 is supposed to be a spectator which continues its initial motion. The projectile a

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is referred to as the particle 1 in the final state. Before the collision, the projectile a has a momentum p_a and the particle 2 a momentum q, which is assumed to be equal to the momentum p_3 of the particle 3. For the notations in the followings see Ref. 1).

In the PWIA, the differential cross section ($\mathrm{d}^5\sigma/\mathrm{d}\Omega_1\mathrm{d}\Omega_2\mathrm{d}T_1$) for the reaction, measured in the laboratory system (LS), is expressed as the product of three terms; the momentum distribution $|\phi(q)|^2$, the differential cross section ($\mathrm{d}\sigma/\mathrm{d}\Omega$) for the two-particle scattering and the kinematic factor. The cross section ($\mathrm{d}\sigma/\mathrm{d}\Omega$) is measured in the rest frame of the (1–2) system (R12) and is a function of the scattering energy and the scattering angle in the R12.

The angles (θ_1, ϕ_1) and (θ_2, ϕ_2) detecting the particles 1 and 2 are selected so that the point $p_3=0$ or $p_3=$minimum (of moderate nonzero value) can be included in measurement regions of the energies T_1 and T_2. Kinematical conditions for these experiments can be divided into four cases. Hereafter, it is supposed the particles 1 and 2 are identical ($m_1=m_2$). In Case (A), the detection angles are coplanar symmetric ($\theta_1=\theta_2$, $\phi_2-\phi_1=180°$) and values of the angles are determined so that the point $p_3=0$ can be included in measurement regions of T_1 and T_2. With maintaining the geometry, the incident energy is varied. As a result, the scattering energy varies with the incident energy and the scattering angle is fixed at 90° for the point $p_3=0$. In Case (B), the detection angles θ_1 and θ_2 are coplanar and are varied in a pair at an incident energy. Values of the paired angles (θ_1, θ_2) are determined to include the point $p_3=0$ in measurement regions of T_1 and T_2. As a result, the scattering angle varies and the scattering energy is constant for the point $p_3=0$. In Case (C), the detection angles θ_1 and θ_2 are coplanar symmetric and varied at an incident energy. The symmetric point $p_1=p_2$ corresponds to $p_3=$minimum (or maximum). For the point, the scattering angle is fixed at 90° and the scattering energy varies with the detection angle. In Case (D), one detection angle θ_1 is fixed and another θ_2 is varied. Usually, this geometry is used in measurements of angular correlations for studying sequential decay process. Since all the kinematic quantities describing the QFS process vary with the detection angle θ_2, this geometry is suitable for looking the reaction as a whole in the initial stage of investigation.

From the experimental data for Cases (A) and (C), one can obtain informations about the scattering energy dependence of the two-particle scattering cross section in the QFS process, at the scattering angle of 90°. For Case (A), these informations are independent of the shape of the momentum distribution. For Case (C), however, it should be known exactly.

III. CALCULATION FORMULAE

1. Scattering Energy and Angle for QFS

For the (1–2) QFS process, the scattering kinetic energy T_{1-2} in the final state prescription is defined as the sum of the kinetic energies of the particles 1 and 2 in the R12 and is given by

$$T_{1-2}=T_1^{R12}+T_2^{R12}=M_{12}-(m_1+m_2), \tag{1}$$

where

$$M_{12}^2 = m_1^2 + m_2^2 + 2E_1E_2 - 2p_1p_2 \cos\theta_{1-2}, \tag{2}$$
$$M_{12}^2 = M_0^2 + m_3^2 - 2E_0E_3 + 2p_0p_3 \cos\theta_3. \tag{3}$$

The scattering angle is defined as the angle $\theta_{a-1}{}^{R12}$ between the momenta of the projectile a and the particle 1 in the R12 and is given by

$$\cos\theta_{a-1}{}^{R12} = (E_a{}^{R12}E_1{}^{R12} - E_aE_1 + p_ap_1\cos\theta_1)/(p_a{}^{R12}p_1{}^{R12}), \tag{4}$$

where

$$E_a{}^{R12} = (M_{12}^2 + m_a^2 - t_{b3})/(2M_{12}), \tag{5}$$
$$E_1{}^{R12} = (M_{12}^2 + m_1^2 - m_2^2)/(2M_{12}), \tag{6}$$
$$p_a{}^{R12} = \lambda^{1/2}(M_{12}^2, m_a^2, t_{b3})/(2M_{12}), \tag{7}$$
$$p_1{}^{R12} = \lambda^{1/2}(M_{12}^2, m_1^2, m_2^2)/(2M_{12}), \tag{8}$$
$$t_{b3} = m_b^2 + m_3^2 - 2m_bE_3. \tag{9}$$

On the assumptions in Chapter II, the rest masses m_i of the participating particles are related as follows,

$$m_a = m_1, \tag{10}$$
$$m_b = m_2 + m_3 + Q, \tag{11}$$

where $Q<0$ and corresponds to the reaction Q-value.

2. Case $\boldsymbol{p}_3=0$

For the point $p_3=0$, the energy-momentum conservation equations are written as

$$E_0 - m_3 = E_1 + E_2, \tag{12}$$
$$\boldsymbol{p}_0 = \boldsymbol{p}_1 + \boldsymbol{p}_2. \tag{13}$$

Then the kinematics for two-particle reaction can be applied to solving $\boldsymbol{p}_1$ as a function of θ_1 and ϕ_1.[1] On that occation, the CMS for two-particle reaction should be replaced by the R12 for three-particle reaction and the target mass by $m_b - m_3$ $(=m_2+Q)$. The direction of velocity $\boldsymbol{v}_{12}$ of the R12 in the LS coincides with the direction of $\boldsymbol{p}_a$. The velocity and the Lorentz factor are given by

$$\boldsymbol{v}_{12} = \boldsymbol{p}_{12}/E_{12}, \tag{14}$$
$$\gamma_{12} = E_{12}/M_{12}, \tag{15}$$
$$\gamma_{12}v_{12} = p_{12}/M_{12}, \tag{16}$$

where

$$E_{12} = E_0 - m_3 = m_a + m_b - m_3 + T_a, \tag{17}$$
$$p_{12} = p_0 = p_a, \tag{18}$$
$$M_{12}^2 = M_0^2 + m_3^2 - 2E_0m_3 \tag{19}$$
$$= (m_a + m_b - m_3)^2 + 2(m_b - m_3)T_a. \tag{20}$$

The solution p_1 is given by

$$p_1{}^{\pm} = p_1{}^{R12}(B_1 \pm \sqrt{D_1})/A_1, \tag{21}$$

where

$$A_1 = \gamma_{12}(1 - v_{12}^2\cos^2\theta_1), \tag{22}$$
$$B_1 = g_1{}^{R12}\cos\theta_1, \tag{23}$$
$$D_1 = \gamma_{12}^2(1 - (g_1{}^{R12})^2) + (g_1{}^{R12})^2(\gamma_{12}/\gamma_1{}^{R12})^2\cos^2\theta_1, \tag{24}$$
$$g_1{}^{R12} = v_{12}/v_1{}^{R12}, \tag{25}$$
$$v_1{}^{R12} = p_1{}^{R12}/E_1{}^{R12}, \tag{26}$$
$$\gamma_1{}^{R12} = E_1{}^{R12}/m_1. \tag{27}$$

The energy E_1 is calculated from the momentum p_1 and the energy E_2 through Eq.

(12).

The relations between the angles (θ_1, θ_2) and $(\theta_{a-1}{}^{R12}, \theta_{a-2}{}^{R12})$ are obtained through the Lorentz transformation

$$\sin\theta_{a-1}{}^{R12}=p_1\sin\theta_1/p_1{}^{R12}, \tag{28}$$
$$\cos\theta_{a-1}{}^{R12}=(\gamma_{12}p_1\cos\theta_1-\gamma_{12}v_{12}E_1)/p_1{}^{R12} \tag{29}$$
$$\sin\theta_1=p_1{}^{R12}\sin\theta_{a-1}{}^{R12}/p_1, \tag{30}$$
$$\cos\theta_1=(\gamma_{12}p_1{}^{R12}\cos\theta_{a-1}{}^{R12}+\gamma_{12}v_{12}E_1{}^{R12})p_1, \tag{31}$$
$$\sin\theta_2=p_1{}^{R12}\sin\theta_{a-1}{}^{R12}/p_2, \tag{32}$$
$$\cos\theta_2=(-\gamma_{13}p_1{}^{R12}\cos\theta_{a-1}{}^{R12}+\gamma_{12}v_{12}E_2{}^{R12})/p_2, \tag{33}$$

where

$$\theta_{a-2}{}^{R12}=180°-\theta_{a-1}{}^{R12}. \tag{34}$$

The relative energy T_{1-3}, for example, is obtained from M_{13}, which is given as

$$M_{13}{}^2=(m_1+m_3)^2+2m_3T_1. \tag{35}$$

(i) Case (A)

For the case $m_1=m_2$,

$$E_1{}^{R12}=E_2{}^{R12}=M_{12}/2, \tag{36}$$

and if the coplanar symmetric geometry $(\theta_1=\theta_2,\ \phi_2-\phi_1=180°)$ is taken, the following relations are obtained;

$$p_1=p_2, \tag{37}$$
$$\theta_{a-1}{}^{R12}=90°. \tag{38}$$

The values of $p_1=p_2$ and $\theta_1=\theta_2$ for $p_3=0$ are obtained through the energy-momentum conservation equations as

$$E_0=2E_1+m_3, \tag{39}$$
$$p_0=2p_1\cos\theta_1, \tag{40}$$

and the results are as follows,

$$E_1=(E_0-m_3)/2, \tag{41}$$
$$T_1=(T_a+Q)/2, \tag{42}$$
$$\cos\theta_1=p_0/(2p_1), \tag{43}$$

where

$$p_0{}^2=(2m_a+T_a)T_a, \tag{44}$$
$$(2p_1)^2=(E_0-m_3)^2-4m_1{}^2, \tag{45}$$
$$=(4m_1+T_a+Q)(T_a+Q). \tag{46}$$

(ii) Case (B)

If θ_1 is given, one can calculate p_1 through Eq. (21) and then $\theta_{a-1}{}^{R12}$ through Eq. (28) and (29) and finally θ_2 through Eqs. (32) and (33). If $\theta_{a-1}{}^{R12}$ is given, one can calculate θ_1 and θ_2 through eqs. (30)–(33) and then p_1 through Eq. (21).

3. Coplanar Symmetric Geometry

Consider the case $m_1=m_2$ in the coplanar symmetric geometry $(\theta_1=\theta_2,\ \phi_1-\phi_2=180°)$. For the point $p_1=p_2$, the (1–2) system is symmetric with respect to the z-axis and therefore θ_3, $\theta_3{}^*=0°$ or $180°$. Giving a value of $p_3{}^*$ for each direction $\theta_3{}^*=0°$ or $180°$, one can calculate the quantities p_3, θ_3, $p_1=p_2$ and $\theta_1=\theta_2$. The value of $p_3{}^*$ is allowed between its minimum and maximum values and they are given as

$$(p_3{}^*)_{\min}=0, \tag{47}$$
$$(p_3{}^*)_{\max}=\lambda^{1/2}(M_0{}^2, 4m_1{}^2, m_3{}^2)/(2M_0), \tag{48}$$

where the latter corresponds to the minimum value of M_{12}, that is, $2m_1$.

The angle θ_3 of particle 3 is given by

$$\sin\theta_3 = p_3^* \sin\theta_3^*/p_3, \tag{49}$$

$$\cos\theta_3 = (\gamma_0 p_3^* \cos\theta_3 + \gamma_0 v_0 E_3^*)/p_3, \tag{50}$$

where

$$\gamma_0 = E_0/M_0, \tag{51}$$

$$\gamma_0 v_0 = p_0/M_0, \tag{52}$$

$$E_0 = m_a + m_b + T_a, \tag{53}$$

$$p_0 = p_a, \tag{54}$$

$$M_0^2 = (m_a + m_b)^2 + 2m_b T_a. \tag{55}$$

Since $\sin\theta_3^* = 0$, then $\sin\theta_3 = 0$. The sign of Eq. (50) decides between two values of θ_3, 0° or 180°.

Since $|\cos\theta_3| = 1$, the momentum p_3 is given with taking the absolute value of Eq. (50) as

$$p_3 = |\gamma_0 p_3^* \cos\theta_3^* + \gamma_0 v_0 E_3^*|, \tag{56}$$

$$= p_3^* \gamma_0 |\cos\theta_3^* + v_0/v_3^*|. \tag{57}$$

For the particle 1, the energy E_1 and the angle θ_1 are obtained from both p_3 and $\cos\theta_3$ $(=\pm 1)$ through the energy-momentum conservation equations as

$$E_0 = 2E_1 + E_3, \tag{58}$$

$$p_0 = 2p_1 \cos\theta_1 + p_3 \cos\theta_3. \tag{59}$$

Then

$$E_1 = (E_0 - E_3)/2, \tag{60}$$

$$T_1 = (T_a + Q - T_3)/2, \tag{61}$$

$$\cos\theta_1 = (p_0 - p_3 \cos\theta_3)/(2p_1). \tag{62}$$

The relative energies T_{1-2} and $T_{1-3} = T_{2-3}$ are obtained from M_{12} and $M_{13} = M_{23}$, respectively, which are given as

$$M_{12}^2 = 4m_1^2 + 4p_1^2 \sin^2\theta_1, \tag{63}$$

$$M_{13}^2 = (M_0^2 + 2m_1^2 + m_3^2 - M_{12}^2)/2. \tag{64}$$

IV. KINEMATIC CONSIDERATIONS

Consider the ^{3}He(p, pp)^{2}H reaction, one of the reactions for four-nucleon system. The detected particles are identical and then the coplanar symmetric geometry is worth-while especially for this reaction. Calculations were done for an incident energy of 65 MeV, as an example, using a programme of three-particle relativistic kinematics.[1]

Figure 1 shows the scattering energy T_{1-2}, the scattering angle θ_{a-1}^{R12}, the energy T_3 of the spectator and two relative kinetic energies T_{1-3} and T_{2-3} as functions of T_1 at the coplanar symmetric angle pair $\theta_1 = \theta_2 = 41.8°$. The energy T_3 is zero at $T_1 = 29.8$ MeV and, on both sides of this point, it increases rapidly. On the other hand, the scattering energy T_{1-2} and the scattering angle θ_{a-1}^{R12} vary showly in the region. Therefore informations about the momentum distribution can be obtained from examinations of the shape of energy-sharing spectrum.

For Case (A), kinematic quantities for the point $T_3 = 0$ are shown in Fig. 2(a) as functions of T_a. For Case (C), kinematic quantities for the point $T_1 = T_2$ are

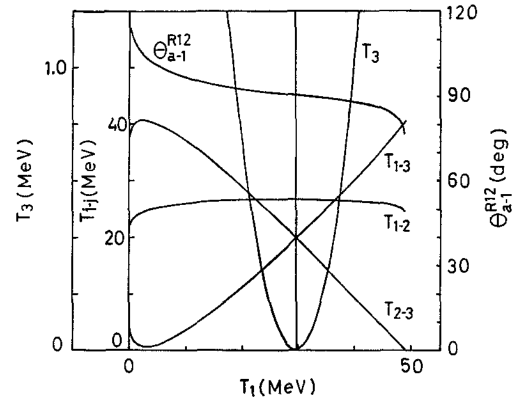

Fig. 1. Kinetic energy T_3, scattering energy T_{1-2}, scattering angle $\theta_{a-1}{}^{R12}$, and relative energies T_{1-3} and T_{2-3} as functions of the kinetic energy T_1 at a coplanar symmetric angle pair $\theta_1=\theta_2=41.8°$ for the $^3He(p, pp)^2H$ reaction at 65 MeV.

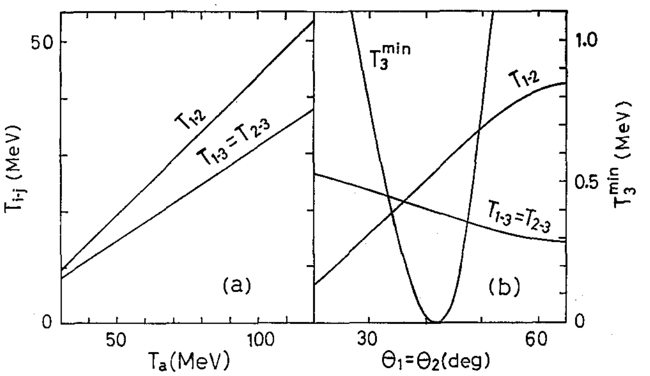

Fig. 2. Scattering energy T_{1-2}, and relative energies T_{1-3} and T_{2-3} as functions of the incident energy T_a for the point $T_3=0$ (a) and as functions of the angle θ_1 for the point T_3=minimum (b) in the coplanar symmetric geometry for the ^3He $(p, pp)^2H$ reaction at 65 MeV.

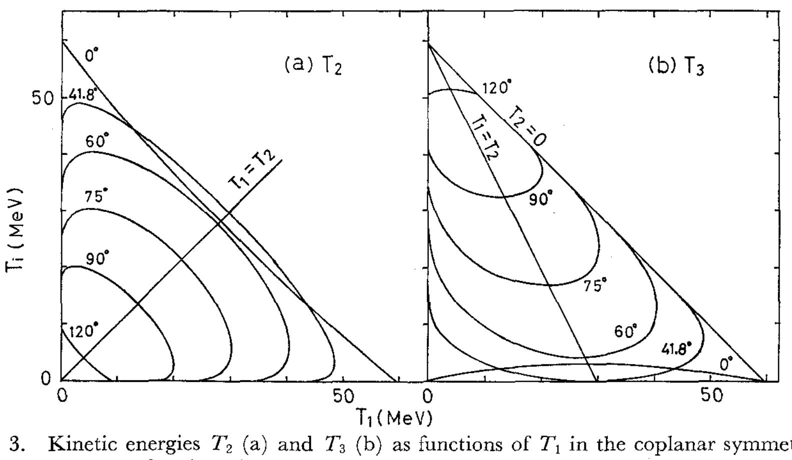

Fig. 3. Kinetic energies T_2 (a) and T_3 (b) as functions of T_1 in the coplanar symmetric geometry for the $^3He(p, pp)^2H$ reaction at 65 MeV. The number adjacent to each curve is the value of the angle $\theta_1(=\theta_2)$.

shown in Fig. 2(b) as functions of $\theta_1=\theta_2$ in the vicinity of the point $T_3=0$. As described in Chapter **I**, multiple scattering effects are not taken into account in the PWIA and if these effects exist, the normalization factor should be no longer constant. This factor seems to vary as a function of T_{1-3} and T_{2-3}.[2] In Case (A), the energies $T_{1-3}=T_{2-3}$ increase together with the scattering energy T_{1-2} as the incident energy T_a increases. In Case (C), near the point $T_3=0$, the energies $T_{1-3}=T_{2-3}$ decrease as the scattering energy T_{1-2} increases. Then, multiple scattering effects and resultant variations of the normalization factor should appear in different way in the experiments in Cases (A) and (C).

Finally, for Case (C), are shown the energies T_2 and T_3, the relative energies T_{1-2} and T_{2-3}, and the angles θ_3 and $\theta_{a-1}{}^{R12}$ as functions of T_1 for several values of

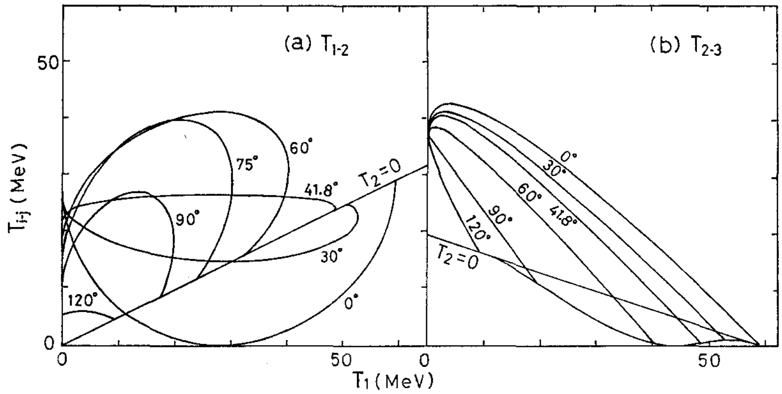

Fig. 4. Scattering energy T_{1-2} (a) and relative energy T_{2-3} as functions of T_1, similar to Fig. 3.

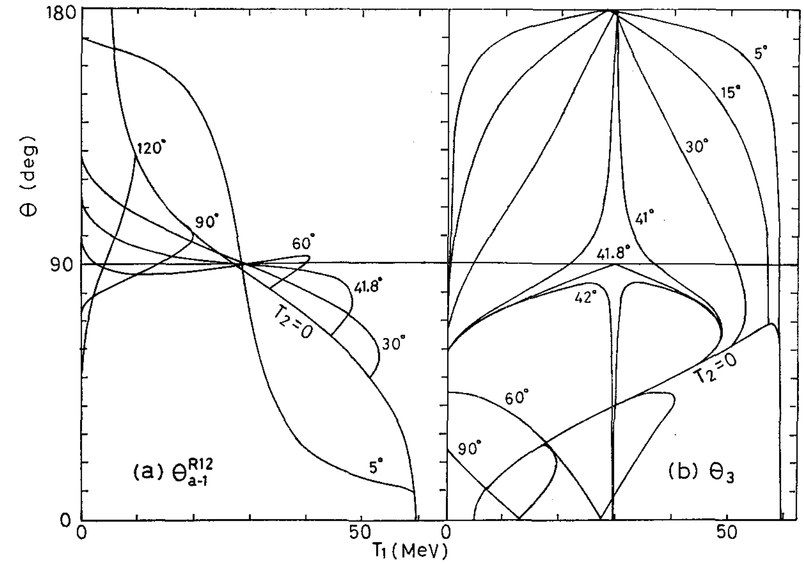

Fig. 5. Scattering angle $\theta_{a-1}{}^{R12}$ (a) and angle θ_3 (b) as functions of T_1, similar to Fig. 3.

$\theta_1 = \theta_2$ in Figs. 3–5.

The numerical calculations were made with FACOM 160M computer at Institute for Chemical Research of Kyoto University.

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