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# Elastic Scattering in Few Nucleon System

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A two body transition amplitude in nuclear reaction is obtained analytically in a Born approximation on the basis of the Faddeev three body theory. Elastic scattering cross sections for $^2H(p, p)^2H$ and $^3He(p, p)^3He$ are calculated and good fits are obtained in comparison with the experimental data.

## I. INTRODUCTION

Faddeev[1] succeeded for the first time in giving a mathematically correct theory of nonrelativistic three particle system. In the case of breakup reaction the Faddeev equation was solved exactly using separable potentials.[2] The incident energy dependence and the angular dependence of the cross section were obtained analytically in the second Born approximation by our group.[3]

After the Faddeev's work, Sloan[4] studied the expanded Faddeev equation in the case of four-particle system. But it is difficult to solve exactly this Faddeev-like four-body equation. Then we atempt to calculate the cross section for $^3H(p, p)^3He$ in a Born approximation on a three body model.

## II. TRANSITION AMPLITUDE IN THREE BODY SYSTEM

In this section we derive the transition amplitude for two body reaction on the basis of the exact three body theory. For the three body system the transition operator $U_{\beta\alpha}$ from the initial channel $\alpha$ to the final channel $\beta$ is given by Alt et al.[5] as,

$$U_{\beta\alpha} = -(1-\delta_{\beta\alpha})(H_0-Z) - \sum_{\gamma\neq\alpha} U_{\beta\gamma} G_0 T_\gamma , \tag{1}$$

and the amplitude for two body reaction can be written as a series of multiple scattering processes as,

$$\begin{aligned}\langle\beta\boldsymbol{q}_f|G_0 U_{\beta\alpha} G_0|\alpha\boldsymbol{q}_i\rangle &= -(1-\delta_{\beta\alpha})\langle\beta\boldsymbol{q}_f|G_0|\alpha\boldsymbol{q}_1\rangle - \sum_{\gamma\neq\alpha}\langle\beta\boldsymbol{q}_f|G_0 U_{\beta\gamma} G_0 T_\gamma G_0|\alpha\boldsymbol{q}_i\rangle \\ &= -(1-\delta_{\beta\alpha})\, g_\beta(\overline{\boldsymbol{p}}_\beta)\, \Phi_\alpha(\overline{\boldsymbol{p}}_\alpha)\frac{1}{|C_{qp}^{\beta\alpha}|^3} \\ &\quad + \sum_{\beta\neq\gamma\neq\alpha}\int \Phi_\beta(\boldsymbol{p}_\beta')\, d\boldsymbol{p}_\beta' \langle\boldsymbol{p}_\gamma'|t_\gamma(z-q_\gamma'^2)|\boldsymbol{p}_\gamma''\rangle\, \Phi_\alpha(\boldsymbol{p}_\alpha'')\frac{1}{|C_{qp}^{\gamma\alpha}|^3} - \cdots ,\end{aligned} \tag{2}$$

where

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$$\bar{\boldsymbol{p}}_\beta = \frac{1}{C^{\beta\alpha}_{qp}} (C^{\beta\alpha}_{qq} \boldsymbol{q}_f - \boldsymbol{q}_i) ,$$

$$\bar{\boldsymbol{p}}_\alpha = \frac{1}{C^{\beta\alpha}_{qp}} (\boldsymbol{q}_f - C^{\beta\alpha}_{qq} \boldsymbol{q}_i) .$$

The notation $C^{\beta\alpha}_{pp}$, $C^{\beta\alpha}_{pq}$, $\cdots$ and $\boldsymbol{p}_\beta$, $\boldsymbol{q}_\beta$, $\cdots$ are same as those in ref. 3. The momentum distribution function $\Phi_\alpha(\boldsymbol{p}_\alpha)$ in the initial channel $\alpha$ can be written with the form factor $g_\alpha(\boldsymbol{p}_\alpha)=N_\alpha/(\beta^2_\alpha+p^2_\alpha)$ and the binding energy $\kappa_\alpha$ as $\Phi_\alpha(\boldsymbol{p}_\alpha)=g_\alpha(\boldsymbol{p}_\alpha)/(\kappa^2_\alpha+p^2_\alpha)$. The factor $\langle \boldsymbol{p} | t_\gamma(\varepsilon) | \boldsymbol{p}' \rangle$ is the two body transition amplitude for $\gamma$ pair with the energy $\varepsilon$ and $\boldsymbol{p}'$ and $\boldsymbol{p}$ are the initial and final momenta, respectively. The momentum distribution $\Phi_\alpha(\boldsymbol{p}_\alpha)$ has the maximun at $\boldsymbol{p}_\alpha=0$. This fact is true in the case of $S$ shell nuclei. Then the most probable values of $\boldsymbol{p}'_\beta$, $\boldsymbol{p}'_\gamma$, $\cdots$ in Eq. 2 can be estimated as,

$$\tilde{\boldsymbol{p}}'_\beta = \frac{1}{C^{\gamma\beta}_{qp}} (C^{\gamma\alpha}_{qq} \boldsymbol{q}_i - C^{\gamma\beta}_{qq} \boldsymbol{q}_f) ,$$

$$\tilde{\boldsymbol{p}}'_\gamma = \frac{1}{C^{\gamma\beta}_{qp}} (C^{\gamma\beta}_{pp} C^{\gamma\alpha}_{qq} \boldsymbol{q}_i - \boldsymbol{q}_f) ,$$

$$\tilde{\boldsymbol{p}}''_\gamma = \frac{1}{C^{\gamma\alpha}_{qp}} \boldsymbol{q}_i ,$$

$$\tilde{\boldsymbol{q}}'_\gamma = C^{\gamma\alpha}_{qq} \boldsymbol{q}_i .$$

Neglecting multiple processes the transition amplitude can be written for the two body reaction from channel $\alpha$ to channel $\beta$ as,

$$\langle \beta \boldsymbol{q}_f | G_0 U_{\beta\alpha} G_0 | \alpha \boldsymbol{q}_i \rangle = -(1-\delta_{\beta\alpha})\, g_\beta(\bar{\boldsymbol{p}}_\beta)\, \Phi_\alpha(\bar{\boldsymbol{p}}_\alpha) \frac{1}{|C^{\beta\alpha}_{qp}|^3} + \sum_{\beta\neq\gamma\neq\alpha} \eta_{\beta\gamma\alpha} \langle \tilde{\boldsymbol{p}}'_\gamma | t_\gamma(z-\tilde{q}'^2_\gamma) | \tilde{\boldsymbol{p}}''_\gamma \rangle \frac{1}{|C^{\gamma\alpha}_{qp}|^3} , \tag{3}$$

where $\eta_{\beta\gamma\alpha}$ is a new factor defined as,

$$\eta_{\beta\gamma\alpha} = \int \Phi_\beta(\boldsymbol{p}'_\beta)\, \Phi_\alpha(\boldsymbol{p}''_\alpha)\, d\boldsymbol{p}'_\beta = 2\pi^2 g_\beta(i\kappa_\beta)\, g_\alpha(i\kappa_\alpha) \left| \frac{C^{\gamma\alpha}_{qp}}{C^{\gamma\beta}_{qp}} \right| \times [I(\kappa_\beta \boldsymbol{q}_f \kappa_\alpha \boldsymbol{q}_i) - I(\kappa_\beta \boldsymbol{q}_f \beta_\alpha \boldsymbol{q}_i) - I(\beta_\beta \boldsymbol{q}_f \kappa_\alpha \boldsymbol{q}_i) + I(\beta_\beta \boldsymbol{q}_f \beta_\alpha \boldsymbol{q}_i)] , \tag{4}$$

$$I(\kappa_\beta \boldsymbol{q}_f \kappa_\alpha \boldsymbol{q}_i) = \frac{|C^{\gamma\alpha}_{qp}|}{\sqrt{C^{\gamma\alpha 2}_{qp} \kappa^2_\alpha + (C^{\gamma\beta}_{qq} \boldsymbol{q}_f - C^{\gamma\alpha}_{qq} \boldsymbol{q}_i)^2} + |C^{\gamma\beta}_{qp} \kappa_\beta|} . \tag{5}$$

In the case of elastic and inelastic scatterings, the final channel $\beta$ is equal to the initial channel $\alpha$ and the first term of Eq. 3 disappears. Thus the transition amplitude is reduced to the amplitude for the subsystem multiplied by the factor $\eta$. Using above the transition amplitude the two body cross section is easily obtained as,

$$\frac{d\sigma}{d\Omega} = (2\pi)^4 \gamma^{-1} \frac{q_f}{q_i} \frac{(m_P\, m_T)^2 (m_P + m_T)}{(4 m_A\, m_B\, m_C)^3} |\langle \beta \boldsymbol{q}_f | G_0 U_{\beta\alpha} G_0 | \alpha \boldsymbol{q}_i \rangle|^2 , \tag{6}$$

$$\gamma = \frac{m}{\hbar^2} \doteqdot \frac{1}{41.6\ \mathrm{MeV}\cdot fm^2} ,$$

where $m_P$, $m_T$, $m_A$, $m_B$ and $m_C$ are masses of the projectile, of the target nucleus, of the detected particle A, of the particle B and C, respectively. The transition amplitude $|\langle \beta \boldsymbol{q}_f | G_0 U_{\beta\alpha} G_0 | \alpha \boldsymbol{q}_i \rangle|^2$ is measured in units of $\mathrm{MeV}^{-1}$, masses in atomic mass units and energies $q_i^2$ and $q_f^2$ in MeV.

Now spins of particles are taken into accounts in the calculation for the transition amplitude. The first and second terms in Eq. 3 must be modefied by spin factor $S_{\beta\alpha}$ and $S_{\beta\gamma_\alpha}$, respectively. For example $S_{\beta\alpha}$ can be calculated by the overapping of two spin functions in the channel $\beta$ and the channel $\alpha$ as,

$$\begin{aligned} S_{\beta\alpha} &= \sum_{J_{TOT}} \langle j_\beta j_\gamma j_\alpha (J_\beta) : J_{TOT} | j_\alpha j_\beta j_\gamma (J_\alpha) : J_{TOT} \rangle \\ &= \sum_{J_{TOT}} (-)^{j_\alpha + 2j_\beta + 2j_\gamma - J_\alpha - J_{TOT}} \sqrt{(2J_\beta + 1)(2J_\alpha + 1)}\; W(j_\alpha j_\gamma J_{TOT} j_\beta | J_\beta J_\alpha) , \end{aligned} \tag{7}$$

where $j_\alpha$, $j_\beta$ and $j_\gamma$ are spins of particles $\alpha$, $\beta$ and $r$, and $J_\alpha$ and $J_\beta$ are spins of pair $\alpha$ and $\beta$, respectively. $W$ is a Racah coefficient. The other spin factor can be also calculated easily.

## III. COMPARISON WITH EXPERIMENTAL DATA AND DISCUSSION

3–1) Elastic scattering of protons on deuteron

Elastic scattering of protons on deuteron is the most simple case in the three body scattering and the cross section can be calculated by Eq. 3 and Eq. 6 with the spin factor. In this case two protons $\alpha$ and $\beta$ are contained in the three body system, then the antisymmetrized amplitude is used to calculate the amplitude as,

$$\begin{aligned} &\langle \beta \boldsymbol{q}_f x_f | G_0 U_{fi} G_0 | \alpha \boldsymbol{q}_i x_i \rangle \\ &= \frac{1}{\sqrt{2}} [\langle \alpha \boldsymbol{q}_f x_f | G_0 U_{\alpha\alpha} G_0 | \alpha \boldsymbol{q}_i x_i \rangle - \langle \alpha \boldsymbol{q}_f x_f | G_0 U_{\beta\alpha} G_0 | \alpha \boldsymbol{q}_i x_i \rangle] , \end{aligned} \tag{8}$$

where $x_i$ and $x_f$ are the spins in the initial and the final channel, respectively. The calculated cross section is compared in Fig. 1 with the experimental data which are obtained from ref. 6. The two nucleon scattering amplitude is calculated with the Yamaguchi type S wave separable potential neglecting higher partial wave components. The potential parameters are same as those of Table III in ref. 7. The calculated curve shows a large forward peak, a small backward peak and a deep minimum at 120 degreees. The forward peak comes from the second terms in $U_{\alpha\alpha}$ and $U_{\beta\alpha}$ and the backward peak comes from the first term of the exchange process $U_{\beta\alpha}$ in Eq. 3. Because the Coulomb interaction is neglected in the calculation, the cross sections at the extremely forward angles are overestimated in comparison with the

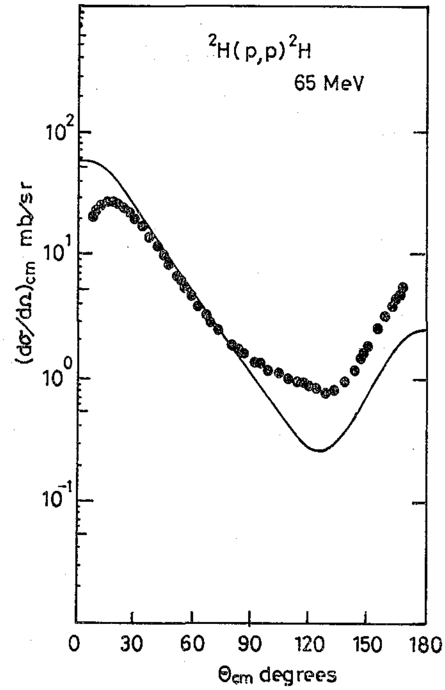


Fig. 1. Differential cross section in elastic *p-d* scattering at 65 MeV. Points are obtained from the experimental data in ref. 6. The curve is calculated analytically using the *S* state separable potential for *N-N* interaction.

experimental data. Furthermore using the first Born approximation the cross sections are underestimated at large angles. Neverthless both the forward and backward rises in the experimental data are well reproduced by the calculation curve.

3–2) Elastic scatterting of protons on helium-3

In this case we assumed that the helium-3 target is composed of one proton and one deuteron. Then the scattering amplitude in Eq. 3 is calculated using the transition amplitude for *p-d* interaction neglecting the *p-p* scattering process. Furthermore the spin factor is neglected. Then the cross section for $^3He(p, p)^3He$ can written by the elastic *p-d* cross section, the factor $|\eta|^2$ and a kinematical factor $K$ as,

$$\left(\frac{d\sigma}{d\Omega}\right)_{p-^3He} = K\cdot|\eta|^2\cdot\left(\frac{d\sigma}{d\Omega}\right)_{p-d}, \tag{9}$$

$$K = \frac{\text{kinematical factor for } p\text{-}^3\text{He}}{\text{kinematical factor for } p\text{-}d} \times \left(\frac{C_{qp}^{\gamma\alpha} \text{ for } p\text{-}d}{C_{qp}^{\gamma\alpha} \text{ for } p\text{-}^3\text{He}}\right)^6 = 1.424$$

In fig. 2 the theoretical curve shows the calculation with Eq. 9. The theoretical curve at forward angles is about 14% smaller than the experimental data.[8] This discrepancy in the absolute cross section may come from neglecting the *p-p* interaction. The angular dependence of the calculated cross section is so simillar to the experimental data.

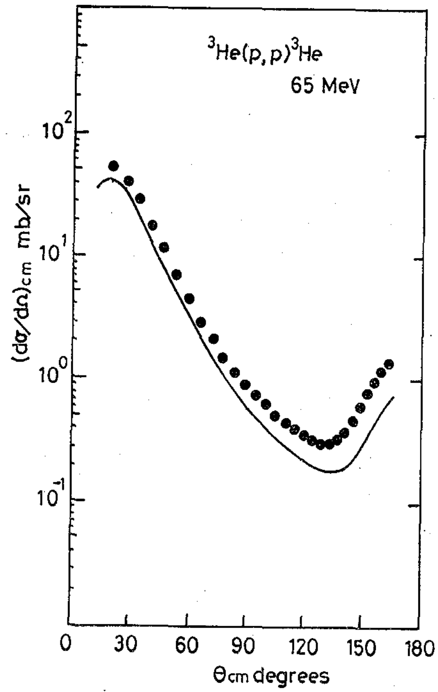


Fig. 2. Differential cross section in elastic $p$-$^3He$ scattering at 65 MeV. Points are obtained from the experimental data in ref. 8. The curve is calculated using Eq. 9 with $|\eta|^2$ and the cross section of elastic $p$-$d$ scattering[6].

In conclusion the simplified transition amplitude is obtained analitically using the Born approximation in the three body theory. The theoretical cross section is proportional to that for the subsystem modefied by the factor $|\eta|^2$, and the theoretical cross section can reproduce the characteristic structure of the experimental data in few nucleon system.

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