Optimization of Wasteload Allocation for River Water Quality Management

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CONTENTS

NOTATIONS

The following symbols are the major symbols used in this thesis.

A	=	cross-sectional area
A_f	=	frequency factor
B	=	channel width
C	=	DO concentration in main stream
$\mathbf{C}$	=	vector of DO concentration in main stream
$\mathbf{C}^L$	=	vector of DO concentration in wastewater
$\mathbf{C}^M$	=	vector of DO concentration at monitoring station
C^L	=	DO concentration in wastewater
C_S	=	saturation value of DO
C_ϕ	=	concentration of water quality index ϕ
C^*	=	upstream boundary value of DO concentration in main stream
C_j^L	=	DO concentration in j-th wastewater
c	=	dissipation parameter
c_k	=	non-dimensional coordinate of k-th integration point
D	=	DO deficit
$D(\mathbf{f})$	=	domination cone or set of domination factors at $\mathbf{f}$
D_x	=	longitudinal dispersion coefficient
D_0	=	DO deficit on upstream boundary in main stream
$D_B^{'}$	=	benthal demand
$\mathbf{d}_f$	=	vector of domination factor
E	=	activation energy in Chapter 2
F_r	=	Froude number
$\overline{F_r^2}$	=	mean value of F_r^2

f_k = k-th objective function
$f(C_\phi)$ = reaction term of water quality index ϕ
f_C^* = dispersive DO flux
f_L^* = dispersive BOD flux
g = gravitational acceleration
h = mean water depth
h^* = downstream boundary value of water depth in main stream
K = reaction rate in Chapter 2/ scenario set in Chapters 5,6,7
K_1 = deoxygenation coefficient
K_2 = reaeration coefficient
K_3 = removal coefficient of BOD by sedimentation and/or absorption
L = BOD concentration in main stream
$\mathbf{L}$ = vector of BOD concentration in main stream
$\mathbf{L}^L$ = vector of BOD concentration in wastewater
$\mathbf{L}^M$ = vector of BOD concentration at monitoring station
L_0, L^* = upstream boundary value of BOD concentration in main stream
L_j^L = BOD concentration in j-th wastewater
N_j = shape function for node j
n = Manning's roughness coefficient
NE = number of element
NL = number of loading point
NN = number of node
P = photosynthesis
P_e = Peclet number
p_s = probability of scenario s
Q = discharge of main stream
Q^* = upstream boundary value of discharge
$\overline{q}$ = lateral discharge per unit width
q_j = total wastewater injected into j-th loading point
R = hydraulic radius

R_g = gas constant

R_P = plant respiration

S = salinity

S_0 = channel slope

S_f = friction slope

T = water temperature

T_a = absolute water temperature

t = travel time of flow

U = velocity

$\mathbf{U}_s^o$ = deviation vector from water quality standard for BOD

$\mathbf{U}_s^{o+}$ = vector of violated deviation for BOD

$\mathbf{U}_s^{o-}$ = vector of surplus deviation for BOD

U_{is}^o = deviation from water quality standard for BOD

$\mathbf{u}_s$ = relaxation vector from water quality standard for BOD

u_{is} = violated deviation from water quality standard for BOD

$\mathbf{V}_s^o$ = deviation vector from water quality standard for DO

$\mathbf{V}_s^{o+}$ = vector of violated deviation for DO

$\mathbf{V}_s^{o-}$ = vector of surplus deviation for DO

V_{is}^o = deviation from water quality standard for DO

$\mathbf{v}_s$ = relaxation vector from water quality standard for DO

v_{is} = violated deviation from water quality standard for DO

W = wind speed

w_j = weighting function

w_k = weighting factor

x = horizontal distance along channel

z_b = elevation of channel bottom above horizontal datum

α = multiobjective weight

β = multiobjective weight

Δx = element length

Γ = scenario set

Γ_d = boundary of domain

Γ_j = j-th boundary of hypothetical river network in Chapters 6 and 7

γ = multiobjective weight

$\epsilon_2, \epsilon_3, \epsilon_4, \hat{\epsilon}_3$ = parameter corresponding to each ϵ-constraint

ζ = velocity-distribution coefficient

$\kappa(i,j)$ = element number of j-th element connected to i-th node

λ = multiobjective weight

λ^*_{1j} = trade-off rate between first objective and j-th objective

$\nu(i)$ = number of elements meeting at i-th node

ξ = velocity-distribution coefficient

Ω = scenario set

ω = multiobjective weight

Superscripts

I = river water except at monitoring station

l = lower limit

M = river water at monitoring station

o = loading point and/or stream junction

u = upper limit

$+$ = violated value

$-$ = surplus value

$*$ = boundary value or optimal value

Subscripts

i = node number

s = scenario

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CHAPTER 1

INTRODUCTION

1.1 Problems on Water Quality Management

Conservation of water quality in water bodies such as rivers, lakes and estuaries is essential for maintaining our good health, improving our quality of life and preserving ecosystems. However water quality deterioration and eutrophication have been appearing with industrial and agricultural development, population growth and advancement of our living conditions. Since the activities of society and conservation of water quality environment are generally in conflict, water quality standards for each standing or flowing water have been settled to manage water quality acceptably. However the specified standards have not been reached yet in many water bodies in the world, in spite of much effort.

Two causes can be considered that make the water pollution control difficult. One is the fact that the demands of wastewater dischargers conflict with the demands of people who use water in the body receiving polluted water. Effluents can be classified into (i) controllable wastewater issued from industrial plants, sewage works and houses, etc., and (ii) difficult-to-control or uncontrollable effluents discharged from agricultural lands, cities and forests, etc. Since all the dischargers in these various types of land use have their own goals on maximizing profits and minimizing costs, total amount of pollutant loadings into the water body resultantly increases. In contrast, water users such as residents near the body of water, fishery and tourism insist restoring water quality environment. Therefore managers of water body should develop plans that are in tune with those contradicting objectives to obtain satisfactory results for both sides.

The other source of difficulty arising in managing water quality is the stochastic nature involved in the physical and biological system being controlled. For example, the decreasing rate of organic pollutants changes due to temporal and spatial variations of water discharge, water temperature and amount of supplied oxygen from exterior, etc.

Thus, effective management plans require accurate predictions of future dynamics of water quality constituents. However, since estimation error or difference between predicted situation and actual one generally arises, there is a risk of incomplete accomplishment of management objectives. Although such a risk could be clarified reactively by the effectiveness evaluation of the implemented management programs, robust plans of water quality remediation to this risk are proactively needed.

Now, the water quality standards and effluent limitations imposed by the Japanese government are reviewed. In Japan, effluent limitations restrict quality of discharged water only from point sources. Additionally, effluent limitations on total amount of COD (Chemical Oxygen Demand) loadings issued from point sources that affect specified closed-type water bodies like gulfs and inner seas are imposed. Besides, there exist water quality standards aiming at both protection of human health and conservation of living environment for all the public waters. With regard to the water quality standards for the latter purpose, controlled water bodies are divided into three classes: 'river', 'lake' and 'sea'. Each class is composed of different numbers (i.e., six, four and three for river, lake and sea, respectively) of subclasses for which limitations on several water quality indices are specified by considering usage of water in the corresponding water bodies. For example, one out of six subclasses (i.e., AA, A, B, C, D and E) is assigned to every river, and daily averaged values of pH, BOD (Biochemical Oxygen Demand), SS (Suspended Solids), DO (Dissolved Oxygen) and coliform group are employed for defining each subclass. However, the attaining rates of the water quality criterion for BOD or COD concentration in 1999 fiscal year are 81.5%, 45.1% and 74.5% in river, lake and sea, respectively [Ministry of Environment(2001)[68]]. Thus more effort should be made to improve water quality, especially in lake.

The problem on the current management policy in Japan is that it is doubtful whether observance of the specified effluent limitations leads to compliance with the water quality standards, because of the vagueness of the relevance between these standards and limitations. In addition, the way of allotting the subclasses on water quality standards to each body of water is not satisfactory, because the local characteristics such as land use and hydrologic conditions in a watershed are not taken into account in creating those subclasses. Furthermore, since uncertainty of self-purification in standing or flowing water is not recognized in the statement of the water quality standards, a measure to evaluate the management risks caused by the uncertainty is necessary.

Considering the above problems related to water quality management, pollution control policies based only on the water quality standards and effluent limitations are not effective. In order to manage water quality rationally and acceptably, harmonizing the abovementioned competing objectives, as well as decreasing the risks pertaining to uncertainty of natural environment, should be achieved. Besides, not only local remediation of water quality but also integrated one for a whole body of water is needed. In this context, a large-scale model-based approach describing both management goals and natural environments quantitatively with probabilistic consideration could be successful, so that the effective strategies for ameliorating water quality can be proposed.

1.2 Necessity of Optimization

Developing simulation models on hydraulics and transport of pollutants is a common approach to produce management plans of water quality. However, social activities affecting water quality should be planned with the concept of optimal management because of the following two reasons. One is the definiteness of environmental capacity [Naito(1987)[71]], and the other is the various conflicting goals of our actions. If the optimization theory is employed, the former can commonly be reflected to constraints in a management model. Certainly, concerning about optimization might not be needed when effects of industrial or agricultural activities on water quality is relatively small compared to the assimilative capacity of the water. However, since these human activities are violating natural environment in reality, effective policies or actions to overcome the deterioration of water quality should be explored by setting some limitations. The latter fact, i.e., the existence of various objectives in conflict, corresponds to several objective functions defined in optimization models. These objectives should be in agreement with each other by discarding inferior solutions, analyzing trade-offs among goals and consulting preference of administrators or decision-makers in water quality management agencies.

Note that optimization models are different from simulation models in that the former can generate management alternatives with a measure of optimality. Actually, both types of models require mathematical modeling of physical phenomena related to water quality. However simulation models are not suited to develop plans of activities in a rational and persuasive way. On the contrary, optimization models can derive noninferior (or nondominated) solutions satisfying both physical law of water quality and artificial standards. Furthermore, since objective criteria are mathematically expressed in optimization models, the decision-making process where management alternatives are evaluated [see Chankong and Haimes(1983)[15] and Djordjević(1993)[22]] could be made explicit, which can lead to highly acceptable final decision.

1.3 Research Aim and Objectives

The aim of this research is to propose rational and acceptable strategies for water quality management by developing optimization models for controlling pollutant loading into river systems. The results obtained from the optimization models could, for example, influence effluent trading [see Nishizawa(2000a,b)[74, 75]] for water quality conservation. Only BOD and DO are considered as water quality indices in this study, as are done by most researchers dealing with the problems of controlling organic pollutants in streams.

The main objectives in this study are:

(1) To develop an optimization model for wasteload allocation, with a prime objective of maximizing total allowable BOD loading into a whole river system subject to both physical rules of BOD and DO transport and artificial regulations such as in-stream water quality standards and effluent limitations.

(2) To formulate a robust optimization model for river water quality management under uncertainty triggered by stochastic changes of hydraulic and environmental variables, so that robust solution in the sense of optimality and feasibility can be obtained.

(3) To modify the robust optimization model by employing the ϵ-constraint method in order to generate noninferior solutions effectively, and to facilitate exploring management alternatives in a multiobjective decision-making process.

1.4 Structure of This Thesis

The remaining portions of this thesis are divided into seven chapters. The subject of each chapter is outlined as follows.

In Chapter 2, the conventional models on BOD and DO transport are reviewed. Streeter-Phelps equations and Camp-Dobbins modification to Streeter-Phelps equations, which have often been employed in the optimization models for water quality management, are described. Formulae to estimate coefficients of those equations are also summarized.

In Chapter 3, a review of literatures related to deterministic, stochastic and fuzzy optimization models for water pollution control is made. Then the framework of robust optimization, multiobjective optimization theory, and multiobjective decision-making process are explained.

In Chapter 4, a deterministic optimization model for water quality management in a river network is presented. The discretization procedure for governing equations are described in detail. The FE (Finite Element) and LP (Linear Programming) method employed in this chapter gives a basis of building optimization models presented in the following chapters.[44]

In Chapter 5, the model developed in the previous chapter is extended to a robust optimization (RO) model, considering uncertain nature of flow and water quality environment.[62]

In Chapter 6, the model shown in the previous chapter is adapted for handling river networks. The way of scenario generation is also revised by introducing upstream boundary values of BOD and DO concentrations to basic uncertain parameters. Additionally, elimination of meaningless constraints reduces the model size and expands its feasible region.[45][61]

In Chapter 7, a modified RO model, ϵ-RO model, is developed where the ϵ-constraint method is adopted to generate noninferior solutions and to obtain trade-off rates among objectives. Minimizing surplus deviations, as well as violated deviations, of imposed in-stream water quality standards is embedded as one of the objective functions in the model.[60]

Finally, summary and conclusions of this study and comments on future works are given in Chapter 8.

CHAPTER 2

WATER QUALITY MODELS

2.1 Introduction

Formulating optimization models for water quality management necessitates modeling in advance the mass balances of water quality constituents. The accuracy of the water quality model embedded in the optimization model can significantly affect the persuasiveness of policies derived. Therefore well-validated water quality model should be incorporated into the optimization model.

Many researches have been done to analyze organic pollution in streams by considering the mass balance of water quality indices, BOD and DO. It is assumed that the dynamics of BOD and DO in a stream can be governed by one-dimensional advection-dispersion differential equations. In Subsections 2.2.1 and 2.2.2, conventionally used Streeter-Phelps equations and Camp-Dobbins modifications to the Streeter-Phelps equations are described, respectively. In Subsection 2.2.3, studies on water quality modeling by other approaches, including the method of discretizing governing equations by numerical scheme such as finite difference method (FDM) or finite element method (FEM), are reviewed. After that, several kinds of formulae for predicting parameters embodied in those water quality models are described in Section 2.3. This chapter ends with remarks in Section 2.4.

2.2 Modeling of BOD and DO Transports

2.2.1 Streeter-Phelps equations

Mass balance of a water quality constituent in rivers or open channels can generally be written in a one-dimensional form

$$\frac{\partial C_\phi}{\partial t} + U\frac{\partial C_\phi}{\partial x} = \frac{\partial}{\partial x}\left(D_x\frac{\partial C_\phi}{\partial x}\right) + f(C_\phi) \tag{2.1}$$

where x = horizontal distance along the channel, t = time, C_ϕ = concentration of water quality index ϕ, U = velocity, D_x = longitudinal dispersion coefficient, and $f(C_\phi)$ = reaction term of water quality index ϕ. In this equation, the mixing in the direction of flow or longitudinal mixing is considered, whereas the mixing across the stream or lateral mixing is neglected. That is, pollutants injected into the stream are assumed to be instantly mixed laterally. If BOD and DO are chosen as such water quality parameters, the last term of the above equation becomes

$$f(L) = -K_1 L \tag{2.2}$$

$$f(C) = -K_1 L + K_2(C_S - C) \tag{2.3}$$

where L = BOD concentration, C = DO concentration, K_1 = deoxygenation coefficient, K_2 = reaeration coefficient, and C_S = saturation value of DO. Mass balances for BOD and DO are then written as follows:

$$\frac{\partial L}{\partial t} + U\frac{\partial L}{\partial x} = \frac{\partial}{\partial x}\left(D_x \frac{\partial L}{\partial x}\right) - K_1 L \tag{2.4}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(D_x \frac{\partial C}{\partial x}\right) - K_1 L + K_2(C_S - C) \tag{2.5}$$

When steady-state condition is considered and the dispersive terms can be neglected (namely, plug flow is assumed)

$$U\frac{dL}{dx} = -K_1 L \tag{2.6}$$

$$U\frac{dC}{dx} = -K_1 L + K_2(C_S - C) \tag{2.7}$$

Eqn.(2.6) can be solved for

$$L = L_0 e^{-\frac{K_1}{U}x} \tag{2.8}$$

where L_0 = BOD concentration on upstream boundary.

DO analysis is often conducted by considering the variation of dissolved oxygen deficit D, which is defined by

$$D = C_S - C \tag{2.9}$$

Then Eqn.(2.7) is rewritten as

$$U\frac{dD}{dx} = K_1 L - K_2 D \tag{2.10}$$

Substituting Eqn.(2.8) into Eqn.(2.10) gives

$$\frac{dD}{dx} = \frac{K_1 L_0}{U} e^{-\frac{K_1}{U}x} - \frac{K_2}{U} D \tag{2.11}$$

If $K_2 \neq K_1$ is supposed, the solution of Eqn.(2.11) is expressed as

$$D = \frac{K_1 L_0}{K_2 - K_1} \left(e^{-\frac{K_1}{U}x} - e^{-\frac{K_2}{U}x} \right) + D_0 e^{-\frac{K_2}{U}x} \tag{2.12}$$

where D_0 = DO deficit on upstream boundary.

The relation $t = x/U$ is substituted into Eqns.(2.8) and (2.12) to give

$$L = L_0 e^{-K_1 t} \tag{2.13}$$

$$D = \frac{K_1 L_0}{K_2 - K_1} \left(e^{-K_1 t} - e^{-K_2 t} \right) + D_0 e^{-K_2 t} \tag{2.14}$$

where t = travel time of flow. Eqns.(2.13) and (2.14) are referred to as Streeter-Phelps equations. These equations are the firstly presented model to analyze the variations of concentrations of organic substance and dissolved oxygen theoretically. In the optimization models for river water quality management, the Streeter-Phelps model is employed by, for example, ReVelle *et al.*(1967)[81], Burn and McBean(1985)[10], and Sasikumar and Mujumdar(1998)[87].

The profile of DO deficit is called DO sag curve. Eqn.(2.14) states that DO deficit increases to a downstream point from upstream boundary, and then decreases again. That is, DO deficit has a maximum D_c at particular time, called critical travel time t_c. The critical travel time can be determined by differentiating Eqn.(2.14) with respect to t, setting the result equal to zero, and solving for

$$t_c = \frac{1}{K_2 - K_1} \ln \left[\frac{K_2}{K_1} \left\{ 1 - \frac{D_0 (K_2 - K_1)}{K_1 L_0} \right\} \right] \tag{2.15}$$

The critical deficit D_c can be determined by solving the equation

$$\left(\frac{dD}{dt} = \right) K_1 L_0 e^{-K_1 t} - K_2 D = 0 \tag{2.16}$$

for D with $t = t_c$. Thus

$$D_c = \frac{K_1 L_0}{K_2} \left[\frac{K_2}{K_1} \left\{ 1 - \frac{D_0(K_2 - K_1)}{K_1 L_0} \right\} \right]^{-\frac{K_1}{K_2 - K_1}} \tag{2.17}$$

DO deficit is the most popular index to describe the state of water quality in the water body. Computing the values of t_c and D_c is important to manage DO concentration in streams.

2.2.2 Camp-Dobbins modification to Streeter-Phelps equations

Dobbins(1964)[23] develops a mathematical model underlying BOD-DO relationships in a river as the Camp-Dobbins modification to the basic Streeter-Phelps equations (2.13) and (2.14), which is often employed in the optimization models in, e.g., ReVelle *et al.*(1968)[82] and Fujiwara *et al.*(1986, 1987, 1988)[31, 32, 33]. Eqns.(2.4) and (2.5) may fairly be simple to represent real mass balances of BOD and DO in some environments. In such cases, consumption of DO by respiration by aquatic lives, photosynthesis and injection of BOD along a river can be considered by adding corresponding terms to these equations. Dobbins(1964)[23] considers the following equations by introducing the parameters K_3, L_a and D_B

$$\frac{\partial L}{\partial t} + U\frac{\partial L}{\partial x} = D_x \frac{\partial^2 L}{\partial x^2} - (K_1 + K_3)L + L_a \tag{2.18}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = D_x \frac{\partial^2 C}{\partial x^2} - K_1 L + K_2(C_S - C) - D_B \tag{2.19}$$

where K_3 = removal coefficient of BOD by sedimentation and/or absorption. Note that the constant K_1 is a biological rate coefficient, whereas K_3 is a measure of physical process. The term L_a accounts for possible addition of BOD along the stretch by the local runoff, artificial injection of wastewater, or the diffusion of partly decomposed organic products from a benthal deposit (i.e., matter lying on the stream bed) into the water above. The term D_B would account for the oxygen demand of benthal deposits, the removal of oxygen by the respiration of algae and attached plants and the introduction of oxygen by photosynthesis. Thus the term D_B could be represented by the expression

$$D_B = D_B' + R_P - P \tag{2.20}$$

where D'_B = benthal demand, R_P = plant respiration, and P = photosynthesis. The expression D_B can be positive or negative, depending on the relative magnitudes of each term in Eqn.(2.20). O'Connor and Toro(1970)[76] presented a mathematical formulation of the effect of factors on the DO concentration in rivers, particularly with respect to the photosynthetic effect.

If steady-state condition is assumed and the dispersive term can be neglected, Eqn.(2.18) can be reduced to

$$U\frac{dL}{dx} = -(K_1 + K_3)L + L_a \tag{2.21}$$

Under the same assumptions, Eqn.(2.19) can be reexpressed as

$$U\frac{dC}{dx} = -K_1L + K_2(C_S - C) - D_B \tag{2.22}$$

If DO concentration is replaced by DO deficit $D = C_S - C$, then

$$U\frac{dD}{dx} = K_1L - K_2D + D_B \tag{2.23}$$

If x/U is replaced with the travel time t of flow from top of the reach to the checking point in Eqns.(2.21) and (2.23), then

$$\frac{dL}{dt} = -(K_1 + K_3)L + L_a \tag{2.24}$$

$$\frac{dD}{dt} = K_1L - K_2D + D_B \tag{2.25}$$

Solving Eqns.(2.24) and (2.25) yields

$$L = \rho L_0 + \theta \tag{2.26}$$

$$D = aL_0 + bD_0 + c \tag{2.27}$$

where

$$\rho = e^{-(K_1+K_3)t}, \quad \theta = \frac{L_a}{K_1 + K_3}(1 - \rho), \quad b = e^{-K_2t}$$

Other terms are defined as follows:
If $K_2 \neq K_1 + K_3$, then

$$a = \frac{K_1}{K_2 - K_1 - K_3}(\rho - b) \tag{2.28}$$

$$c = \left\{ \frac{K_1 L_a}{K_2(K_1 + K_3)} + \frac{D_B}{K_2} \right\} (1 - b) - \frac{K_1 L_a(\rho - b)}{(K_2 - K_1 - K_3)(K_1 + K_3)} \tag{2.29}$$

If $K_2 = K_1 + K_3$, then

$$a = K_1 t b \tag{2.30}$$

$$c = \frac{1}{K_2}\left(D_B + \frac{K_1 L_a}{K_2}\right)(1 - b) - \frac{K_1 L_a t b}{K_2} \tag{2.31}$$

Eqns.(2.26) and (2.27) are called Streeter-Phelps equations with Camp-Dobbins modification. Note that when L_a, K_3 and D_B are set equal to zero, Eqns.(2.26) and (2.27) revert to the Streeter-Phelps equations (2.13) and (2.14). Eqns.(2.27) is sometimes called Camp's equation [Ebise(1989)[25]]. Lohani and Thanh(1978, 1979)[55, 56] developed CCP models for river water quality management based on the Camp's model. Namely, the Camp's model combined with water quality standard for DO is employed as probabilistic constraints in the optimization models.

2.2.3 Other models

The resulting equations (2.26) and (2.27), as well as Streeter-Phelps equations, depend on the assumption that the effect of longitudinal dispersion on BOD and DO profiles can be omitted. This assumption holds in most fresh water streams [Dobbins(1964)[23]]. There are, however, practical cases where longitudinal dispersion is important. A common example is the daily cyclic variation of output from a sewage treatment plant [Fisher *et al.*(1979)[30]]. The importance of longitudinal dispersion effect in slow-moving, highly mixed streams, such as estuaries is also demonstrated [Dobbins(1964)[23]]. It is noted that other approaches without the assumption that the dispersive term is negligible in the transport equation do exist. The method is the one that the advection-dispersion equations of BOD and DO, Eqns.(2.18) and (2.19), are discretized by appropriate numerical scheme like FDM or FEM. For instance, Dresnack and Dobbins(1968)[24] employs FDM and Futagami *et al.*(1976)[34] FEM.

Banks(1976)[2] computes BOD and DO distribution in a river and a lake using mixing cell model of water quality. Bedford(1983)[4] simulates profiles of eight water quality indices with Holley-Preissmann nonlinear formulation, after solving governing equations of river flow. Canale *et al.*(1995)[12] develops a water quality model with respect to BOD and DO by multiple-box approach, and applies the model to Seneca River, N.Y. Lung and Sobeck(1999)[59] modifies a BOD and DO simulation model for planning water quality management in Roanoke River, Virginia. Meadows *et al.*(1978)[67] estimates nonpoint water quality in Brush Creek in First Tennessee Development District using monitoring data and mathematical model.

2.3 Formulae for Parameter Evaluation

2.3.1 Longitudinal dispersion

Several formulae are available to estimate the longitudinal coefficient D_x for streams and rivers. Three simple methods for predicting the coefficient are shown in this section. McQuivey and Keefer(1974)[66] proposes the formula

$$D_x = 0.058\frac{Q}{S_0 B} \tag{2.32}$$

where Q = discharge (m^3/s), S_0 = channel slope (dimensionless), and B = channel width (m). They studied rivers with flows ranging from 0.98 to 924m^3/s. They limit use of this formulation to systems with Froude number less than 0.5

Fisher(1967)[29] presents the following formula.

$$D_x = mnR^{5/6}\frac{Q\sqrt{g}}{A} \tag{2.33}$$

where m = undetermined parameter that varies within the range of 50 to 700 in natural streams, n = Manning's roughness coefficient ($s/m^{1/3}$), R = hydraulic radius (m), g = gravitational acceleration (m/s^2), and A = cross-sectional area (m^2).

Fisher *et al.*(1979)[30] develops the following.

$$D_x = 0.011\frac{Q^2B^2}{\sqrt{gS_0}h^{3/2}A^2} \tag{2.34}$$

where h = mean water depth (m).

2.3.2 Dissolved oxygen saturation

Saturation concentration of dissolved oxygen can ideally be calculated by Henry's law. In reality, it can be affected by several environmental factors including water temperature, salinity, and partial pressure variations due to elevation [Chapra(1997)[16]]. Saturation value of DO decreases with increasing temperature, salinity and elevation.

Several formulae were presented to predict saturation level of DO. For example, Lee *et al.*(1991)[52] used the following.

$$\begin{aligned} C_S = 1.43\{&10.291 - 0.2809T + 0.006009T^2 - 0.0000632T^3 \\ &-0.607(0.1161 - 0.003922T + 0.0000631T^2)S\} \quad \text{(mg/L)} \end{aligned} \tag{2.35}$$

where T = water temperature (°C), and S = salinity (g/L).

2.3.3 Deoxygenation and reaeration

The rates of most reactions in natural waters increase with temperature. A general rule of thumb is that the rate will approximately double for a temperature rise of 10 °C [Chapra(1997)[16]].

A more rigorous quantification of the temperature dependence is provided by the Arrhenius equation [for example, Gouda(1985)[36]]

$$K(T_a) = A_f e^{\frac{-E}{R_g T_a}} \tag{2.36}$$

where K = reaction rate at T_a K, A_f = frequency factor, E = activation energy (J mole^{-1}), R_g = the gas constant (8.314 J mole^{-1} K^{-1}), and T_a = absolute temperature (K). Eqn.(2.36) is often used to compare the reaction rate constant at two different temperatures. This can be done by expressing the ratio of the rates, as in

$$\frac{K(T_{a2})}{K(T_{a1})} = e^{\frac{E(T_{a2}-T_{a1})}{R_g T_{a2} T_{a1}}} \tag{2.37}$$

Since the following can be defined as a constant:

$$\theta \equiv e^{\frac{E}{R_g T_{a2} T_{a1}}} \tag{2.38}$$

where θ = temperature correction coefficient, Eqn.(2.37) can be reexpressed as

$$\frac{K(T_2)}{K(T_1)} = \theta^{T_2 - T_1} \tag{2.39}$$

where the temperature is expressed in °C.

In water quality modeling, many reactions are reported at 20℃. Therefore, Eqn.(2.39) is usually expressed as

$$K(T) = K(20)\theta^{T-20} \tag{2.40}$$

where T = water temperature (°C). With respect to the value of θ in Eqn.(2.40), 1.047 is typically used for BOD decomposition and 1.024 for oxygen reaeration. For example, Lee *et al.*(1991)[52] employed the following formulae to evaluate deoxygenation rate K_1 and oxygen reaeration rate K_2 in the form of Eqn.(2.40)

$$K_1 = K_1(20)(1.047)^{T-20} = 0.23(1.047)^{T-20} \quad (\text{day}^{-1}) \tag{2.41}$$

where $K_1(20)$ = deoxygenation rate at 20 °C (day^{-1}); and O'Connor-Banks formula

$$\begin{aligned} K_2 &= K_2(20)(1.024)^{T-20} \\ &= \left\{ 3.9\frac{U^{0.5}}{h^{1.5}} + \frac{0.728W^{0.5} - 0.317W + 0.0372W^2}{h} \right\} (1.024)^{T-20} \quad (\text{day}^{-1}) \end{aligned} \tag{2.42}$$

where $K_2(20)$ = reaeration rate at 20 °C (day^{-1}), U = velocity (m/s), and W = wind speed (m/s).

In order to describe gas transfer in natural streams, the two-film theory and the surface renewal theory are widely employed. Although both are used in streams, estuaries and lakes, the two-film theory is more widely used in standing waters such as lakes, whereas the surface renewal model is more commonly used in flowing waters such as streams [Chapra(1997)[16]]. With respect to the details of these theories, see, for example, Chapra(1997)[16] and Tanigaki(1990)[92].

One of the most important requirements in stream pollution analysis is an accurate estimate of the reaeration rate. Many investigators have developed formulae for predicting reaeration in streams and rivers conceptually or empirically. Comprehensive reviews and analyses of predictive equations can be found, for instance, in Rathbun(1977)[79]. Among those formulae, three are very commonly used: the O'Connor-Dobbins, Churchill, and Owens-Gibbs formulae [Chapra(1997)[16]].

- O'Connor-Dobbins formula (Conceptual equation based on the surface renewal model)

$$K_2 = 3.93\frac{U^{0.5}}{h^{1.5}} \quad (\text{day}^{-1})$$

- Churchill formula (Empirical equation)

$$K_2 = 5.026\frac{U}{h^{1.67}} \quad (\text{day}^{-1})$$

- Owens-Gibbs formula (Empirical equation)

$$K_2 = 5.32\frac{U^{0.67}}{h^{1.85}} \quad (\text{day}^{-1})$$

where U = velocity (m/s) and h = water depth (m). Ranges of depth and velocity used to develop these three formulae for stream reaeration are summarized in Table 2.1. The O'Connor-Dobbins formula has the widest applicability of the three, being appropriate for moderate to deep streams with moderate to low velocities. The Churchill formula applies for similar depths but for faster streams. The Owens-Gibbs formula is used for shallower systems.

2.4 Remarks

Numerical models describing BOD and DO transport in streams are summarized in this chapter. The Streeter-Phelps model and its modification have been widely adopted in optimization models for river water quality management. However the assumption in those models that the influence of longitudinal dispersion is negligible may cause modeling error in some situations. In contrast to those classic models, the direct approach discretizing the governing differential equations of BOD and DO can avoid such error. This method can be thus considered superior to the former methods not only in the relative correctness of modeling physical phenomena but also in its systematic way of handling resultant equations in the whole water body. In this study, therefore, the finite element model based on the full BOD and DO transport equations is employed as the water quality model

Table 2.1: Ranges of depth and velocity used to develop formulae [Chapra(1997)[16]]

Parameter	O'Connor-Dobbins	Churchill	Owens-Gibbs
Depth h(m)	0.30 - 9.14	0.61 - 3.35	0.12 - 0.73
Velocity U(m/s)	0.15 - 0.49	0.55 - 1.52	0.03 - 0.55

embedded in an optimization model for wasteload allocation.

Chapter 3

OPTIMIZATION AND DECISION-MAKING

3.1 Introduction

Optimization models can play a key role in decision-making process due to their definite description of both management objectives and constraints with their rigorous solution algorithm. With the rapid deterioration of water quality in rivers, lots of researches for controlling water pollution have been done using optimization theory. Those are reviewed in Section 3.2. In Section 3.3, robust optimization (RO) is described as one of the frameworks to build stochastic models for mitigating the effects of uncertain input. A RO model is compared with a stochastic programming model and a chance-constrained programming model, so that the RO model would be well differentiated. The content of this section gives a basis of extending the deterministic optimization model, which is presented in Chapter 4, to a RO model for river water quality management developed in subsequent chapters.

Note that this extension, i.e., building the RO model, means not only the conversion from a deterministic model to a stochastic one but also the reformation from a single objective model to a multiobjective one. Therefore vector optimization theory should be employed to analyze the solutions obtained. Since in general those objectives are conflicting each other, solutions of a multiobjective optimization model are, at best, noninferior (or nondominated). In Subsection 3.4.1, the noninferior solution in vector optimization theory is defined. A multiobjective optimization model can be solved by various methods. In Subsection 3.4.2, popular three methods of them are introduced. In Section 3.5, the function of an optimization model in multiobjective decision-making process (MDMP) is explained. Finally, remarks are given in Section 3.6.

3.2 Literature Review

3.2.1 Deterministic optimization models

The problem of determining an optimal wasteload allocation for a series of point source discharges to a river has often been discussed since the middle of the 20th century. Various types of models have been presented using some mathematical programming technique. In the early stage developing models on this subject, the stochastic conditions of stream are neglected in the optimization models due probably to simplicity. Most of those models, often called deterministic models, therefore employ the traditional 'design' values, e.g., the lowest 7-day moving average of daily flow rate over a 10-yr period, and the highest recorded daily temperature. In this subsection, the deterministic optimization models based on 'design' parameter values are reviewed.

Many research workers [e.g., Kerri(1966)[47], Loucks *et al.*(1967)[58], ReVelle *et al.* (1967, 1968)[81, 82], Arbabi and Elzinza(1975)[1], deLucia *et al.*(1978)[20], and Wen(1989) [102]] develop linear programming models to manage river water quality. Liebman and Lynn (1966)[53], Bayer(1974)[3], Ecker(1975)[26], and Bishop and Grenney(1976)[6] employ dynamic programming, nonlinear programming, geometric programming, and integer programming, respectively. Nakayama *et al.*(1980a)[73] develops an interactive optimization technique applicable to a multiobjective optimization problem on water quality control in Yodo River, Japan. Ikeda(1994)[43] presents a multiobjective optimal control model. Burn and Yulianti(2001)[11] presents a multiobjective optimization problem and solves the problem by genetic algorithm. Futagami *et al.*(1976)[34] presents FE and LP method where mass balance equations on water quality indices discretized by the FEM are directly used as linear constraints in a linear programming (LP) model on wasteload allocation in rivers.

All the optimization and optimal control models mentioned above are dealing with information on pollutant loading only from point sources to streams as decision variables. In contrast, Ejaz and Peralta(1995)[27] develops a linear programming model that can abate pollutant loading from agricultural and domestic lands, i.e., nonpoint sources. However this kind of researches that considers an optimal allocation of wasteload from distributed sources is very few.

3.2.2 Stochastic optimization models

There has been growing interest in considering uncertainty due to randomness associated with various components of a water quality system [e.g., Loucks and Lynn(1966)[57], Kothandaraman and Ewing(1969)[49], Padgett and Rao(1979)[77], and Ward and Loftis (1983)[99]]. Many researchers have presented optimization models where uncertainties of flow and/or environmental variables are stochastically handled. Sobel(1965)[88] presents stochastic quadratic programming model, and Lohani and Hee(1983)[54] and Cardwell and Ellis(1993)[13] present stochastic dynamic programming models for managing water quality. Takyi and Lence(1999)[90] insists that, with few exceptions [e.g., Sobel (1965) [88], Lohani and Hee(1983)[54], and Cardwell and Ellis(1993)[13]], there are three widely used methods for incorporating input information uncertainty in water quality management models. These are (i) chance-constrained optimization, (ii) combined simulation-optimization, and more recently, (iii) multiple realization-based approaches.

Chance-constrained programming (CCP) is widely used technique on this subject [Lohani and Thanh(1978, 1979)[55, 56], Burn and McBean(1985)[10], Fujiwara *et al.*(1986, 1987, 1988)[31, 32, 33], Ellis(1987)[28]]. Lohani and Thanh(1978, 1979)[55, 56] formulate chance constraints that represent regulation of DO deficit with a risk probability. They treat only flows probabilistically. Fujiwara *et al.*(1986, 1987)[31, 32] use mathematical model developed using the Camp-Dobbins modification to the Streeter-Phelps model to impose chance constraints for the same type of regulations as in Lohani-Thanh model. However the same limitation exists in the model by Fujiwara *et al.*(1986, 1987)[31, 32] as in the Lohani-Thanh model: the assumption that the parameters such as travel time t, reaction kinetics K_1, K_2, K_3, and L_a and D_B in Eqns.(2.24) and (2.25) are independent of the stochastic variations in flows. In contrast, Burn and McBean(1985)[10] develops a CCP model where uncertainties present in the level of flow, the pollutant loading, the travel time of flow, and the reaction coefficients for the Streeter-Phelps equation are characterized using first-order uncertainty analysis. The optimization model with two objectives of maximizing the total sum of DO concentrations at given checking points and minimizing treatment costs is solved by the constraint method.

The simulation-optimization approach for water quality management utilize Monte Carlo simulation or a long record of historical information to generate several possible scenarios of hydrologic-, hydraulic-, and pollution-loading conditions of the water quality system. Each created scenario or realization of the water quality conditions is incorporated into an optimization model as a set of constraints. Fujiwara *et al.*(1988)[33] modifies the method introduced by Burn and McBean(1985)[10] by using Monte Carlo simulation instead of first-order uncertainty analysis, and by adopting an iterative scheme of CCP and simulation analysis to maintain violation of water quality standards within maximum allowable probability levels. Burn(1989)[8] develops a modeling technique for river water quality management based on the simulation-optimization approach. A Monte Carlo simulation model is used 5,000 times in the illustrative example to generate a series of water quality responses that lead to the formulation of a constraint set for an integer programming model. A trade-off curve between total expenditures on removal of BOD and a probability level that can be used to reflect the degree of risk aversion of the decision-maker is provided.

Multiple realization-based approach is based on the use of multiple scenarios that reflect possible combinations of hydrologic, meteorologic, and pollutant loading design conditions. Burn and Lence(1992)[9] develops four types of optimization model formulations, namely, to minimize the maximum violation from the imposed water quality standard, to minimize the maximum regret, to minimize the total violation, and to minimize the total regret. Every model contains a single objective function. In their study, a scenario is composed of a flow value, a water temperature, and a pollutant loading impact from nonpoint source contributions. The methodology is applied to a case study based on the Willamette River in Oregon. Five scenarios are assumed using collected data with equal probability of occurrence.

Robust optimization (RO), which is described in Section 3.3, is one of the frameworks in this category of multiple realization-based approach, employing a robustness concept [e.g., Rosenhead *et al.*(1972)[83] and Mulvey *et al.*(1995)[70]]. The work by Burn and Lence(1992)[9] contrasts with a RO model in the fact that their model does not consider adjustment of robustness proactively (i.e., before optimization). Takyi and Lence(1999)[90] develops a multiple realization model for stream water quality management. The techniques developed are used for generating cost-reliability trade-off relationships for the management system. One main difference between the RO models and the multiple realization models presented by Takyi and Lence(1999)[90] is that the former obtains individual solutions for each realization and consolidates these solutions into an overall policy decision, while the latter produces results that simultaneously satisfy constraints representing a unique set of realizations.

3.2.3 Fuzzy optimization models

Water quality management problems can be characterized by various kinds of uncertainties at different stages of the decision-making process. Chang *et al.*(1997)[14], Chen and Chang(1998)[17], and Sasikumar and Mujumdar(1998)[87] point out two types of uncertainties. The one is the uncertainty due to randomness associated with river flow and effluent flow, which has received much attention. The stochastic optimization models reviewed in the previous section treat this kind of uncertainty in the modeling process. The other type of uncertainty prominent in the management of water quality systems is the uncertainty caused by vagueness or ambiguity related to description of the goals pertaining to water quality and pollutant abatement. Namely, fuzziness, or ambiguity that can be found in the linguistic description of a concept or feeling, takes in another aspect of uncertainty.

In recent years, several models embedding fuzzy sets theory for management problems of river water quality have been developed to cope with the latter type of uncertainty. Sasikumar and Mujumdar(1998)[87] considers the uncertainty due only to ambiguous goals in their fuzzy wasteload allocation model. The goals of pollution control agency and the dischargers, which are conflicting, are transformed to fuzzy goals using fuzzy sets, and fuzzy decision is conducted. Chang *et al.*(1997)[14] develops water quality model in rivers, employing fuzzy sets theory and gray systems theory to handle uncertainties included in the decision problem. Both the fuzzy goals pertaining to the decision-maker's aspiration levels and gray messages related to imprecision of the input parameter values are treated within a multiobjective analytical framework. The other feature of Chang *et al.*(1997)[14] is that it explicitly refers to the participation of decision-makers and their roles in the operating process of the developed methodology. The method presented is used to determine wastewater treatment levels within the Tseng-Wen River basin in Taiwan, aiming at minimizing total cost and maximizing BOD loading to the river. Chen and Chang(1998)[17] proposes a nonlinear multiobjective optimization model for water pollution control using fuzzy mathematical programming. Three objectives, i.e., the maximization of assimilative capacity in the river, the minimization of treatment cost for water pollution control, and the maximization of economic value of river flow with regard to recreation aspect, are considered in the model. Genetic algorithm is employed to solve the formulated model for the case study in Tseng-Wen River basin in Taiwan.

3.3 Framework of Robust Optimization

Optimization models that have two distinct components are dealt with in the RO framework [Mulvey *et al.*(1995)[70], Watkins and McKinney(1997)[100], and Vladimirou and Zenios(1997a, b)[97, 96]]. The components are (a) a *structural* component that is fixed and free of any noise in its input data, and (b) a *control* component that is subjected to noisy input data. To define the appropriate model, two sets of variables are introduced:

(a) $\mathbf{x} \in \mathbf{R}^{n_1}$ denotes the vector of decision variables whose optimal value is not conditioned on the realization of the uncertain parameters. These are the *design* variables. Variables in this set cannot be adjusted once a specific realization of the data is observed.

(b) $\mathbf{y} \in \mathbf{R}^{n_2}$ denotes the vector of *control* decision variables that are subjected to adjustment once the uncertain parameters are observed. Their optimal value depends both on the realization of uncertain parameters, and on the optimal value of the design variables.

The optimization model has the following structure.

LP

$$\begin{aligned} \text{Minimize} \quad & \mathbf{c}^T\mathbf{x} + \mathbf{d}^T\mathbf{y} && (3.1)\\ \text{subject to} \quad & A_0\mathbf{x} = \mathbf{b} && (3.2)\\ & B\mathbf{x} + C\mathbf{y} = \mathbf{e} && (3.3)\\ & \mathbf{x}, \mathbf{y} \geq 0 && (3.4)\\ & \mathbf{x} \in \mathbf{R}^{n_1}, \mathbf{y} \in \mathbf{R}^{n_2} \end{aligned}$$

Eqn.(3.2) denotes the structural constraints whose coefficients are fixed and free of noise. Eqn.(3.3) denotes the control constraints. The coefficients of this constraint set are subject to noise.

To define the robust optimization problem, a set of scenarios $\Omega = \{1, 2, 3, \cdots, S\}$ is introduced. With each scenario $s \in \Omega$, the set $\{\mathbf{d}_s, B_s, C_s, \mathbf{e}_s\}$ of realizations for the coefficients of the control constraints is associated. The probability of the scenario $p_s(\sum_{s=1}^{S} p_s = 1)$ is also associated with each scenario s. The optimal solution of the mathematical program **LP** will be robust with respect to optimality if it remains 'close' to optimal for any realization of the scenario $s \in \Omega$. It is then termed *solution robust.* That is, an optimal policy is solution robust if it remains optimal or nearly optimal for all scenarios. Thus, Watkins and McKinney(1997)[100] terms solution robustness as optimality robustness. In Figure 3.1, an example is given which compares a solution of solution robust with that of not solution robust in a linear RO model with two decision variables and two scenarios under equal probability of occurrence. In this figure, z = aggregate optimal objective value, and z_1 and z_2 = optimal objective value at scenario 1 and 2, respectively.

The solution is also robust with respect to feasibility if it remains 'almost' feasible for any realization of s. It is then termed *model robust*. In other words, an optimal policy is model robust if it remains feasible or nearly feasible for all scenarios. Watkins and McKinney(1997)[100] referrers to model robustness as feasibility robustness. Model robustness can usually be measured by the magnitude of violence or relaxation of the original constraints (Figure 3.2). The more relaxed the constraints are, the less model robust the solution becomes. The notions of 'close' and 'almost' are made precise through the choice of norms, as is shown later in this section.

It is unlikely that any solution to the program **LP** will remain both feasible and optimal for all scenario indices $s \in \Omega$. If the system that is being modeled has substantial redundancies built in, then it might be possible to find solutions that remain both feasible and optimal. Otherwise, a model is needed that will allow us to measure the trade-off between solution and model robustnesses. The robust optimization model given next formalizes a way to measure this trade-off.

A set $\{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_S\}$ of control variables for each scenario $s \in \Omega$ is first introduced. A set $\{\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_S\}$ of error vectors that will measure the infeasibility allowed in the control constraints under scenario s is also introduced. In other point of view, control constraints can be relaxed in order to assure feasibility of the robust optimization model. Consider now the following formulation of the robust optimization model.

Model ROBUST

$$\text{Minimize} \quad \sigma(\mathbf{x}, \mathbf{y}_1, \cdots, \mathbf{y}_S) + \omega\rho(\mathbf{z}_1, \cdots, \mathbf{z}_S) \tag{3.5}$$

$$\text{subject to} \quad A_0\mathbf{x} = \mathbf{b} \tag{3.6}$$

$$B_s\mathbf{x} + C_s\mathbf{y}_s + \mathbf{z}_s = \mathbf{e}_s \qquad \forall s \tag{3.7}$$

$$\mathbf{x} \geq 0, \ \mathbf{y}_s \geq 0 \qquad \forall s \tag{3.8}$$

where $\sigma(\cdot)$ = aggregate objective function, $\rho(\cdot)$ = feasibility penalty function, and ω = weight.

With multiple scenarios, the objective function $\xi = \mathbf{c}^T\mathbf{x} + \mathbf{d}^T\mathbf{y}$ in Eqn.(3.1) becomes a random variable taking the value $\xi_s = \mathbf{c}^T\mathbf{x} + \mathbf{d}_s^T\mathbf{y}_s$, with probability p_s. Hence, there is no longer a single choice for an aggregate objective. The mean value

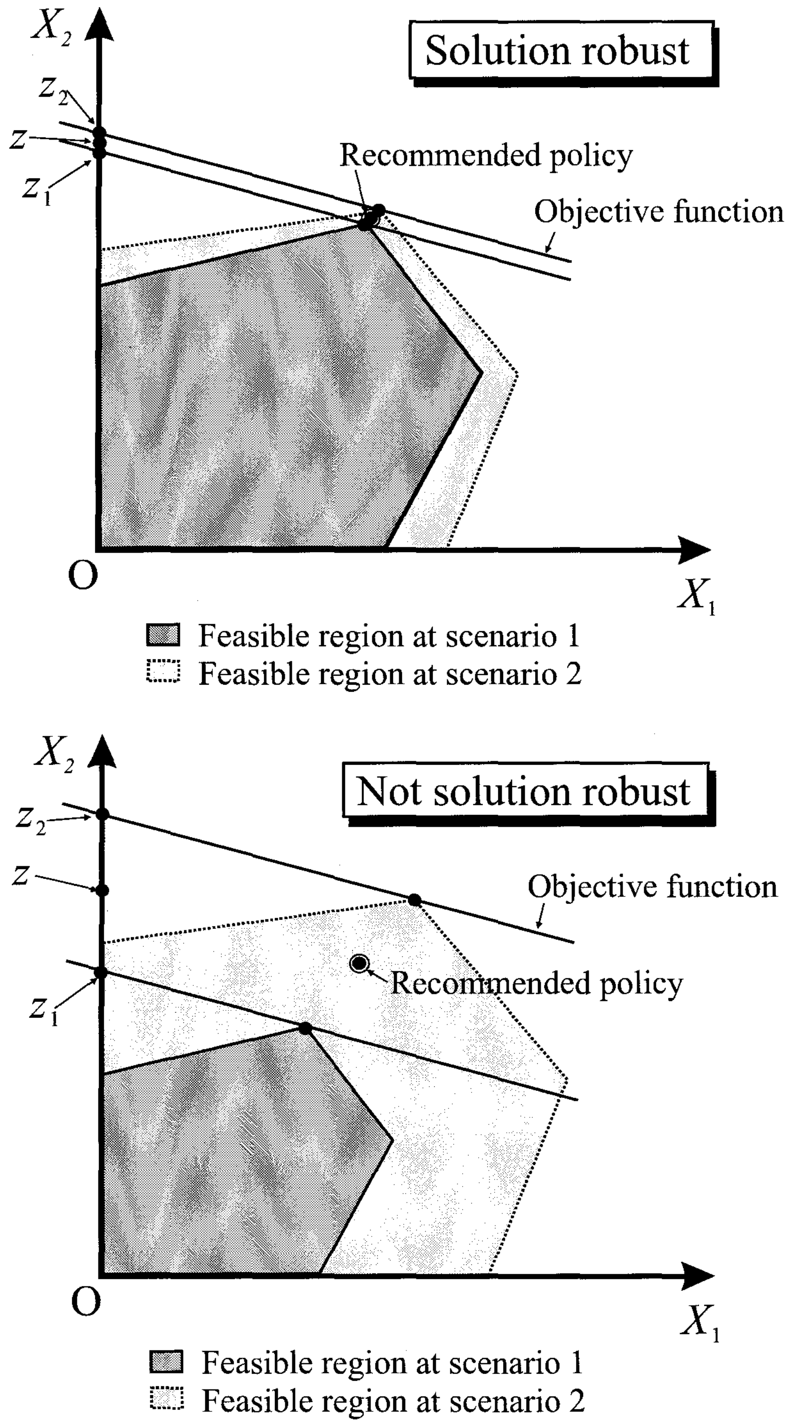

Figure 3.1: Solution robust and not solution robust

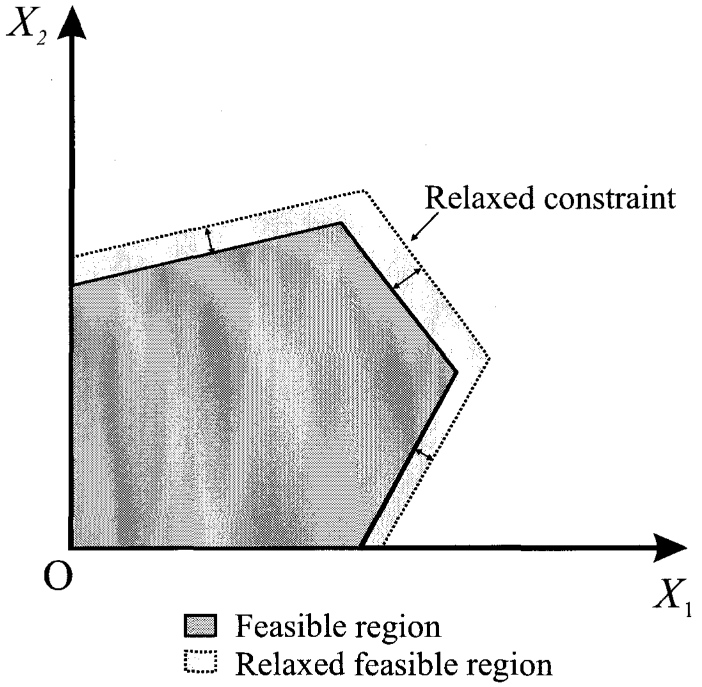

Figure 3.2: Relaxation of constraints

$$\sigma(\cdot) = \sum_{s \in \Omega} p_s \xi_s \tag{3.9}$$

could be used, which is the function used in stochastic linear programming formulations. In worst-case analysis the model minimizes the maximum value, and the objective function is defined by

$$\sigma(\cdot) = \max_{s \in \Omega} \xi_s \tag{3.10}$$

Both of these choices are special cases of RO, but they are nevertheless standard in the literature. One novelty of the RO formulation is that it allows the introduction of higher moments of the distribution of ξ_s in the optimization model. For example, a utility function [e.g., Kubo and Miyamoto(1980)[50] and Chankong and Haimes(1983)[15]] that embodies a trade-off between mean value and variability in this mean value could be introduced. Other formulation is mean/variance, in which

$$\sigma(\cdot) = \mathrm{E}_s(\cdot) + \lambda \mathrm{Var}_s(\cdot) \tag{3.11}$$

where $\mathrm{E}_s(\cdot)$ and $\mathrm{Var}_s(\cdot)$ are the expected value and variance of the objective function over all scenarios, and λ represents the relative importance of the variance compared to the expected value [Watkins and McKinney(1997)[100]]. Indeed, the introduction of higher moments is one of the distinguishing features of RO from stochastic linear programming. In Chapter 5, a function which comprises mean value and value of maximum expected absolute deviation from the mean is employed for $\sigma(\cdot)$ in Eqn.(3.5) in a RO model for water quality management.

The second term in the objective function of **Model ROBUST**, $\rho(\mathbf{z}_1, \cdots, \mathbf{z}_s)$, is a feasibility penalty function. It is used to penalize violations of the control constraints under some of the scenarios. The introduction of the penalty function distinguishes the RO model from existing approaches for dealing with noisy data. The RO model will generate solutions with the least amount of violations of constraints originated from the **LP** model.

Relaxing some constraints that are related to water quality standards sometimes takes place in optimization models built in other framework in the context of managing river water quality. For example, the CCP model developed by Burn and McBean(1985)[10] embraces the set of probabilistic constraints in the form

$$\Pr[A_0\mathbf{x} \leq \mathbf{b}] \geq \boldsymbol{\alpha}_p \qquad (3.12)$$

where Pr[ ] = probability with which the relation in [ ] holds, A_0 = deterministic matrix, $\mathbf{x}$ = decision vector, $\mathbf{b}$ = vector of random variables, and $\boldsymbol{\alpha}_p$ = vector of exceedance probabilities. Therefore $\boldsymbol{\alpha}_p$ should be determined in advance in the CCP model, whereas in RO, there is no need to give exceedance probabilities.

The merit of using CCP is that the method does not increase the model size from the size of the basic deterministic model in spite of taking parameter uncertainty into account. One of the drawbacks of the RO model is its large model size caused by the incorporation of constraints under all scenarios. Such a large-scale model requires much computational effort, which results in long computational time.

The specific choice of penalty function is problem dependent, and it also has implications for the accompanying solution algorithm. Mulvey *et al.*(1995)[70] proposes two alternative penalty functions:

(a) $\rho(\mathbf{z}_1, \cdots, \mathbf{z}_S) = \sum_{s\in\Omega} p_s \mathbf{z}_s^T \mathbf{z}_s$. This quadratic penalty function is applicable to equality constrained problems where both positive and negative violations of the control constraints are equally undesirable.

(b) $\rho(\mathbf{z}_1, \cdots, \mathbf{z}_S) = \mathbf{p}_s^T \max\{\mathbf{0}, \mathbf{z}_s\}$. This exact penalty function applies to inequality control constraints when only positive violations are of interest.

It is noted that the RO model takes a multicriteria objective form. The first term measures solution robustness, whereas the penalty term model robustness. The weight ω is used to derive a spectrum of answers that trade-off solution for model robustness. In order to reconcile the effects of uncertain information embedded in the optimization model, RO adopts a proactive approach: The values of weights attached to some of the terms in the objective function are chosen before solving the optimization problem. This means that the model analysts can adjust the impact of data uncertainties, not just discover the influence by reactive approach like sensitivity analysis.

3.4 Multiobjective Optimization

3.4.1 Noninferior solution

One of the critical points of the RO problem is that it is a multiobjective optimization problem. Since management goals are usually in conflict each other in a multiobjective problems, the concept of optimal solution in such problems differs from that in a single-objective problem. Thus vector optimization theory should be applied to analyze its solutions. In this section, a general vector optimization model is presented and its meaningful set of solutions, i.e., noninferior solutions, are mathematically defined after Chankong and Haimes(1983)[15].

Let $\mathbf{x}$ be an N-dimensional vector of decision variables. For $i = 1, \cdots, m$, the symbol $g_i(\mathbf{x})$ is reserved to denote the real-valued function defined on R^n that represents the i-th system constraint. Any other form of constraint (i.e., those which cannot be expressed as a g function) can be included in the set $S \subseteq R^n$. The *decision space* or the *feasible region* of the system will be characterized by the set

$$X = \{\mathbf{x} | g_i(\mathbf{x}) \leq 0, i = 1, \cdots, m \text{ and } \mathbf{x} \in S\} \tag{3.13}$$

Note that $X \subseteq R^N$. Likewise for each $j = 1, \cdots, n$ the symbol $f_j(\mathbf{x})$ is reserved to denote the real-valued function defined on X that represents the j-th attribute (or objective function or decision criterion). For compact notation, the multiobjective function (or vector-valued criterion) will be denoted by

$$\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \cdots, f_n(\mathbf{x})) \tag{3.14}$$

i.e., f: $X \to R^n$ for $X \subseteq R^N$. Notice that the objective function (3.5) in the general RO model can be rewritten in the form of Eqn.(3.14). Correspondingly, the *objective space* (or criterion space) refers to the set $F = \{\mathbf{f}(\mathbf{x})|\mathbf{x} \in X\}$. Thus $F \subseteq R^n$. In summary, the decision space belongs to R^N and the objective space belongs to R^n. A vector optimization problem (VOP) is then formulated as

$$\underset{\mathbf{x} \in X}{\text{Minimize}} \quad [f_1(\mathbf{x}), \cdots, f_n(\mathbf{x})] \tag{3.15}$$

Solving a VOP entails finding its set of noninferior solutions. Conceptually, a noninferior solution is one which is not dominated by any other feasible solution. Precisely what is meant by "one solution dominates the other" depends on the type of analysis being used (which, in turn, depends on the manner in which the decision-maker interacts with the model). Intuitively, the domination structure of a multiobjective decision problem is a structure related to the decision-maker's preference, which determines how one alternative dominates another alternative. $\mathbf{x}^1$ *dominates* $\mathbf{x}^2$ means

$$\mathbf{x}^1 \geq \mathbf{x}^2 \quad \text{if and only if} \quad v(\mathbf{f}(\mathbf{x}^1)) \geq v(\mathbf{f}(\mathbf{x}^2)) \tag{3.16}$$

where v is the value function. It can be said that Eqn.(3.16) defines the domination structure for this decision problem. In general, Yu(1973)[103] uses the so-called domination cone, which is a convex cone $D(\mathbf{f})$ in R^n, to define the domination structure. For $\mathbf{x}^1$ and $\mathbf{x}^2$ in X, alternative $\mathbf{x}^1$ dominates alternative $\mathbf{x}^2$ if and only if

$$\mathbf{f}^2 - \mathbf{f}^1 \in D(\mathbf{f}^1) \tag{3.17}$$

where $\mathbf{f}^1 = \mathbf{f}(\mathbf{x}^1)$, $\mathbf{f}^2 = \mathbf{f}(\mathbf{x}^2)$, and $D(\mathbf{f})$ is the domination (convex) cone at $\mathbf{f}$. Consequently, $\mathbf{x}$ can be said nondominated if it is not dominated by any $\mathbf{x}$ in X. Bergstresser *et al.*(1976)[5] later generalized the concept and used a convex set $D(\mathbf{f})$, rather than a convex cone, to represent the domination structure. They call $D(\mathbf{f})$ the set of domination factors at $\mathbf{f}$. In terms of this definition and Eqn.(3.17), the domination structure reflected by Eqn.(3.16) can be represented by the convex set $D(\mathbf{f})$, where, for each $\mathbf{f} \in F$,

$$D(\mathbf{f}) = \{\mathbf{d}_f | v(\mathbf{f}) > v(\mathbf{f} + \mathbf{d}_f)\} \tag{3.18}$$

The use of the generalized domination structure to define the concept of nondominated solution opens up opportunities to develop theoretical results that are applicable to more than one type of preference structure. The specific class of domination problem of Eqn.(3.15) is focused on. The implicit preference structure, which underlies this formulation, is in tune with *monotonicity* of preference. It states that, for each objective function $f_j, j = 1, \cdots, n$, an alternative having a smaller value of f_j is always preferred to an alternative having a larger value of f_j, with all other objective functions being equal. The corresponding domination structure for VOP in Eqn.(3.15) is thus represented by a constant convex cone of the form

$$D = \{\mathbf{d}_f | \mathbf{d}_f \in R^n, \mathbf{d}_f \geq \mathbf{0}\} \tag{3.19}$$

The general nondominated (or noninferior) solution defined by Eqns.(3.17) and (3.19) then becomes the familiar Pareto-optimal solution.

Definition. $\mathbf{x}^*$ is said to be a *noninferior solution* of VOP if there exists no other feasible $\mathbf{x}$ (i.e., $\mathbf{x} \in X$) such that $\mathbf{f}(\mathbf{x}) \leq \mathbf{f}(\mathbf{x}^*)$, meaning that $f_j(\mathbf{x}) \leq f_j(\mathbf{x}^*)$ for all $j = 1, \cdots, n$ with strict inequality for at least one j.

Intuitively the alternative $\mathbf{x}^*$ in X is noninferior if and only if any other alternative $\mathbf{x}$ in X cannot be found such that some objective functions at $\mathbf{x}$ improve (i.e., decrease) from those at $\mathbf{x}^*$ without degrading at least one of the other objective functions.

3.4.2 Methods for generating noninferior solutions

In order to operationalize the concept of noninferior solutions, it should be related to a familiar concept. The most common strategy is to characterize noninferior solutions in terms of optimal solutions of appropriate scalar optimization problems. Among the many possible ways of obtaining a scalar problem from a VOP, the following are common [Chankong and Haimes(1983)[15]].

1) The weighting method: Let $W = \{\mathbf{w}|\mathbf{w} \in R^n, w_j \geq 0 \text{ and } \sum_{j=1}^n w_j = 1\}$ be the set of nonnegative weights. The weighting problem is defined for some $\mathbf{w} \in W$:

$$\underset{\mathbf{x}\in X}{\text{Minimize}} \quad \sum_{j=1}^{n} w_j f_j(\mathbf{x}) \tag{3.20}$$

2) The kth-objective Lagrangian method:

$$\underset{\mathbf{x}\in X}{\text{Minimize}} \quad f_k(\mathbf{x}) + \sum_{j\neq k} u_j f_j(\mathbf{x}) \tag{3.21}$$

where u_j = weight, $u_j \geq 0$ for each $j \neq k$

3) The kth-objective ϵ-constraint method:

$$\underset{\mathbf{x}\in X}{\text{Minimize}} \quad f_k(\mathbf{x}) \tag{3.22}$$

$$\text{subject to} \quad f_j(\mathbf{x}) \leq \epsilon_j, \quad j = 1, \cdots, n, \; j \neq k \tag{3.23}$$

where ϵ_j = parameter.

In the RO framework described in the previous section, it can be interpreted that the optimization problem [Eqns.(3.5)-(3.8)] is formulated by the Lagrangian method. However, the ϵ-constraint method [Haimes *et al.*(1971)[40]] will be adopted in a RO problem in Chapter 7 because of its advantages over the Lagrangian method.

3.5 Multiobjective Decision-Making Process

A multiobjective decision-making process (MDMP) with single decision-maker can be assumed that it consists of the following three steps [Haimes and Chankong(1979)[37]]:

1) Analysts generate noninferior solutions of a multiobjective optimization problem;

2) Obtain meaningful information to interact with a decision-maker (DM); and

3) Use information obtained in Step 2 to interact with the DM and select the final solution based on the DM's preference response.

Step 1 serves as a preliminary screening process designed to reduce the originally large set of feasible alternatives by eliminating inferior ones from further consideration. What remains is a set of noninferior alternatives whose number is still large in general. Then

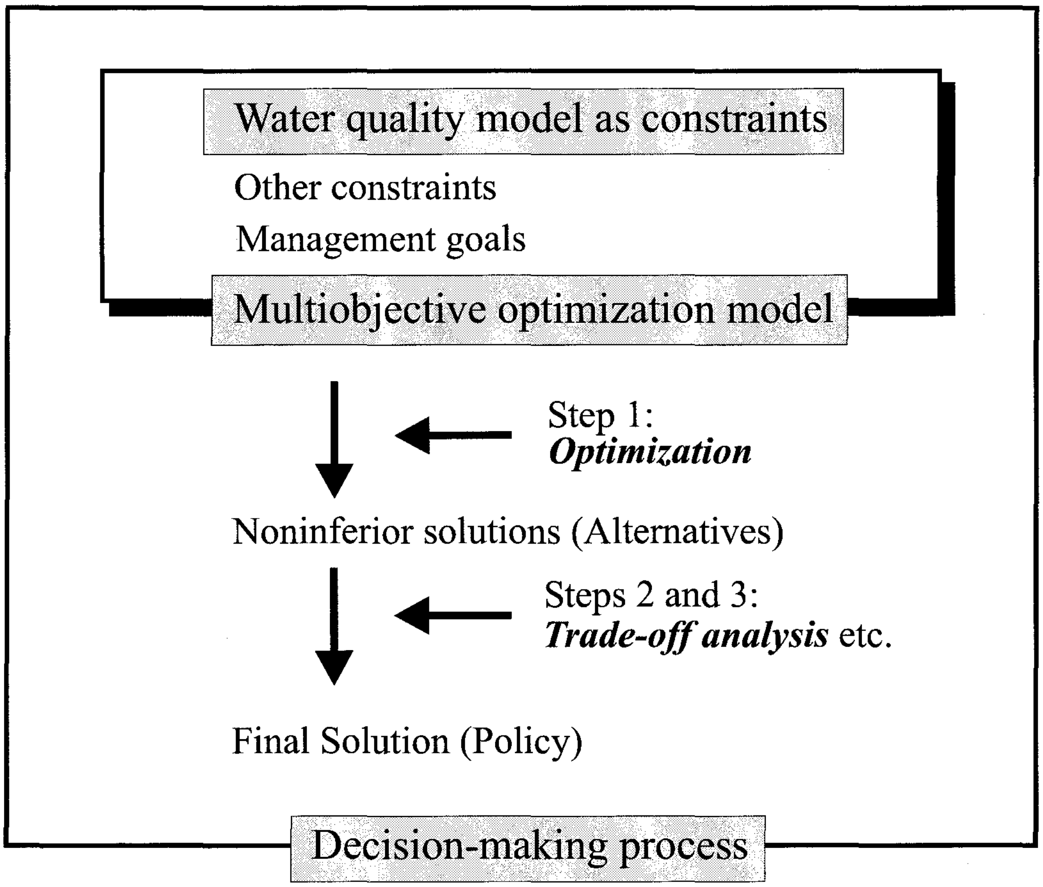

Figure 3.3: Multiobjective decision-making

the next step is to select the 'best' alternative from noninferior ones. Contrary to Step 1, the task in Steps 2 and 3 is not routine, and sometimes requires careful and elaborated analysis and execution.

Trade-off analysis, that is, the investigation to know how to balance one objectives against another except in a subjective way, is often conducted in Step 2 [e.g., Major(1969) [63], Cohon and Marks(1973)[18], and Monarchi *et al.*(1973)[69]]. One of the merits of the ϵ-constraint method is obtaining trade-off rates [e.g., Sakawa(1986)[86] and Haimes *et al.*(1990)[41]] that are produced as byproducts of operating the optimization model.

Most multiobjective optimization models for water quality management have not ever been built in the context of the MDMP. In such models, generating noninferior solutions and obtaining trade-off information (i.e., Steps 1 and 2) are implemented, but Step 3 in the decision-making process is out of consideration. This may be due partly to the large model size considered.

Several frameworks to handle the overall MDMP have been developed. Among them, the surrogate worth trade-off (SWT) method developed by Haimes and Hall(1974)[38] and its extensions using interactive methods [e.g., Haimes *et al.*(1975)[39], Tarvainen(1984)[93], Tamura(1986)[91], and Haimes *et al.*(1990)[41]] are some of the common approaches. The SWT method is applied to various management fields [e.g., Sakawa(1978)[84], Das and Haimes(1979)[19], Kim(1998)[48], and Dhillon and Kothari(2000)[21]]. Other approaches are taken by Zionts and Wallenius(1976)[104] and Nakayama *et al.*(1980a, b)[73, 72], and Gershon and Duckstein(1983)[35]. Diagram that shows the outline of multiobjective decision-making is given in Figure 3.3

3.6 Remarks

Robust optimization model can be categorized as stochastic optimization model employing multiple realization-based approach. The proactive procedure of obtaining robust solution in both optimality and feasibility under uncertainty is a distinguished feature of RO. In the latter half of this chapter, the RO framework is reviewed from the viewpoint of vector optimization. In order to develop effective and tractable decision support systems, the role which an optimization model plays in a whole decision-making process should be considered in the development of the model.

CHAPTER 4

DETERMINISTIC OPTIMIZATION MODEL FOR WATER QUALITY MANAGEMENT IN RIVER NETWORK

4.1 Introduction

In order to control water quality in such bodies of water as streams, lakes and estuaries, it is an imperative need to develop a model-based methodology that can give an allowable maximum amount of pollutant loadings in terms of abiding by some predetermined standards. In model building, of central importance is the simple but pertinent representation of the phenomenal aspects predominantly contributory to water quality events. For the stream water quality control herein envisaged, a few modeling attempts have been made in the context of optimal control or management of the streams. These are, however, unsatisfactory because of less pertinent formulation of the phenomena concerned. ReVelle *et al.*(1968)[82] considers BOD and DO as water quality constituents, however the oxygen sag equation and the BOD decay equation are fairly simple and not directly used as constraints of an optimization model. The optimization models that are developed by Burn and McBean(1985)[10] and Fujiwara *et al.*(1988)[33] can in some degree reflect uncertainties present in water quality events. They don't employ the equations that govern BOD and DO concentrations as constraints, either. On the contrary, Futagami *et al.*(1976)[34] presents the finite element (FE) and linear programming (LP) method where the equations discretized by the finite element method are directly used as equality constraints of an optimization model. The main feature of the method is the tractability of both boundary conditions and constraints. An application is made to the systems of two-dimensional convection-diffusion phenomena, and it is showed that the calculated results agree well with those by an analytical method based on double Fourier series. The optimization calculation is, however, carried out under the spatially uniform conditions on flow velocity and diffusion coefficient.

In this chapter, a deterministic optimization model for water pollution control in a network of streams using the FE and LP method is developed. For more detailed analysis than that of Futagami *et al.*(1976)[34], non-uniform open channel flow is computed to obtain spatially distributed values of water depth, cross-sectional discharge and cross-sectional area. Some other unknown parameters such as longitudinal dispersion coefficient, deoxygenation coefficient, reaeration coefficient, removal coefficient of BOD by sedimentation and/or absorption, and saturation level of DO are evaluated by the empirical formulae. The objective of the model is to maximize the allowable total quantity of BOD loadings from outfalls, subject to water quality constraints, and to obtain an optimal allocation of BOD loadings from outfalls.

In the following sections, first, governing equations and parameter evaluation formulae are given. Second, the methodology for flow analysis is described in detail, and then constraints and an objective function are defined to formulate an optimization problem. Finally, the optimization model so obtained is applied to a hypothetical network of streams to demonstrate the validity of the model.

4.2 Governing Equations

Steady-state gradually varied flow in streams or open channels is governed by dynamic and continuity equations. The dynamic equation which holds along the channel can be expressed as

$$(1 - F_r^2)\frac{dh}{dx} + \frac{dz}{dx} + S_f = 0 \tag{4.1}$$

where x = horizontal distance along the channel, z = elevation of channel bottom above a horizontal datum, h = water depth, F_r = Froude number and S_f = friction slope. F_r and S_f are given by

$$F_r = F_r(h, Q) = Q\sqrt{\frac{\xi}{gA^3}\frac{\partial A}{\partial h}} \tag{4.2}$$

and

$$S_f = S_f(h, Q) = \frac{n^2 Q|Q|}{A^2 R^{\frac{4}{3}}} \tag{4.3}$$

where Q = cross-sectional discharge, ξ = velocity-distribution coefficient, g = gravitational acceleration, A = cross-sectional area, n = Manning's roughness coefficient, and R = hydraulic radius.

The continuity equation is expressed as

$$\frac{dQ}{dx} - \overline{q} = 0 \tag{4.4}$$

where $\overline{q}$ = lateral discharge per unit width.

Let us assume that stream water quality is represented by BOD and DO concentrations. It is assumed that injected solutes are well-mixed laterally and vertically. Then the steady-state BOD and DO profiles along a stretch of the polluted stream can be expressed by the equations

$$\Phi_b = Q\frac{dL}{dx} - \frac{d}{dx}\left(AD_x\frac{dL}{dx}\right) + A(K_1 + K_3)L + \overline{q}(L - L^L) = 0 \tag{4.5}$$

$$\Phi_d = Q\frac{dC}{dx} - \frac{d}{dx}\left(AD_x\frac{dC}{dx}\right) + AK_1L - AK_2(C_S - C) + \overline{q}(C - C^L) = 0 \tag{4.6}$$

where L and C = concentrations of BOD and DO in the main stream water, respectively, L^L and C^L = concentrations of BOD and DO in the laterally injected water, respectively, D_x = longitudinal dispersion coefficient, K_1 = deoxygenation coefficient, K_2 = reaeration coefficient, K_3 = removal coefficient of BOD by sedimentation and/or absorption and C_S = saturation level of DO. It should be noted that in Eqn.(4.6) the removal of oxygen by the respiration of algae and attached plants and the supply of oxygen by photosynthesis are neglected, and that the last terms in Eqns.(4.5) and (4.6) are those related to the injected wastewater from nonpoint sources (see Eqns.(2.18) and (2.19)). The method of treatment of the injected wastewater from point sources is described later.

The parameters D_x, K_1, K_2, K_3, C_S in Eqns.(4.5) and (4.6) are evaluated as follows.

$$D_x = mnR^{\frac{5}{6}}\frac{Q\sqrt{g}}{A} \quad (\mathrm{m^2/s})[29] \tag{4.7}$$

$$K_1 = 0.23(1.047)^{T-20} \quad (\mathrm{day^{-1}})[52] \tag{4.8}$$

$$K_2 = \left\{\frac{3.9}{h^{\frac{3}{2}}}\sqrt{\frac{Q}{A}} + \frac{0.728\sqrt{W} - 0.317W + 0.0372W^2}{h}\right\}(1.024)^{T-20} \quad (\mathrm{day^{-1}})[52] \tag{4.9}$$

$$K_3 = 0.25 \quad (\mathrm{day^{-1}})[24] \tag{4.10}$$

$$C_S = 1.43\{10.291 - 0.2809T + 0.006009T^2 - 0.0000632T^3$$
$$-0.607(0.1161 - 0.003922T + 0.0000631T^2)S\} \quad (\text{mg/L})[52] \qquad (4.11)$$

where m = undetermined parameter which varies within the range of 50 to 700 in natural streams [Fischer(1967)[29]], T = water temperature (°C), W = wind speed (m/s), and S = salinity (g/L).

The procedure to obtain equality constraints in the optimization model under consideration is shown in Figure 4.1. It should be noted that (i) the equality constraints are those obtained from the BOD and DO transport equations; and (ii) the stream flow equation (the dynamic and continuity equations) is not considered as a part of equality constraints, however the solutions of these equations play an important role in making more accurate representation of constraints. A detailed explanation of the procedure is given in the following sections.

4.3 Gradually Varied Flow Simulation

In order to obtain the solutions to Eqns.(4.1) and (4.4), the following boundary conditions are considered:

$$Q = Q^* \quad \text{on upstream boundary} \qquad (4.12)$$
$$h = h^* \quad \text{on downstream boundary} \qquad (4.13)$$

where Q^* and h^* = specified boundary values for discharge and water depth, respectively.

Regarding the simulation of the stream flow governed by Eqns.(4.1) and (4.4) with the boundary conditions Eqns.(4.12) and (4.13), the numerical model presented by Kawachi *et al.*(1996)[46] can be adopted. A one-dimensional stream network to be analyzed is divided into *NE* elements by *NN* nodes so that any junction point falls on one of the nodes.

4.3.1 Discretization of dynamic equation

The dynamic equation (4.1) is discretized by the finite volume method (FVM) [Kawachi *et al.*(1996)[46]]. In a generic element bounded by two nodes, the unknown h and the bottom elevation z are approximated by linear functions, whereas the unknown Q is assumed constant. The weighted residual form of Eqn.(4.1) is given by

$$\int \left\{ (1 - F_r^2)\frac{dh}{dx} + \frac{dz}{dx} + S_f \right\} \psi_m dx = 0 \tag{4.14}$$

where ψ_m = weighting function, and the path of integral is along the whole channel where Eqn.(4.1) holds. Substituting ψ_{mi} $(i = 1 \sim NN)$ defined by

$$\psi_{mi} = \begin{cases} 1 & (x_l(i) < x < x_r(i)) \\ 0 & \text{(otherwise)} \end{cases} \tag{4.15}$$

where $x_l(i)$ = inferior of the i-th element, and $x_r(i)$ = superior of the i-th element, into ψ_m of Eqn.(4.14) results in element equations

$$(1 - \overline{F_r^2}) \int_{x_l(i)}^{x_r(i)} \frac{dh}{dx} dx + \int_{x_l(i)}^{x_r(i)} \frac{dz}{dx} dx + \int_{x_l(i)}^{x_r(i)} S_f dx = 0 \tag{4.16}$$

that is

$$(1 - \overline{F_r^2})(h_r(i) - h_l(i)) + z_r(i) - z_l(i) + \int_{x_l(i)}^{x_r(i)} S_f dx = 0 \tag{4.17}$$

where $\overline{F_r^2}$ = mean value of F_r^2, $h_l(i)$ = nodal value of h at $x_l(i)$, and $h_r(i), z_l(i), z_r(i)$ are similarly defined. $\overline{F_r^2}$ and the integral in Eqn.(4.17) can be calculated using the 4-point Gauss quadrature rule as follows.

$$\overline{F_r^2} = \frac{1}{x_r(i) - x_l(i)} \int_{x_l(i)}^{x_r(i)} F_r^2 dx \approx \frac{1}{2} \sum_{k=1}^{4} w_k F_r(h_k, Q(i))^2 \tag{4.18}$$

where w_k = weighting factor, $Q(i)$ = i-th element value of Q, and

$$h_k = \frac{1 - c_k}{2} h_l(i) + \frac{1 + c_k}{2} h_r(i) = c_k \frac{h_r(i) - h_l(i)}{2} + \frac{h_r(i) + h_l(i)}{2} \tag{4.19}$$

where c_k = non-dimensional coordinate of the k-th integration point, and

$$\int_{x_l(i)}^{x_r(i)} S_f dx \approx \frac{x_r(i) - x_l(i)}{2} \sum_{k=1}^{4} w_k S_f(h_k, Q(i)) \tag{4.20}$$

The values of w_k and c_k are shown in Table 4.1. These numerical evaluations enable

Table 4.1: Constants w_k and c_k in 4-point Gauss quadrature rule

k	w_k	c_k
1	0.34785485	0.86113631
2	0.34785485	−0.86113631
3	0.65214515	0.33998104
4	0.65214515	−0.33998104

Eqn.(4.1) to be applied to sudden horizontal transitions where $x_l(i) = x_r(i)$, because any zero division does not appear.

4.3.2 Discretization of continuity equation

The continuity equation (4.4) is discretized by the finite element method [Kawachi *et al.*(1996)[46]]. The weighted residual form of the continuity equation Eqn.(4.4) is further reduced to a weak form

$$\int \left(\frac{dQ}{dx} - \overline{q} \right) \psi_c dx = [Q\psi_c] - \int_{\Gamma_d} Q \frac{d\psi_c}{dx} dx - \int \overline{q} \psi_c dx = 0 \tag{4.21}$$

where the path of integration is along the whole channel, ψ_c = weighting function, and Γ_d = boundary of the domain. Substituting ψ_{ci} $(i = 1 \sim NN)$ defined by

$$\psi_{ci} = \begin{cases} \dfrac{x - x_L(i)}{x_i - x_L(i)} & (x_L(i) < x \leq x_i) \\ \dfrac{x_R(i) - x}{x_R(i) - x_i} & (x_i < x < x_R(i)) \\ 0 & (\text{otherwise}) \end{cases} \tag{4.22}$$

where x_i = coordinate of the i-th node, $x_L(i)$ = coordinate of the node which is connected to x_i by a particular element, $x_R(i)$ = coordinate of the node which is connected to x_i by another particular element, if any, results in the nodal equations

$$\sum_{j=1}^{\nu(i)} \sigma(i,j) Q(\kappa(i,j)) - q_i = 0 \tag{4.23}$$

where

$$q_i = \hat{q}_i + \sum_{j=1}^{\nu(i)} \frac{1}{2}\overline{q}(\kappa(i,j))l(\kappa(i,j)) \tag{4.24}$$

$\nu(i)$ = number of elements meeting at the i-th node, $\kappa(i,j)$ = element number of the j-th element connected to x_i, $\hat{q}_i$ = inflow discharge of wastewater from point sources into the i-th node from the exterior, $\overline{q}(\kappa(i,j))$ = injected wastewater from nonpoint sources into the $\kappa(i,j)$-th element, $l(\kappa(i,j))$ = length of the $\kappa(i,j)$-th element, and $\sigma(i,j) = -1$ if $+x$ of the $\kappa(i,j)$-th element is directed toward x_i, otherwise, $\sigma(i,j) = +1$. The relation represented in Eqn.(4.23) is illustrated in Figure 4.2.

4.3.3 Newton-Raphson method

It is noted that only one node is settled even for a junction where streams are interconnected, and that the number of Eqns.(4.17) and (4.23) is equal to that of variables included in these equations. Eqns.(4.17) and (4.23) can be solved by the Newton-Raphson method. The method is an iterative procedure by successively calculating points that yields improved approximations to the solution to a system of algebraic equations. Eqns.(4.17) and (4.23) can be rewritten with $i = 1, 2, \cdots, NE, \quad j = 1, 2, \cdots, NN$, respectively, as

$$\begin{cases} f_i(h_j, Q_i) &= 0 \\ g_j(h_j, Q_i) &= 0 \end{cases} \tag{4.25}$$

Considering Taylor series of those equations near h_j^{k-1} and Q_i^{k-1} gives

$$f_i(h_j^k, Q_i^k) = f_i(h_j^{k-1}, Q_i^k) - \frac{\partial f_i}{\partial h}(h_j^{k-1}, Q_i^k)\Delta h_j^k \approx 0, \quad \Delta h_j^k = h_j^{k-1} - h_j^k \tag{4.26}$$

$$g_j(h_j^k, Q_i^k) = g_j(h_j^k, Q_i^{k-1}) - \frac{\partial g_j}{\partial Q}(h_j^k, Q_i^{k-1})\Delta Q_i^k \approx 0, \quad \Delta Q_i^k = Q_i^{k-1} - Q_i^k \tag{4.27}$$

Then the following matrix-vector form is derived.

$$\begin{bmatrix} \frac{\partial f_i}{\partial h}(h_j^{k-1}, Q_i^k) & 0 \\ 0 & \frac{\partial g_j}{\partial Q}(h_j^k, Q_i^{k-1}) \end{bmatrix} \begin{bmatrix} \Delta h_j^k \\ \Delta Q_i^k \end{bmatrix} = \begin{bmatrix} f_i(h_j^{k-1}, Q_i^k) \\ g_j(h_j^k, Q_i^{k-1}) \end{bmatrix} \tag{4.28}$$

where

$$\frac{\partial f_i}{\partial h}(h_j^{k-1}, Q_i^k) = \frac{f_i(h_j^{k-1} + \Delta h_j^k, Q_i^k) - f_i(h_j^{k-1} - \Delta h_j^k, Q_i^k)}{2\Delta h_j^k} \tag{4.29}$$

$$\frac{\partial g_j}{\partial Q}(h_j^k, Q_i^{k-1}) = \frac{g_j(h_j^k, Q_i^{k-1} + \Delta Q_i^k) - g_i(h_j^k, Q_i^{k-1} - \Delta Q_i^k)}{2\Delta Q_i^k} \tag{4.30}$$

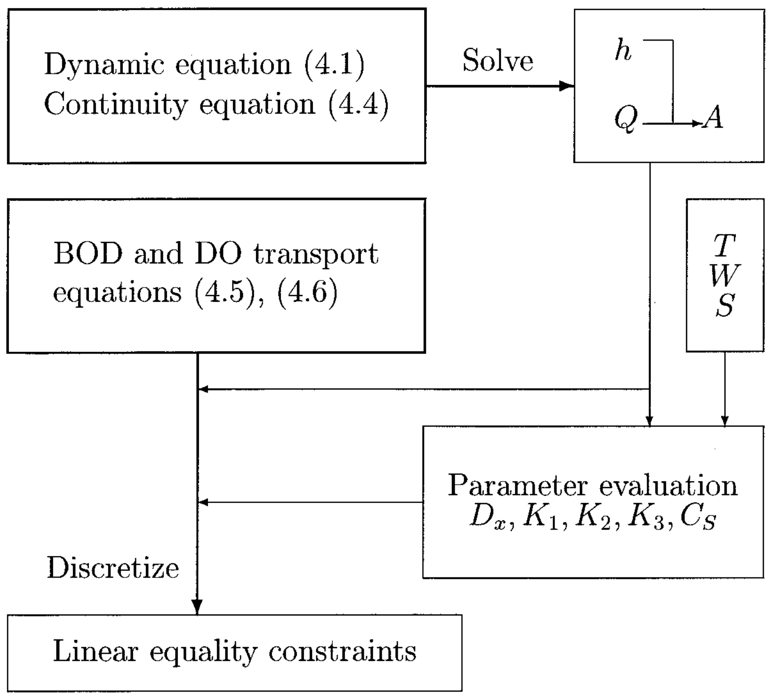

Figure 4.1: Derivation of equity constraints in linear programming problem

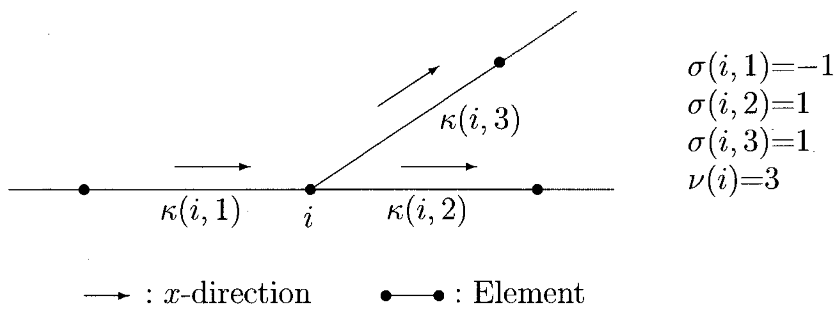

Figure 4.2: Relation between node i and elements that are connected with the node

Eqn.(4.28) can be solved by using the Gauss elimination method. After that, the k-th approximations of h_j and Q_i are computed as follows.

$$h_j^k = h_j^{k-1} - \Delta h_j^k \tag{4.31}$$

$$Q_i^k = Q_i^{k-1} - \Delta Q_i^k \tag{4.32}$$

If the differences Δh_j^k and ΔQ_i^k are sufficiently small, the iterative procedure terminates. Nodal values of h and A and elemental values of Q are used to estimate the coefficients of BOD and DO transport equations in the subsequent section.

4.4 Optimization Model

Exterior boundary conditions of Eqn.(4.5) which is not solved but discretized only are imposed as

$$L = L^* \quad \text{on upstream boundary} \tag{4.33}$$

and

$$AD_x \frac{dL}{dx} = -f_L^* \quad \text{on downstream boundary} \tag{4.34}$$

Exterior boundary conditions of Eqn.(4.6) are similarly specified as

$$C = C^* \quad \text{on upstream boundary} \tag{4.35}$$

and

$$AD_x \frac{dC}{dx} = -f_C^* \quad \text{on downstream boundary} \tag{4.36}$$

where L^* and C^* = specified boundary values for BOD and DO concentrations, respectively, and f_L^* and f_C^* = dispersive BOD and DO fluxes (outward positive), respectively. At a junction, compatibility conditions must be imposed that the individual concentrations at the end of streams toward the junction are the same. Usually special treatments are needed to specify all such conditions as interior boundary conditions. In the present model, however, these conditions are completely satisfied without such treatments since the junction is regarded as a point junction and thus a common node is placed at the point where streams meet.

Next, the finite element method is employed to cast the Eqns.(4.5) and (4.6) into a system of linear algebraic equations.

4.4.1 Discretization of BOD transport equation

Applying the weighted residual method to Eqns.(4.5) gives the weighted residual form

$$\int \psi_b \left\{ Q\frac{dL}{dx} - \frac{d}{dx}\left(AD_x\frac{dL}{dx}\right) + A(K_1+K_3)L + \overline{q}(L-L^L) \right\} dx = 0 \tag{4.37}$$

where ψ_b = weighting function for Eqn.(4.5), and the path of integral is along the whole channel where Eqn.(4.5) holds. A weak form of Eqn.(4.5) is written as

$$\int \psi_b \left\{ Q\frac{dL}{dx} + A(K_1+K_3)L + q(L-L^L) \right\} dx - \left[\psi_b AD_x\frac{dL}{dx}\right] + \int AD_x\frac{d\psi_b}{dx}\frac{dL}{dx}dx = 0 \tag{4.38}$$

By considering the boundary conditions (4.33) and (4.34), the following equation is obtained.

$$\int \left\{ Q\psi_b\frac{dL}{dx} + AD_x\frac{d\psi_b}{dx}\frac{dL}{dx} + A(K_1+K_3)\psi_b L + q\psi_b(L-L^L) \right\} dx = -\psi_b f_L^* \tag{4.39}$$

Since this equation has an advective term, a more sophisticated scheme than the standard Galerkin scheme is required. Thus, the upwind scheme presented by Unami *et al.*(1996)[95] is now introduced. A weighting function w_j, which is a function of the local Peclet number P_e given by

$$P_e = \frac{Q\Delta x}{AD_x} \tag{4.40}$$

where Δx = element length, is defined with a dissipation parameter c ($|c| \geq 1$) (Table 4.2 and Figure 4.3). The weighting functions $w_j (j = 1 \sim NN)$ are substituted for ψ_b in Eqn.(4.39).

In a generic element bounded by two nodes 1 and 2, the unknown L is approximated by the linear shape functions as

Table 4.2: Definition of weighting function w_j

	x directs the node j	$-x$ directs the node j
$P_e \geq 0$	$\left(\frac{x}{\Delta x}\right)^{\frac{1}{1+\lvert cP_e \rvert}}$	$\left(\frac{\Delta x - x}{\Delta x}\right)^{1+\lvert cP_e \rvert}$
$P_e < 0$	$\left(\frac{x}{\Delta x}\right)^{1+\lvert cP_e \rvert}$	$\left(\frac{\Delta x - x}{\Delta x}\right)^{\frac{1}{1+\lvert cP_e \rvert}}$

$$L = \sum_{k=1}^{2} N_k L_k, \quad k = 1, 2 \tag{4.41}$$

$$N_1 = \frac{x_2 - x}{x_2 - x_1}, \quad N_2 = \frac{x - x_1}{x_2 - x_1} \tag{4.42}$$

where x_1 and x_2 = coordinates for node 1 and 2, respectively, N_1 and N_2 = shape functions for node 1 and node 2, respectively, and L_k = nodal values of BOD concentration. In this element, a weighting function w is approximated with arbitrary constants b_1 and b_2 as follows:

$$w = \sum_{j=1}^{2} b_j w_j \tag{4.43}$$

Then the left hand side of Eqn.(4.39) can be separated into term integrations

$$\int \psi_b \Phi_b(L) dx = \sum_l \int_l \psi_b \Phi_b(L) dl = \sum_l \int_l \sum_j b_j w_j \Phi_b(\sum_k N_k L_k) dl \tag{4.44}$$

Namely the following equation holds in a general element l

$$b_1 \int_l w_1 \Phi_b(\sum_k N_k L_k) dl + b_2 \int_l w_2 \Phi_b(\sum_k N_k L_k) dl = 0 \tag{4.45}$$

Since the constants b_1 and b_2 are arbitrary in the equation above, the following equations are deduced.

$$\int_l w_1 \Phi_b(\sum_k N_k L_k) dl = 0 \tag{4.46}$$

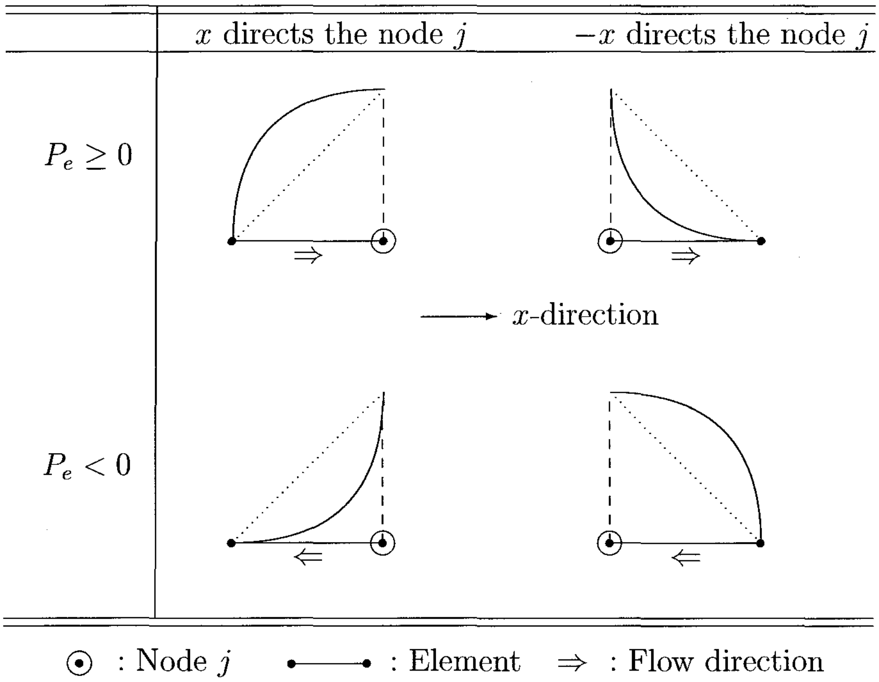

Figure 4.3: Shape of weighting function w_j

$$\int_l w_2 \Phi_b(\sum_k N_k L_k) dl = 0 \tag{4.47}$$

It is noted that the term including the lateral discharge $\overline{q}$ is temporarily neglected here, and later on it is considered again. The element that includes the downstream boundary is not considered herein, either. Then combining Eqns.(4.39), (4.46) and (4.47) results in

$$Q \int_l w_1 \frac{d(\sum N_k L_k)}{dx} dl + AD_x \int_l \frac{dw_1}{dx} \frac{d(\sum N_k L_k)}{dx} dl + A(K_1 + K_3) \int_l w_1 \sum N_k L_k dl = 0 \tag{4.48}$$

$$Q \int_l w_2 \frac{d(\sum N_k L_k)}{dx} dl + AD_x \int_l \frac{dw_2}{dx} \frac{d(\sum N_k L_k)}{dx} dl + A(K_1 + K_3) \int_l w_2 \sum N_k L_k dl = 0 \tag{4.49}$$

These equations are consolidated into the following finite element equation.

$$\left\{ Q \int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \begin{bmatrix} \frac{dN_1}{dx} & \frac{dN_2}{dx} \end{bmatrix} dl \;+\; AD_x \int_l \begin{bmatrix} \frac{dw_1}{dx} \\ \frac{dw_2}{dx} \end{bmatrix} \begin{bmatrix} \frac{dN_1}{dx} & \frac{dN_2}{dx} \end{bmatrix} dl \right.$$

$$+A(K_1+K_3)\int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} [N_1 \quad N_2]\, dl \Bigg\} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = 0 \quad (4.50)$$

This can be rewritten in a compact form

$$E^L_{jk} L_k = 0, \quad j,k = 1,2 \tag{4.51}$$

where E^L_{jk} = coefficient matrix (2×2).

4.4.2 Discretization of DO transport equation

The DO transport equation (4.6) is also discretized by the finite element method in the same way described in the last section. The finite element equation is then given by

$$\Bigg\{ Q \int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \begin{bmatrix} \frac{dN_1}{dx} & \frac{dN_2}{dx} \end{bmatrix} dl + AD_x \int_l \begin{bmatrix} \frac{dw_1}{dx} \\ \frac{dw_2}{dx} \end{bmatrix} \begin{bmatrix} \frac{dN_1}{dx} & \frac{dN_2}{dx} \end{bmatrix} dl + AK_2 \int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} [N_1 \quad N_2]\, dl \Bigg\} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$
$$+ \Bigg\{ AK_1 \int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} [N_1 \quad N_2]\, dl \Bigg\} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} - AK_2 C_s \int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} dl = 0 \tag{4.52}$$

or in a compact form

$$E^C_{jk} C_k + \overline{E_{jk}} L_k - b_k = 0, \quad j,k = 1,2 \tag{4.53}$$

where E^C_{jk} and $\overline{E_{jk}}$ = coefficient matrices (2×2), respectively, and b_k = constant vector.

4.4.3 Numerical integration

The terms

$$\int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \begin{bmatrix} \frac{dN_1}{dx} & \frac{dN_2}{dx} \end{bmatrix} dl, \quad \int_l \begin{bmatrix} \frac{dw_1}{dx} \\ \frac{dw_2}{dx} \end{bmatrix} \begin{bmatrix} \frac{dN_1}{dx} & \frac{dN_2}{dx} \end{bmatrix} dl, \quad \int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} [N_1 \quad N_2]\, dl, \quad \int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} dl \tag{4.54}$$

that are included in Eqns.(4.50) and (4.52) are concretely integrated here. The values of w_1 and w_2 are given as

$$w_1 = \begin{cases} N_1^{\frac{1}{p}} & (P_{el} \geq 0) \\ N_1^p & (P_{el} < 0) \end{cases}, \quad w_2 = \begin{cases} N_2^p & (P_{el} \geq 0) \\ N_2^{\frac{1}{p}} & (P_{el} < 0) \end{cases} \tag{4.55}$$

where

$$P_{el} = \frac{Q_l \Delta x_l}{A_l D_{x_l}}, \quad p = \frac{1}{1 + |cP_{el}|} \tag{4.56}$$

and P_e = Peclet number in element l.

Numerical integrations of those terms are conducted using the Simpson's 1/3 rule that is expressed as

$$\int_{x_1}^{x_2} f(x)dx = \frac{l/2}{3}\left\{f(x_1) + 4f\left(\frac{x_1 + x_2}{2}\right) + f(x_2)\right\} \tag{4.57}$$

where x_1 and x_2 = values of nodes between which exists the element l, respectively. The results of calculating the terms in the state $P_{el} \geq 0$ are summarized as follows.

(i)

$$\int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \begin{bmatrix} \dfrac{dN_1}{dx} & \dfrac{dN_2}{dx} \end{bmatrix} dl = \frac{1}{6} \begin{bmatrix} -\{1 + (\frac{1}{2})^{\frac{1}{p}-2}\} & 1 + (\frac{1}{2})^{\frac{1}{p}-2} \\ -\{1 + (\frac{1}{2})^{p-2}\} & 1 + (\frac{1}{2})^{p-2} \end{bmatrix} \tag{4.58}$$

(ii)

$$\int_l \begin{bmatrix} \frac{dw_1}{dx} \\ \frac{dw_2}{dx} \end{bmatrix} \begin{bmatrix} \dfrac{dN_1}{dx} & \dfrac{dN_2}{dx} \end{bmatrix} dl = \frac{1}{6l} \begin{bmatrix} \frac{1}{p}\{1 + (\frac{1}{2})^{\frac{1}{p}-3}\} & -\frac{1}{p}\{1 + (\frac{1}{2})^{\frac{1}{p}-3}\} \\ -p\{1 + (\frac{1}{2})^{p-3}\} & p\{1 + (\frac{1}{2})^{p-3}\} \end{bmatrix} \tag{4.59}$$

(iii)

$$\int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} [N_1 \quad N_2]\, dl = \frac{l}{6} \begin{bmatrix} 1 + (\frac{1}{2})^{\frac{1}{p}-1} & (\frac{1}{2})^{\frac{1}{p}-1} \\ (\frac{1}{2})^{\frac{1}{p}-1} & 1 + (\frac{1}{2})^{\frac{1}{p}-1} \end{bmatrix} \tag{4.60}$$

(iv)

$$\int_l \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} dl = \frac{l}{6} \begin{bmatrix} 1 + (\frac{1}{2})^{\frac{1}{p}-2} \\ 1 + (\frac{1}{2})^{p-2} \end{bmatrix} \tag{4.61}$$

If $P_{el} < 0$, then p is replaced by $\dfrac{1}{p}$ in the above results.

4.4.4 Matrix-vector form of discretized BOD and DO transport equations

The finite element equations of the BOD and DO transport equations are written as

$$E_{jk}^{L} L_k = 0, \quad j, k = 1, 2 \tag{4.62}$$

$$E^C_{jk} C_k + \overline{E_{jk}} L_k - b_k = 0, \quad j,k = 1,2 \tag{4.63}$$

or in a matrix-vector form

$$\begin{bmatrix} E^L_{jk} & 0 \\ \overline{E_{jk}} & E^C_{jk} \end{bmatrix} \begin{bmatrix} L_k \\ C_k \end{bmatrix} = \begin{bmatrix} 0 \\ b_k \end{bmatrix} \tag{4.64}$$

Assembling the finite element equations for all the elements yields the global equations system as follows:

$$\begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1,2NN} \\ e_{21} & e_{22} & \cdots & e_{2,2NN} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ e_{2NN,1} & e_{2NN,2} & \cdots & e_{2NN,2NN} \end{bmatrix} \begin{bmatrix} L_1 \\ C_1 \\ L_2 \\ C_2 \\ \vdots \\ L_{NN} \\ C_{NN} \end{bmatrix} = \begin{bmatrix} 0 \\ b_1 \\ 0 \\ b_2 \\ \vdots \\ 0 \\ b_{NN} \end{bmatrix} \tag{4.65}$$

where $e_{ij} = ij$-element of the global matrix. Since the wastewater loading and boundary conditions should be taken into account in this stage, the left hand side of this equation is modified as

$$\left[\begin{array}{cccccc|cccc} e_{11} & e_{12} & \cdots & & \cdots & e_{1,2NN} & 0 & 0 & \cdots & 0 \\ e_{21} & e_{22} & \cdots & & \cdots & e_{2,2NN} & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & e_{2i-1,2i-1} + q_j & & \cdots & \cdots & -q_j & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & e_{2i,2i} + q_j & & \cdots & 0 & -q_j & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & -q_{NL} \\ e_{2NN,1} & e_{2NN,2} & \cdots & & \cdots & e_{2NN,2NN} & 0 & 0 & \cdots & 0 \end{array}\right] \begin{bmatrix} L_1 \\ C_1 \\ L_2 \\ C_2 \\ \vdots \\ L_{NN} \\ C_{NN} \\ \hline L^L_1 \\ C^L_1 \\ \vdots \\ L^L_{NL} \\ C^L_{NL} \end{bmatrix} \tag{4.66}$$

where q_j = total wastewater discharged into the j-th node defined by Eqn.(4.24) ($1 \leq j \leq NL$, NL = number of loading point), and the matrix in Eqn.(4.66) is $(2NN) \times$

(2NN+2NL). The values of q_j are added to the (2i-1,2i-1)-th, (2i,2i)-th, (2i-1,2j-1)-th and (2i,2j)-th components of the matrix. The resulting equation can be reexpressed as

$$E\mathbf{L} + F\mathbf{L}^L = \mathbf{b}, \quad G\mathbf{C} + H\mathbf{C}^L = \mathbf{d} \tag{4.67}$$

where $\mathbf{L}$ and $\mathbf{C}$ = vectors whose components are BOD and DO concentrations of river water, respectively, $\mathbf{L}^L$ and $\mathbf{C}^L$ = vectors whose j-th components are BOD and DO concentrations of injected wastewater L_j^L and C_j^L, respectively, E, F, G and H = coefficient matrices, and $\mathbf{b}$ and $\mathbf{d}$ = constant vectors.

4.4.5 Formulation of optimization model

Long-range planning problem, not the daily or weekly operational one, of optimal wasteload allocation for a river system is considered. This assumed control horizon is different from that in the studies by Spear and Hornberger(1983)[89] and Rauch and Harremoës(1999)[80]. By the FE and LP method, Eqn.(4.67) is used directly as equality constraints of a linear programming problem developed herein. Inequality constraints consist of some water quality limitations for the injected wastewater and at the selected monitoring stations, and nonnegative conditions for all variables. This implies that the stream water at all monitoring stations and the wastewater must meet the in-stream water quality standards and effluent limitations, respectively, and therefore the stream water in the channel except at the monitoring stations dose not always need to satisfy the standards. The objective of the problem is to maximize the total BOD loadings from the loading points under aforementioned conditions. From the viewpoint of the assimilation capacity of the environment, such an objective function may give the upper limit of the total acceptable loading in a network of streams. Thus the complete linear programming model is written as

$$\text{Maximize} \quad z = \sum_{j=1}^{NL} q_j L_j^L \tag{4.68}$$

subject to

$$E\mathbf{L} + F\mathbf{L}^L = \mathbf{b}, \quad G\mathbf{C} + H\mathbf{C}^L = \mathbf{d} \tag{4.69}$$

$$\underline{L_j^L} \leq L_j^L \leq \overline{L_j^L}, \; \underline{C_j^L} \leq C_j^L \leq \overline{C_j^L} \quad \forall j \tag{4.70}$$

$$0 \leq L_k \leq \overline{L_k}, \; 0 \leq \underline{C_k} \leq C_k \quad \forall k \tag{4.71}$$

$$\mathbf{L}, \mathbf{L}^L, \mathbf{C}, \mathbf{C}^L \geq 0 \tag{4.72}$$

where j = node number associated with the j-th decision variable, k = node number at which water quality is monitored, q_j = lateral discharge at the node j, $\underline{L_j^L}$ and $\overline{L_j^L}$ = lower and upper limit of BOD concentration in injected wastewater through outfall, respectively, $\underline{C_j^L}$ and $\overline{C_j^L}$ = lower and upper limit of DO concentration in injected wastewater, respectively, $\overline{L_k}$ = upper limit of BOD concentration in the river water at monitoring station, and $\underline{C_k}$ = lower limit of DO concentration in the river water at monitoring station. Since this problem has a linear objective function and linear constraints, it can be solved by the simplex method [e.g., Ibaraki and Fukushima(1991)[42] and Sakawa(1984)[85]]. The optimal value of total BOD loadings and all values of BOD and DO concentrations in the injected water at all loading points, i.e., decision variables, are determined. Furthermore, BOD and DO concentrations in the main stream water at all nodes in the channel, i.e., state variables, are obtained.

4.5 Demonstrative Example

To demonstrate the validity of the proposed methodology, a water pollution control problem in a hypothetical network of streams is solved. The arrangement of channel reaches, boundaries, loading points (LPs) and monitoring stations (MSs) are shown in Figure 4.4. The network that comprises 8 reaches (R-1 to R-8) with a uniform bed slope of 1/10,000 is divided into 34 elements with 34 nodes for finite element discretization. Each element is 500m long. The boundary values specified at the boundaries 1, 2, 3 and 4 are taken as: $Q_1^* = 50.0(\mathrm{m}^3/\mathrm{s})$, $L_1^* = 1.5(\mathrm{mg/L})$, $C_1^* = 8.0(\mathrm{mg/L})$, $h_i^* = 2.0(\mathrm{m})$ and $f_{Li}^* = f_{Ci}^* = 0$ $(i = 2, 3, 4)$. Velocity-distribution coefficient $\xi = 1.1$, Manning's roughness coefficient $n = 0.03(\mathrm{s/m}^{1/3})$, undetermined parameter $m = 200$, dissipation parameter $c = 1.0$, salinity $S = 0$, water temperature $T = 15.0(^\circ\mathrm{C})$ and wind speed $W = 3.0(\mathrm{m/s})$ are assumed to be constant along all the reaches in the network. The wastewaters only from point sources are considered and those from nonpoint sources are neglected in this example. The wastewater flowing into the loading points and the stream water at the monitoring stations must meet the water quality standards given in Tables 4.3 and 4.4. The outfall discharge at the j-th loading point is $q_j = 0.5(\mathrm{m}^3/\mathrm{s})$ for j = 1, 2, 3, 4 and 5, and $q_j = 1.5(\mathrm{m}^3/\mathrm{s})$ for $j = 6$ and 7. The objective function is defined by $z = \sum_{j=1}^{7} q_j L_j^L$. Then, the linear programming problem contains 82 variables. This problem is solved by the simplex method to obtain not only the optimal BOD and DO concentrations in the injected water, but also the profiles of these concentrations in the network. The optimal solution obtained is consolidated in Table 4.5 and Figure 4.5. The discharge profile and the BOD and DO concentration profiles along a stretch of the reaches R-1, R-2, R-6 and R-8 are shown in Figure 4.6, 4.7 and 4.8, respectively. It can be confirmed that water

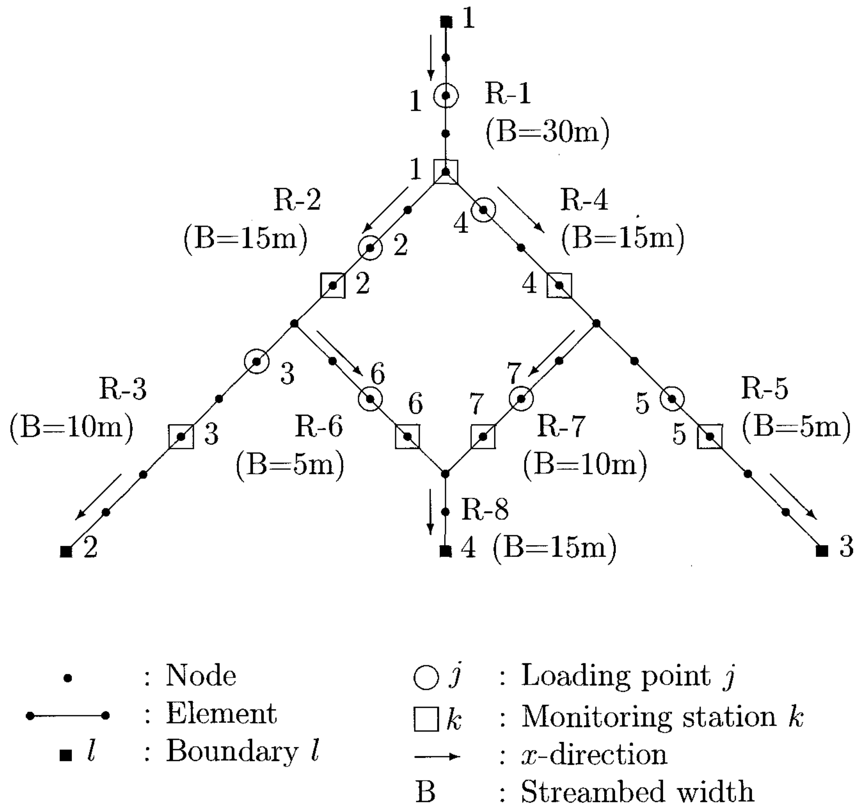

Figure 4.4: Topological sketch of stream network

quality standards at the monitoring stations are satisfied, but are not always satisfied at the other points.

4.6 Conclusions

The optimization model for water pollution control in a network of interconnected streams has been formulated using the FE and LP method. The numerical example for the hypothetical network demonstrates that the model successfully determines allowable maximum pollutant (BOD) loading from each of the outfalls.

Since phenomenal aspects of the methodology herein proposed are described by coupled BOD and DO transport equations, reliability of the optimization model obtained is

Table 4.3: Conditions for wastewater

Loading point number j	$\underline{L_j^L}$ (mg/L)	$\overline{L_j^L}$ (mg/L)	$\underline{C_j^L}$ (mg/L)	$\overline{C_j^L}$ (mg/L)
1,2,3,4,5	1.5	30.0	1.0	5.0
6,7	1.0	30.0	1.0	5.0

Table 4.4: Conditions at monitoring stations

Monitoring station number k	$\overline{L_k}$ (mg/L)	$\underline{C_k}$ (mg/L)
1,2,4	2.0	7.5
3,5,6,7	2.5	7.0

Table 4.5: Optimal solution

Loading point number j	L_j^L (mg/L)	C_j^L (mg/L)	BOD loading (g/s)
1	30.000	1.000	15.000
2	8.701	1.000	4.350
3	17.179	1.000	8.589
4	8.805	1.000	4.402
5	9.221	1.000	4.611
6	5.802	5.000	8.703
7	9.780	1.000	14.670

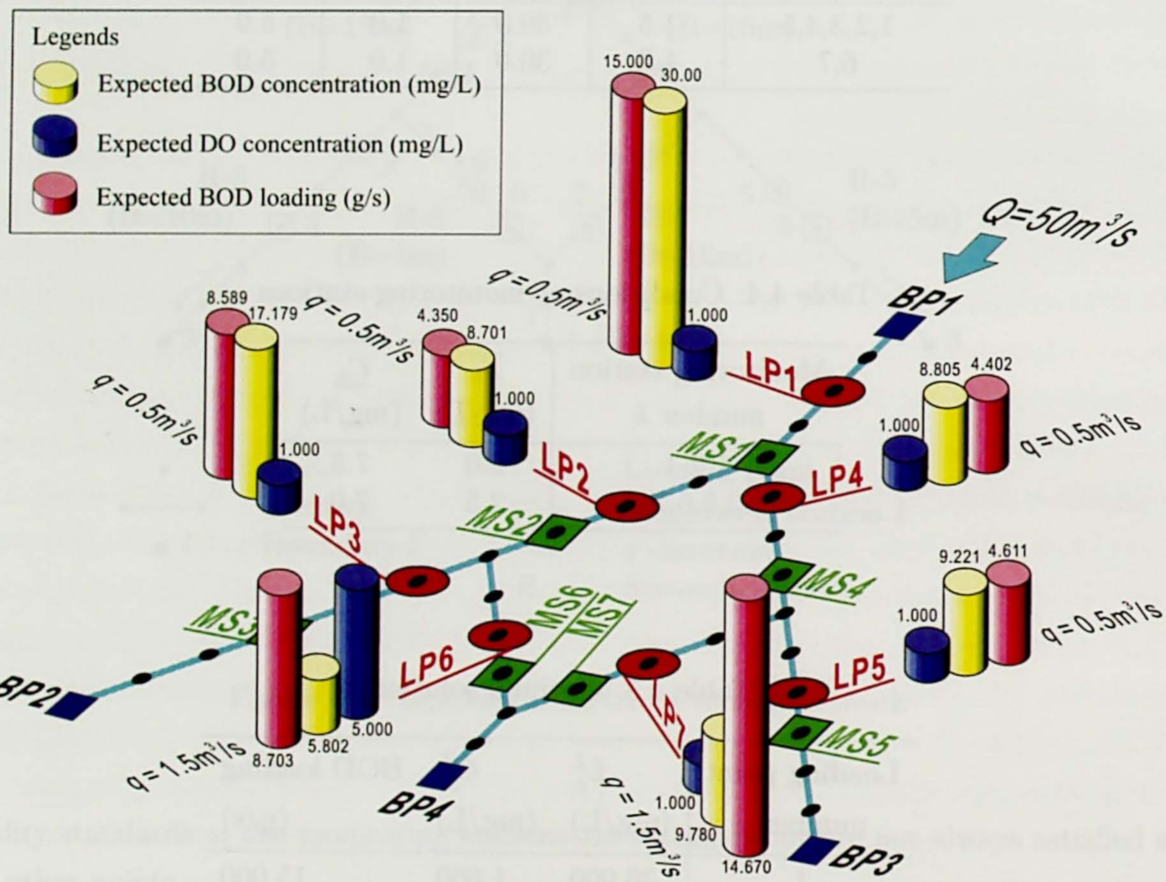

Figure 4.5: Optimal wasteload allocation

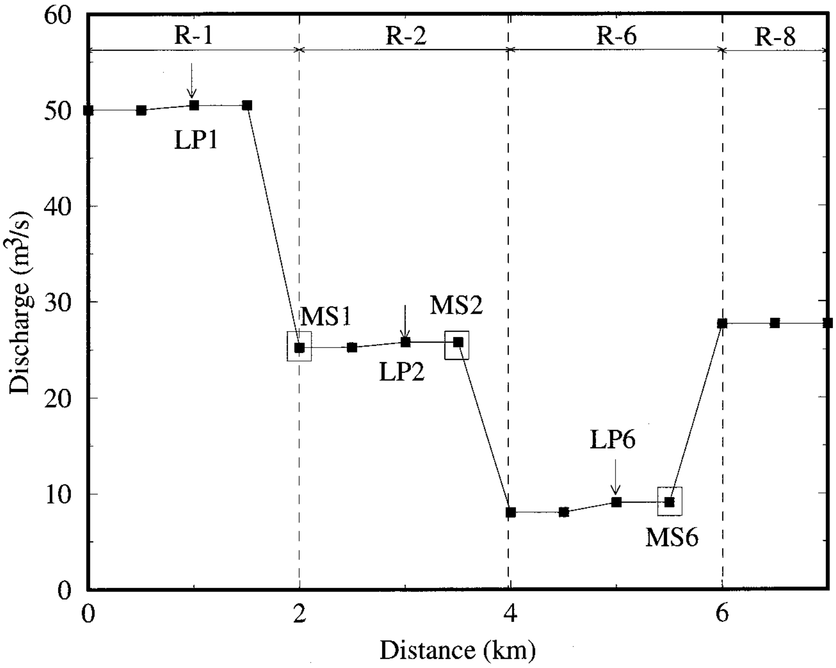

Figure 4.6: Discharge profile (LP: Loading point, MS: Monitoring station)

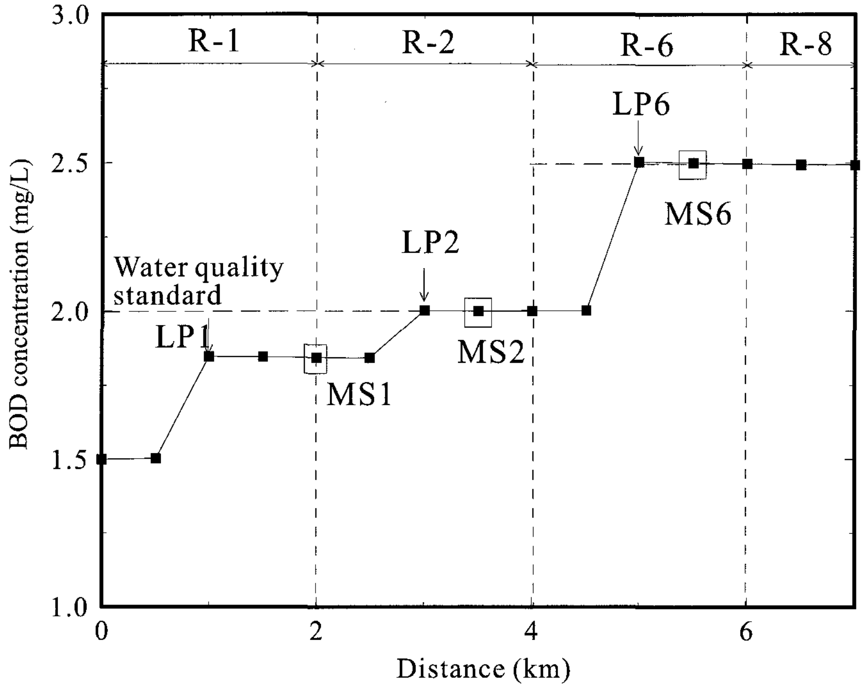

Figure 4.7: BOD concentration profile (LP: Loading point, MS: Monitoring station)

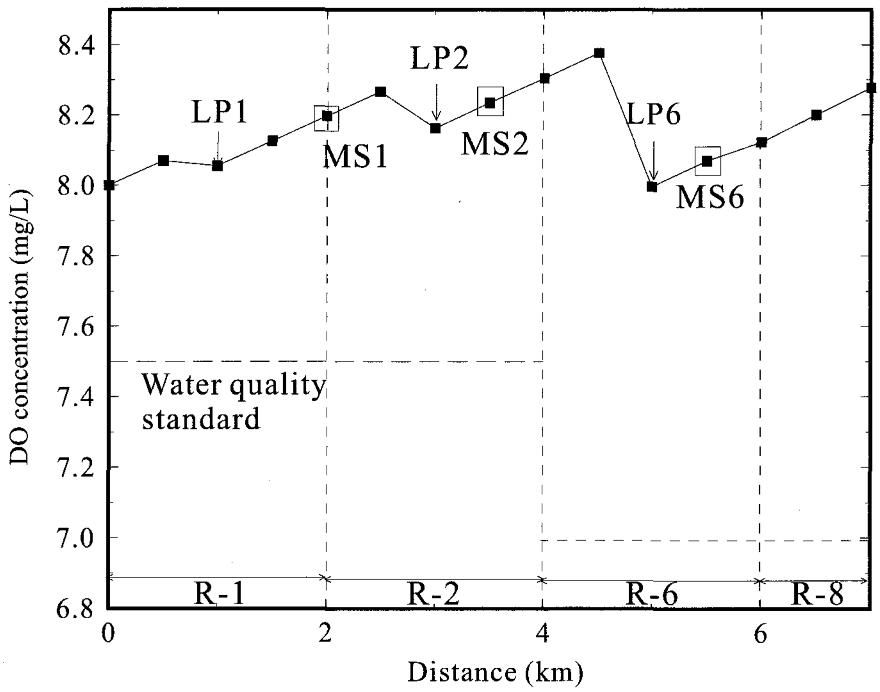

Figure 4.8: DO concentration profile (LP: Loading point, MS: Monitoring station)

importantly affected by evaluated values of the predetermined parameters in the equations as well as by level of the numerical approximation to the equations. In the present study, two approaches to evaluation of the parameters are employed; simulation-based and empirical formula-based approaches. The former provides water depth, cross-sectional discharge and cross-sectional area in streams through a prior practice of gradually varied flow simulation, while the latter directly evaluates longitudinal dispersion coefficient, deoxygenation coefficient, reaeration coefficient, removal coefficient of DO and saturation level of DO. Use of a more refined simulation model for stream flow and evolved empirical formulae for the transport and reaction coefficients might alleviate the discrepancies between real and computed events that are encountered in practical use of the transport equations, thus ameliorating the potentialities of the optimization model.

CHAPTER 5

ROBUST OPTIMIZATION OF RIVER WATER QUALITY MANAGEMENT

5.1 Introduction

Mathematical programming can be a useful technique to make decisions on river water quality management, because it enables us to treat water quality regulations, physical phenomena in rivers and our economic activities quantitatively. However the management strategies derived from traditional deterministic models, including the model presented in Chapter 4, are not robust enough to be applied to real world problems. It is indispensable to consider uncertainties that are inevitably included in the data for mathematical programming models. Therefore various stochastic optimization models and fuzzy optimization models have been developed. For example, Burn and McBean(1985)[10] develops an optimization model with an ability to reflect uncertainties present in water quality problem, employing chance constrained programming technique. Chang *et al.*(1997)[14] applies an interactive fuzzy interval multiobjective mixed integer programming model for water pollution control in a river basin.

A framework of robust optimization (RO) for achieving robustness in management plans is developed by Mulvey *et al.*(1995)[70]. This approach integrates goal programming formulations with a scenario-based description of problem data, in order to generate a series of solutions that are less sensitive to realizations of the data from a scenario. Watkins and McKinney(1997)[100] applies the RO to the two example issues of urban water transfer planning and groundwater quality management to demonstrate its ability.

Most of the researches dealing with optimal water quality management or control in such bodies of water as rivers, lakes and estuaries have been devoted to minimizing costs in the context of planning for the investment in water quality control projects. In this context, the treatment curve is often considered, which plays a major role in the optimization model formulation. However Sasikumar and Mujumdar(1998)[87] points out that this type of formulation is disadvantaged due to two reasons: nonconvexity and uncertainty of the cost curve. An alternative optimization perspective is associated with searching for allowable pollutant loads from outfalls to conserve or improve water qualities in a body of water. A deterministic optimization model in this category is formulated for water quality control by Futagami *et al.*(1976)[34] using the finite element (FE) and linear programming (LP) approach. The optimization model developed in the last chapter is also included in this class.

The aim of this chapter is to make an improvement over the work presented in Chapter 4 by using the RO concept, i.e., to build a non-deterministic optimization model with probabilistic consideration to relevant uncertainties. First a brief review of the deterministic optimization model is made. After descriptions of uncertainties to be considered in the current problem, and their incorporation into the optimization model by the RO approach, the ability of the RO model developed is demonstrated through a sample optimization in a hypothetical river system.

5.2 Optimization Model

5.2.1 Deterministic model

To develop an RO model for stream water quality management, the deterministic linear programming model formulated in the previous chapter is briefly reviewed by reason of explanation. The role of the linear programming model is to maximize the total BOD loading injected into streams or open channels, and to obtain an optimal allocation of BOD loadings from outfalls. As equality constraints for the optimization model, the BOD and DO transport equations [Eqns.(4.5) and (4.6)], which are discretized by the finite element method (FEM) later, are employed.

The coefficients of these equations are evaluated resorting to steady-state open channel flow simulation and empirical formulae. It is assumed that the lateral discharge is perpendicular to the channel and the pressure distribution is hydrostatic. Then hydraulic variables are obtained by solving the following continuity and momentum equations using FEM and FVM, respectively [Unami(1998)[94]].

$$\frac{dQ}{dx} - \overline{q} = 0 \tag{5.1}$$

$$\frac{dF_M(h,Q)}{dx} + S_M(h,Q) = 0 \tag{5.2}$$

where

$$F_M = \frac{\zeta Q^2}{A(h)} + g\int_0^h A(z)dz \tag{5.3}$$

$$S_M = -g\int_0^h \frac{\partial A(z)}{\partial x}dz + gA(h)\frac{\partial z_b}{\partial x} + g\frac{n^2 Q|Q|}{A(h)R^{4/3}} \tag{5.4}$$

x = horizontal distance along the channel, Q = cross-sectional discharge, A = cross-sectional area, $\overline{q}$ = lateral discharge per unit width, z = vertical distance originated at a horizontal datum, z_b = elevation of channel bed, h = water depth, g = gravitational acceleration, ζ = velocity-distribution coefficient, n = Manning's roughness coefficient, and R = hydraulic radius. The parameters D_x, K_1, K_2, K_3 and C_S in the transport equations of BOD and DO [Eqns.(4.5) and (4.6)] are evaluated by Eqns.(4.7)-(4.11).

Effluent limitation standards, river water quality standards and non-negative conditions are used as inequality constraints for the deterministic optimization model. The complete linear programming model is then expressed as follows.

$$\text{Maximize} \quad \sum_j q_j L_j^L \tag{5.5}$$

subject to
(i) BOD and DO transport equations that are discretized by the FEM

$$E\mathbf{L} + F\mathbf{L}^L = \mathbf{b}, \quad G\mathbf{C} + H\mathbf{C}^L = \mathbf{d} \tag{5.6}$$

(ii) Effluent limitation standards

$$\mathbf{L}^{Ll} \leq \mathbf{L}^L \leq \mathbf{L}^{Lu}, \quad \mathbf{C}^{Ll} \leq \mathbf{C}^L \leq \mathbf{C}^{Lu} \tag{5.7}$$

(iii) River water quality standards at monitoring stations

$$0 \leq \mathbf{L}^M \leq \mathbf{L}^u, \quad 0 \leq \mathbf{C}^l \leq \mathbf{C}^M \tag{5.8}$$

(iv) Nonnegativity

$$\mathbf{L}, \mathbf{L}^L, \mathbf{L}^M, \mathbf{C}, \mathbf{C}^L, \mathbf{C}^M \geq 0 \tag{5.9}$$

where superscripts L, M, u and l stand for injected wastewater, river water at monitoring stations, upper limit and lower limit, respectively, $\mathbf{L}$ and $\mathbf{C}$ = vectors whose components are BOD and DO concentrations at the nodes in the river, respectively, $\mathbf{L}^L$ and $\mathbf{C}^L$ =

vectors whose components are BOD and DO concentrations in the wastewater that is injected into the j-th node of loading point, L_j^L and C_j^L, respectively, $\mathbf{L}^M$ and $\mathbf{C}^M$ = vectors whose components are BOD and DO concentrations at the monitoring stations where water quality standards should be strictly observed in the river, respectively, q_j = discharge of the wastewater injected into the loading point j, E and G = state matrices corresponding to the global stiffness matrices of the FEM, F and H = matrices associated with decision variables, and $\mathbf{b}$ and $\mathbf{d}$ = constant vectors.

5.2.2 Uncertainty and robustness

The deterministic optimization model [Eqns.(5.5)-(5.9)] is improved using the RO formulation developed by Mulvey *et al.*(1995)[70]. The coefficient matrices E, F, G and H and constant vectors $\mathbf{b}$ and $\mathbf{d}$ in the constraints (5.6) include some parameters that will vary uncertainly for the duration of controlling water quality. Hence, it should be noted that an optimal solution derived from the model [Eqns.(5.5)-(5.9)] in one situation may not be optimal in other situations. To depress this high sensitivity of the model, all the situations which will occur uncertainly are gathered into a plausible set of scenarios in the RO framework. Each scenario s is assumed to take place with the probability of the scenario p_s, $\sum p_s = 1$.

A process of scenario generation is very important in RO. Uncertain parameters are specified in the first phase of making an RO model. In this problem, discharge Q, water depth h and water temperature T are specified as uncertain parameters, because these three parameters play important roles in deciding the coefficients of the BOD and DO transport equations (4.5) and (4.6). The influence of other parameters such as wind speed W and salinity S included in the coefficients of those equations can be considered much smaller than that of Q, h and T.

Realizations of all boundary conditions, with which water depth h in every node and discharge Q in every element are determined by solving Eqns.(5.1) and (5.2), can be assumed to constitute a set of scenarios Γ. A set K is also defined as a scenario set that includes all the realizations of water temperature T. For simplicity, the set Γ is assumed independent of K. The scenario space for this model is thus $\Omega = \Gamma \times K$ and assumed to be generated from past outcomes.

In the deterministic optimization model [Eqns.(5.5)-(5.9)], the constraints which represent water quality standards are prepared only at monitoring stations, not along the whole river. Besides, the scale of violations of those standards except at monitoring stations is not measured in the model. In this chapter, therefore, the relaxed constraints that represent the standards except at monitoring stations, as well as the constraints that require strict observance of the standards at monitoring stations, are considered by introducing relaxation vectors $\mathbf{u}_s$ and $\mathbf{v}_s$ and by adding a penalty term to the objective function to keep the violation of the standards small. More strategic policies can also be found by this relaxation.

Mulvey *et al.*(1995)[70] introduces two different robustness concepts in the RO formulation. Remember that a solution to the mathematical program is *solution robust* if it remains close to optimal for any input data scenario to the model. Besides, a solution is *model robust* if it remains almost feasible for any scenario realization. In this problem, the RO model considers solution robustness to be inversely related to a standard deviation of the total BOD loading, and model robustness to be inversely related to the magnitude of violating water quality standards.

5.2.3 Robust optimization model formulation

The RO model for stream water quality management can be expressed mathematically as follows.

$$\text{Minimize} \quad \left[-\sum_s p_s x_s + \lambda \max_s \left\{p_s |x_s - \sum_s p_s x_s|\right\} + \omega \sum_s \sum_i p_s \left(u_{is} + v_{is}\right)\right] \tag{5.10}$$

subject to

(i) BOD and DO transport equations that are discretized by the FEM under all scenarios

$$E_s \mathbf{L}_s + F_s \mathbf{L}_s^L = \mathbf{b}_s, \; G_s \mathbf{C}_s + H_s \mathbf{C}_s^L = \mathbf{d}_s \quad \forall s \tag{5.11}$$

(ii) Effluent limitation standards under all scenarios

$$\mathbf{L}_s^{Ll} \leq \mathbf{L}_s^L \leq \mathbf{L}_s^{Lu}, \quad \mathbf{C}_s^{Ll} \leq \mathbf{C}_s^L \leq \mathbf{C}_s^{Lu} \quad \forall s \tag{5.12}$$

(iii) River water quality standards at monitoring stations under all scenarios

$$0 \leq \mathbf{L}_s^M \leq \mathbf{L}_s^u, \quad 0 \leq \mathbf{C}_s^l \leq \mathbf{C}_s^M \quad \forall s \tag{5.13}$$

(iv) River water quality standards except at monitoring stations under all scenarios

$$0 \leq \mathbf{L}_s^I \leq \mathbf{L}_s^u + \mathbf{u}_s, \; 0 \leq \mathbf{C}_s^l - \mathbf{v}_s \leq \mathbf{C}_s^I \quad \forall s \tag{5.14}$$

(v) Nonnegativity

$$\mathbf{L}_s, \mathbf{L}_s^I, \mathbf{L}_s^L, \mathbf{L}_s^M, \mathbf{C}_s, \mathbf{C}_s^I, \mathbf{C}_s^L, \mathbf{C}_s^M, \mathbf{u}_s, \mathbf{v}_s \geq 0 \quad \forall s \tag{5.15}$$

where the subscript s stands for scenario, the superscript I represents river water except at monitoring stations, $x_s = \sum_j q_j L_{js}^L$ = total BOD loading at scenario s, $\mathbf{u}_s$ and $\mathbf{v}_s$ = relaxation vectors whose i-th component is the variable relaxing water quality standards for BOD and DO concentrations at the i-th node, u_{is} and v_{is}, respectively, and λ and ω = multiobjective weights.

The objective function of the RO model, represented by Eqn.(5.10), consists of the following three terms: (i) an expected total BOD loading with the minus sign; (ii) a constant (λ) times a maximum expected absolute deviation of a total BOD loading under each scenario from an expected total BOD loading; and (iii) a constant (ω) times an expected value of total violations of river water quality standards for BOD and DO, that is, a penalty for not maintaining the standards. It is noted that the first, second and third terms in Eqn.(5.10) are related to a prime objective, solution robustness and model robustness, respectively. The second term in Eqn.(5.10) is used as a substitute for a standard deviation of a total BOD loading to make the objective function linear. To convert the RO model into a linear programming problem, the absolute value such that $|x_s - \sum_s p_s x_s|$ should be changed into a linear function of the variables. Variables y_s^+, y_s^- that are defined by

$$y_s^+ = \frac{1}{2}\left\{|x_s - \sum_s p_s x_s| + x_s - \sum_s p_s x_s\right\} \quad \forall s \tag{5.16}$$

$$y_s^- = \frac{1}{2}\left\{|x_s - \sum_s p_s x_s| - (x_s - \sum_s p_s x_s)\right\} \quad \forall s \tag{5.17}$$

are now introduced [Sakawa(1984)[85]]. Then, the following equations are obtained.

$$y_s^+ + y_s^- = |x_s - \sum_s p_s x_s| \quad \forall s \tag{5.18}$$

$$y_s^+ - y_s^- = x_s - \sum_s p_s x_s \quad \forall s \tag{5.19}$$

$$y_s^+,\ y_s^- \geq 0 \quad \forall s \tag{5.20}$$

Furthermore a variable w which is given by

$$w = \max_s \left\{p_s|x_s - \sum_s p_s x_s|\right\} = \max_s \left\{p_s(y_s^+ + y_s^-)\right\} \tag{5.21}$$

is introduced. Consequently if the relations

$$p_s(y_s^+ + y_s^-) - w \leq 0 \qquad \forall s \tag{5.22}$$

$$y_s^+ - y_s^- = x_s - \sum_s p_s x_s \qquad \forall s \tag{5.23}$$

$$y_s^+,\ y_s^- \geq 0 \qquad \forall s \tag{5.24}$$

are added into the set of constraints in the RO model, and the objective function in Eqn.(5.10) is rewritten as

$$-\sum_s p_s x_s + \lambda w + \omega \sum_s \sum_i p_s(u_{is} + v_{is}) \tag{5.25}$$

then the RO model becomes a linear programming model that has effective algorithms to be solved.

The deterministic optimization model [Eqns.(5.5)-(5.9)] can also be extended to a conventional stochastic programming (SP) [e.g., Wagner *et al.*(1992)[98]] model whose formulation is the same as that of the RO model except that the objective function of the SP model is equal to the first term of the objective function of the RO model and $\mathbf{u}_s$=$\mathbf{v}_s$=$\mathbf{0}$. However the SP model can be considered as a special case of the RO model because setting λ equal to zero and ω equal to the infinity in the RO model yields the SP model. In general, there is a trend that setting a big value to λ results in solution robust and setting a big value to ω results in model robust. By adjusting values of weights λ and ω, users of the model can obtain optimal solutions related to their preference level for each objective. The RO model can be solved by the simplex method, and the optimal expected BOD loading for each loading point j, i.e., $\sum_s p_s q_j L_{js}^L$, is obtained.

5.3 Demonstrative Example

5.3.1 Hypothetical river system

To demonstrate the ability of the RO model described above, optimization computation is carried out in a hypothetical river system shown in Figure 5.1. The river length is 7.5km, the river bottom slope is 1/10,000, the bottom width is 10m and the shape of cross section is rectangular. The river is divided into 21 line elements with 22 nodes. The numbers of loading points (LPs) and monitoring stations (MSs) are both 3 and their locations are shown in Figure 5.1. Wastewater discharges injected into LP1, LP2 and LP3 are 0.5, 1.0

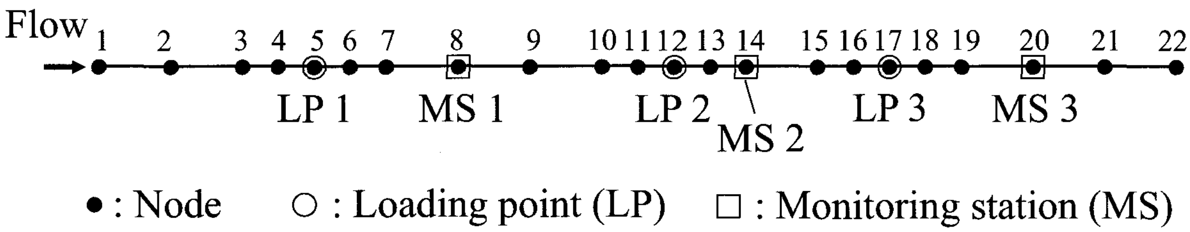

Figure 5.1: Hypothetical river system

and 0.8(m^3/s), respectively. Effluent limitations for these wastewaters corresponding to Eqn.(5.12) are as follows: $L_{js}^{Ll} = 2.0$(mg/L), $L_{js}^{Lu} = 160.0$(mg/L), $C_{js}^{Ll} = 1.0$(mg/L) and $C_{js}^{Lu} = 5.0$(mg/L), j = 1,2,3 under all scenarios. Water quality standards for in-stream water are $\mathbf{L}_s \leq 2.0$(mg/L) and $\mathbf{C}_s \geq 7.5$(mg/L) at nodes 1, 2, $\cdots$, 14 and $\mathbf{L}_s \leq 3.0$(mg/L) and $\mathbf{C}_s \geq 5.0$(mg/L) at nodes 15, 16, $\cdots$, 22 under all scenarios.

The boundary values are taken as: BOD and DO concentrations on the upstream boundary $L_s^* = 1.5$(mg/L), $C_s^* = 8.0$(mg/L) and dispersive BOD and DO fluxes on the down stream boundary $f_{Ls}^* = f_{Cs}^* = 0$ under all scenarios. Manning's roughness coefficient $n = 0.03$(s/$m^{1/3}$), velocity-distribution coefficient $\zeta = 1.1$, undetermined parameter $m =$ 200, salinity $S = 0$, and wind speed $W = 3.0$(m/s) are assumed constant along the river. Five realizations are considered for both the coming discharge at the upstream boundary and the water depth at the downstream boundary. In addition, three realizations of water temperature in the river are considered. Then the total number of scenarios is $5 \times 3 = 15$, and the data for scenarios are listed in Table 5.1. Generating such scenarios is supposed possible by classifying historical data on boundary values in the real world.

First, steady-state open channel flow is simulated under every scenario. Then values of multiobjective weights λ and ω are prescribed, and the RO model established is solved by the simplex method.

5.3.2 Results and discussion

The computational results of the RO model in the objective space are summarized in Table 5.2. Optimal expected BOD and DO concentrations in the injected water and optimal expected BOD loading at each loading point are consolidated in Table 5.3 and Figure 5.2.

In Table 5.2, small values of the standard deviation of the total BOD loading and the expected total violation of river water quality standards imply that the optimal solution is solution robust and model robust, respectively. Since the three objectives such as maximizing an expected total BOD loading, minimizing a standard deviation and an expected total violation of river water quality standards are generally in conflict, balancing these objectives in various levels may be required in decision-making. About Solution F which is identical with the solution of the SP model, the value of the expected total BOD loading is the largest and that of the expected violation is the smallest (zero) in all solutions shown in Table 5.2. However, the standard deviation, which is often important for a risk averse decision-maker, is the largest. A large standard deviation means that the outcome is much in doubt. Neither model robustness nor solution robustness are investigated in the SP model, while their importance is taken into account in the RO model.

Consider Solutions A, B and C in Table 5.2 that have the same values of the multi-objective weight ω. Comparing these solutions each other shows that the large value of λ can derive the small standard deviation. Solution C is solution robust, while the expected total BOD loading is fairly small. Such a trade-off among the objectives can be evaluated by the RO model.

Let us consider Solutions A, D and E in Table 5.2 that result from the same values of λ. The table indicates that the larger the value of ω, the smaller the expected violation of river water quality standards.

As a result, DO concentration in every wastewater at every scenario takes either 1.0 or 5.0mg/L of its prescribed limitations, and as shown in Table 5.3 all the solutions of A to F have the same values of the expected concentrations of DO in wastewaters. The fact that DO concentrations in wastewaters are not included in the objective function may lead to those results.

Table 5.1: Scenarios for realizations of discharge, water depth and water temperature

Scenario number s	Discharge on upstream boundary $Q^*(\mathrm{m}^3/\mathrm{s})$	Water depth on downstream boundary $h^*(\mathrm{m})$	Water temperature $T(°\mathrm{C})$	Probability of scenario s p_s
1	30.0	3.7	15.0	0.0250
2	25.0	3.3	15.0	0.0625
3	20.0	2.9	15.0	0.0750
4	15.0	2.5	15.0	0.0625
5	10.0	1.9	15.0	0.0250
6	30.0	3.7	13.0	0.0500
7	25.0	3.3	13.0	0.1250
8	20.0	2.9	13.0	0.1500
9	15.0	2.5	13.0	0.1250
10	10.0	1.9	13.0	0.0500
11	30.0	3.7	11.0	0.0250
12	25.0	3.3	11.0	0.0625
13	20.0	2.9	11.0	0.0750
14	15.0	2.5	11.0	0.0625
15	10.0	1.9	11.0	0.0250

Table 5.2: Optimal solutions in objective space under various values of weights

Solution	Multiobjective weights λ, ω	Expected total BOD loading (g/s)	Standard deviation of total BOD loading (g/s)	Expected violation of river water quality standards (mg/L)
A	10, 5	28.709	5.744	0.0039
B	20, 5	21.263	1.494	0.0013
C	30, 5	18.858	0.009	0.0021
D	10, 0.1	28.712	5.744	0.0083
E	10, 100	28.553	5.850	0.0000
F	0, 100,000	30.462	6.661	0.0000

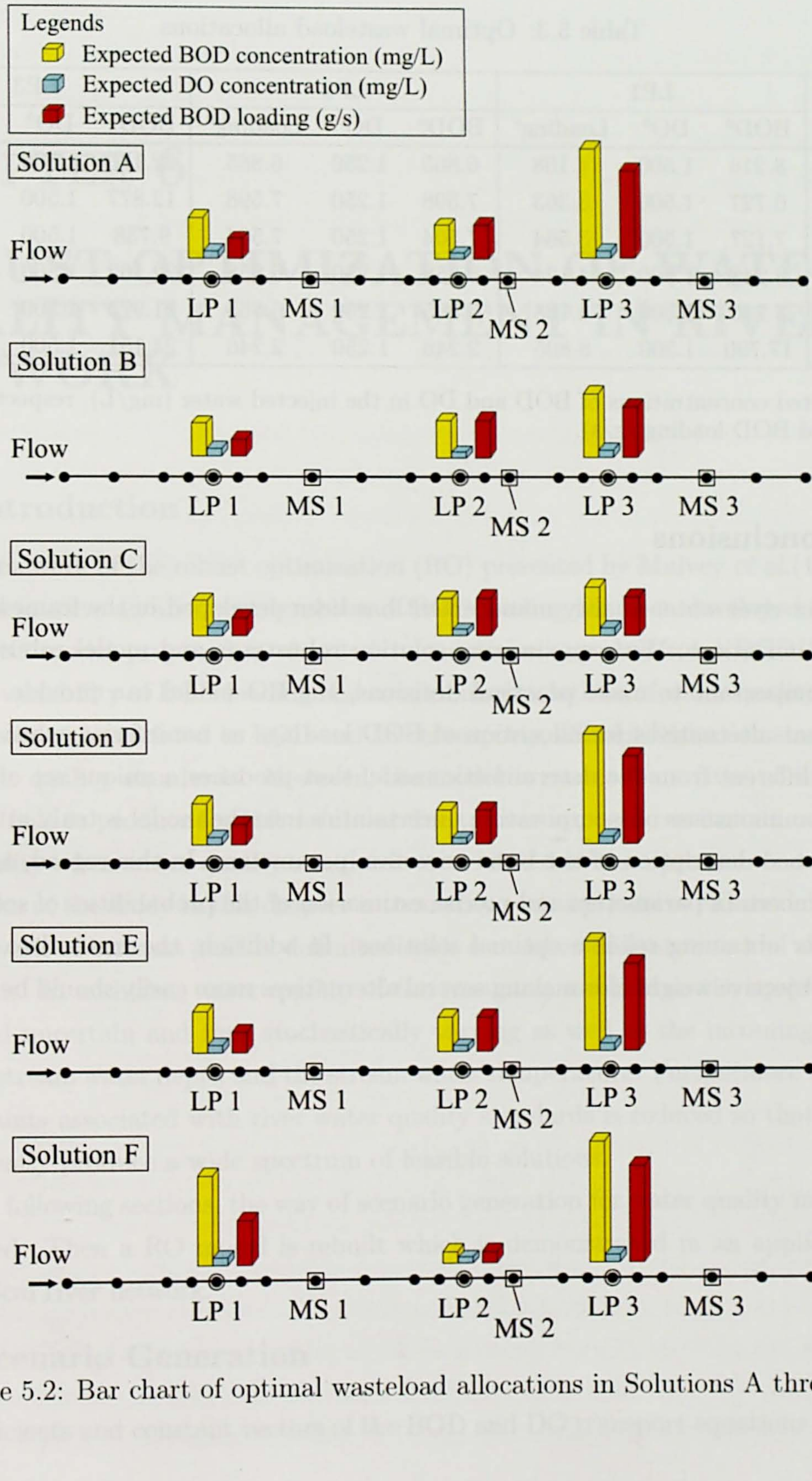

Figure 5.2: Bar chart of optimal wasteload allocations in Solutions A through F

Table 5.3: Optimal wasteload allocations

Solution	LP1			LP2			LP3		
	BOD[a]	DO[b]	Loading[c]	BOD[a]	DO[b]	Loading[c]	BOD[a]	DO[b]	Loading[c]
A	8.216	1.500	4.108	6.865	1.250	6.865	22.170	1.500	17.736
B	6.727	1.500	3.363	7.598	1.250	7.598	12.877	1.500	10.302
C	7.127	1.500	3.564	7.504	1.250	7.504	9.738	1.500	7.791
D	8.248	1.500	4.124	6.858	1.250	6.858	22.162	1.500	17.729
E	8.216	1.500	4.108	6.865	1.250	6.865	21.975	1.500	17.580
F	17.790	1.500	8.895	2.246	1.250	2.246	24.151	1.500	19.321

a, b: Expected concentrations of BOD and DO in the injected water (mg/L), respectively.
c: Expected BOD loading (g/s).

5.4 Conclusions

A model for river water quality management has been developed in the framework of the robust optimization. With maximizing solution robustness and model robustness that are often important to make practical decisions, the RO model can provide a series of management alternatives for allocation of BOD loadings as noninferior solutions, which is totally different from the deterministic model that produces a unique set of solutions. The magnanimousness of incorporating uncertainties into the model is truly attributed to scenario-based description of the basic uncertain parameters. In this regard, appropriate choice of uncertain parameters and precise estimation of the probabilities of scenarios are essential for obtaining reliable optimal solutions. In addition, the method to determine the multiobjective weights for making several alternatives more easily should be discussed.

CHAPTER 6

ROBUST OPTIMIZATION OF WATER QUALITY MANAGEMENT IN RIVER NETWORK

6.1 Introduction

In the framework of the robust optimization (RO) presented by Mulvey *et al.*(1995)[70], a RO model to solve an allocation problem of BOD loading has been developed in Chapter 5. The RO model proposed is objected to maximize expected total BOD loading, to minimize variability of BOD loading and to minimize violations of water quality standards with the constraints related to BOD and DO transports, effluent limitation standards and river water quality standards. However, the model is meant for a river of single reach, and often fails to produce feasible solutions due to over-imposed constraints on in-stream water quality when water quality standards are violated at the upstream end of a river.

The aim of the study in this chapter is to modify such a primitive RO model applicable for a network of streams that is delimited with a number of upstream and downstream boundaries. In addition, water quality of the incoming water from the upstream end is considered uncertain and thus stochastically varying as well as the incoming discharge, the downstream water depth and the stream water temperature. Furthermore the number of constraints associated with river water quality standards is reduced so that the model can efficiently produce a wide spectrum of feasible solutions.

In the following sections, the way of scenario generation for water quality management is modified. Then a RO model is rebuilt which is demonstrated in an application to a hypothetical river network.

6.2 Scenario Generation

The coefficients and constant vectors of the BOD and DO transport equations [Eqns.(4.5) and (4.6)] include some parameters (hereafter, called *basic uncertain parameters*) that will vary uncertainly during the period of controlling pollutant loading. The parameters such as discharge Q, water depth h and stream water temperature T are considered as most important triggers of such uncertainty and are stochastically treated in Chapter 5. In this chapter, BOD and DO concentrations (L_u^* and C_u^*, respectively) of the incoming water from the upstream ends that delimit a river system, taken as unchangeable deterministic boundary values in the last chapter, are also included in the basic uncertain parameters to embrace their time-varying nature in a stochastic sense. It is then allowed that some realizations of these external concentrations violate prescribed in-stream water quality standards.

Suppose that coherent relations exist between discharge and BOD concentration of the incoming water and between BOD and DO concentrations of the same. It is thus assumed that a set of realizations of the incoming discharge, its BOD and DO concentrations and the downstream boundary values of water depth is a constituent of a scenario set Γ. Namely the following is specified as a realization of a scenario in the scenario set Γ.

$$(Q_{u_1}^*, \cdots, Q_{u_U}^*, h_{d_1}^*, \cdots, h_{d_D}^*, L_{u_1}^*, \cdots, L_{u_U}^*, C_{u_1}^*, \cdots, C_{u_U}^*)$$

where the subscripts u and d indicate boundary numbers that are placed on upstream and downstream boundaries, respectively, and the subscripts U and D the numbers of upstream and downstream boundaries, respectively. A set K is also defined as a scenario set that includes all the realizations of water temperature T, which is assumed constant along the stream network. For simplicity, let the set Γ be independent of the set K, and the whole scenario space is then defined as $\Omega = \Gamma \times K$. Scenario sets Γ and K are subject to discrete stochastic distributions which are *a priori* known, or obtained from analyzing historical data in the real world, and thus a scenario $s \in \Omega$ varies with a probability p_s.

6.3 Robust Optimization Model

A RO model can progressively generate noninferior solutions less sensitive to uncertainty of the input data. For that purpose two types of robustness are defined: solution robustness and model robustness. Two objectives corresponding to solution robustness and model robustness are considered, and those and a prime objective are synthesized into a scalar objective function with two weights. The RO model for water quality management in an interconnected stream network is then formulated as follows.

$$\text{Minimize} \quad \left[-\sum_s p_s x_s + \lambda \max_s \left\{p_s |x_s - \sum_s p_s x_s|\right\} + \omega \sum_s \sum_i p_s \left(u_{is}^o + v_{is}^o\right)\right] \tag{6.1}$$

subject to

(i) BOD and DO transport equations that are discretized by the FEM

$$E_s \mathbf{L}_s + F_s \mathbf{L}_s^L = \mathbf{b}_s, \ G_s \mathbf{C}_s + H_s \mathbf{C}_s^L = \mathbf{d}_s \quad \forall s \tag{6.2}$$

(ii) Effluent limitation standards at all scenarios

$$\mathbf{L}_s^{Ll} \leq \mathbf{L}_s^L \leq \mathbf{L}_s^{Lu}, \quad \mathbf{C}_s^{Ll} \leq \mathbf{C}_s^L \leq \mathbf{C}_s^{Lu} \quad \forall s \tag{6.3}$$

(iii) River water quality standards at loading points and/or stream junctions at all scenarios

$$0 \leq \mathbf{L}_s^o \leq \mathbf{L}_s^{ou} + \mathbf{u}_s^o, \ 0 \leq \mathbf{C}_s^{ol} - \mathbf{v}_s^o \leq \mathbf{C}_s^o \quad \forall s \tag{6.4}$$

(iv) Nonnegativity at all scenarios

$$\mathbf{L}_s, \mathbf{L}_s^L, \mathbf{L}_s^o, \mathbf{C}_s, \mathbf{C}_s^L, \mathbf{C}_s^o, \mathbf{u}_s^o, \mathbf{v}_s^o \geq 0 \quad \forall s \tag{6.5}$$

In the foregoing equations, the superscripts L, o, u and l stand for injected wastewater, loading point and/or stream junction, upper limit, and lower limit, respectively, the subscripts s and i stand for scenario and node number at a loading point and/or a stream junction, respectively, p_s = probability of a scenario s, $x_s = \sum_j q_j L_{js}^L$ = total BOD loading under a scenario s, q_j = discharge of wastewater injected into the loading point j, $\mathbf{L}_s$ and $\mathbf{C}_s$ = vectors whose components are BOD and DO concentrations of river water at the nodes, respectively, $\mathbf{L}_s^L$ and $\mathbf{C}_s^L$ = vectors whose components are BOD and DO concentrations in wastewater that are injected into the j-th loading point, L_{js}^L and C_{js}^L, respectively, $\mathbf{L}_s^o$ and $\mathbf{C}_s^o$ = vectors whose components are BOD and DO concentrations of river water at a loading point and/or a stream junction, respectively, E_s and G_s = state matrices corresponding to the global stiffness matrices that arise from application of the FEM, F_s and H_s = matrices associated with $\mathbf{L}_s^L$ and $\mathbf{C}_s^L$, respectively, $\mathbf{b}_s$ and $\mathbf{d}_s$ = constant vectors, $\mathbf{u}_s^o$ and $\mathbf{v}_s^o$ = relaxation vectors whose i-th component is the variable relaxing water quality standards for BOD and DO concentrations at the i-th node, u_{is}^o and v_{is}^o, respectively, and λ and ω=weights. It is noted that $\mathbf{L}_s$, $\mathbf{L}_s^o$, $\mathbf{C}_s$ and $\mathbf{C}_s^o$ are state variable vectors, $\mathbf{L}_s^L$ and $\mathbf{C}_s^L$ are decision variable vectors, while $\mathbf{L}_s^{Ll}$, $\mathbf{L}_s^{Lu}$, $\mathbf{C}_s^{Ll}$, $\mathbf{C}_s^{Lu}$, $\mathbf{L}_s^{ou}$ and $\mathbf{C}_s^{ol}$ are prescribed constant vectors. The coefficient matrices E_s, F_s, G_s and H_s and constant vectors $\mathbf{b}_s$ and $\mathbf{d}_s$ in Eqn.(6.2) are determined by steady-state open channel flow simulation using Eqns.(5.1) and (5.2) and by evaluation using empirical formulae [Eqns.(4.7)-(4.11)].

The RO model [Eqns.(6.1)-(6.5)] is different in the objective function (6.1) and the constraint set (6.4) from that presented in the preceding chapter. The objective Eqn.(6.1) means (i) maximizing an expected total BOD loading, which is a prime objective; (ii) minimizing a maximum expected absolute deviation of a total BOD loading under each scenario from an expected total BOD loading; and (iii) minimizing an expected sum of deviations from river water quality standards for BOD and DO concentrations at loading points and/or stream junctions.

A concept of monitoring station (MS) where state of the system (i.e., stream water quality) is measured and regulated is often employed in optimization models. However, since a whole body of water in a river system is actually required to satisfy prescribed water quality standards and solutions of an optimization model may be heavily affected by locations of MSs [Lee and Deininger(1992)[51]], the concept of MS is not introduced in the model presently developed. Moreover, the constraints that require observance of water quality standards for the whole network of streams, as employed in the last chapter, are so strict that the model may often have no feasible solution. Thus, imposition of the relevant constraints is limited to loading points and stream junctions [Eqn.(6.4)]. Since the water quality varies seasonally in the real world, it can happen that water quality standards are violated only in short duration on upstream boundary despite of satisfying them for most durations. The new technique presented here, however, can avoid to fail in computing optimal solution in such a situation. Additionally, these treatments lead to reduction of the number of constraints, and therefore to saving computational efforts necessary for solving the linear programming problem.

The RO model, which includes an absolute term in Eqn.(6.1), can be handled as a linear programming problem by introducing some appropriate variables and constraints, as mentioned in the last chapter. The values of weights λ and ω in Eqn.(6.1) are arbitrarily selected, and iterative determination of the values may be needed to supply sufficient alternatives to decision-makers. Optimal BOD loading at loading point j suggested by the RO model is computed as $\sum_s p_s q_j L_{js}^L$

6.4 Demonstrative Example

Ability and implication of the RO model described above are demonstrated through its application to a hypothetical river network, shown in Figure 6.1, that comprises five prismatic reaches of R-1 to R-5. The river is divided into 29 line elements with 30 nodes. Γ_1, Γ_2 and Γ_3 are upstream boundaries and Γ_4 is a downstream boundary. The number of loading points (LPs) is 5, and their locations and the streambed width of each reach are designated in Figure 6.1. The values of effluent limitations associated with Eqn.(6.3) and injected wastewater discharges are listed in Table 6.1. The river water quality standards are $\mathbf{L}_s \leq$2.0(mg/L) and $\mathbf{C}_s \geq$7.5(mg/L) along the reaches R-1, -2 and -3 and $\mathbf{L}_s \leq$3.0(mg/L) and $\mathbf{C}_s \geq$5.0(mg/L) along the reaches R-4 and -5. Velocity-distribution coefficient $\zeta = 1.1$, undetermined parameter $m = 200$, Manning's roughness coefficient $n = 0.03(\mathrm{s/m^{1/3}})$, salinity $S = 0$, and wind speed $W = 3.0(\mathrm{m/s})$ are assumed constant along all the river reaches. The scenario sets Γ and K are taken as six and two, respectively. Then the total number of scenarios is $6\times2 = 12$, and the data for all scenarios with their probabilities are given in Table 6.2.

In order to figure out the hydraulic ingredients in the RO model in advance, steady-state open channel flow simulation using Eqns.(5.1) and (5.2) is carried out for each of all the scenarios generated. In optimization practice, some adjusted combinations of the values of weights λ and ω are predetermined to obtain a wide spectrum of solutions to the problem, and finally the respective particularized linear programming problems are solved with the aid of the simplex method. Here, λ = 1, 15, 20, ω = 50, 100, 150, 1000 and their appropriate combinations are considered to produce five sample solutions of A to E. The solution and model robustnesses, achieved by the model, can actually be quantified by measuring a standard deviation of total BOD loading and an expected sum of deviations from BOD and DO standards over all nodes, respectively. The results are shown in Table 6.3, including the expected total BOD loadings. Remember that less standard deviation of a total BOD loading implies higher solution robustness (Solution D possesses the highest solution robustness), and less expected sum of deviations from prescribed water quality standards implies higher model robustness (Solution E is slightly high in model robustness compared with the others). In relation to the magnitude of the weights, generally the increasing values of λ and ω increase the solution robustness and the model robustness, respectively.

In deciding a practical management strategy, of central importance is the fact that characteristically the RO model yields the solutions with a trade-off especially between the expected total BOD loading and the standard deviation of total BOD loading. When the solution robustness is intensified to obtain solutions closer to optimal under every scenario, the allowable total BOD loading for the entire river system is correspondingly reduced. The utmost solution is Solution D where its noninferior solution is exactly optimal under every scenario, while the expected total BOD loading is reduced to less than half of that in the other solutions.

Table 6.4 summarizes the noninferior solutions of major concern for the whole spectrum of solutions; expected BOD and DO concentrations of wastewaters injected at the loading points, and subsequent expected BOD loadings (i.e., allocated BOD loadings). The result shows that a relatively large amount of BOD loading is allowed at LP5 most downstream located, and at all or a few of the remaining loading points the loading is depressed to an amount of its given lower limit. This is primarily because prescribed water quality standards at the downstream reaches are not so strict as those at the upstream reaches. Figures 6.2 - 6.6 show optimal wasteload allocation in Solution A through E, respectively.

To take a look at expected stream water qualities (i.e., state variables) as a result of strategic water quality management, their profiles along a stretch of R-1 to R-5 for Solution A and Solution D are delineated in Figure 6.7, 6.8 and Figure 6.9, 6.10, respectively. These two solutions are poles apart in their strategies. Solution A is an aggressive strategy that plots to increase BOD loadings with the lowest tolerance for uncertainty (i.e., with the highest sensitivity to uncertainty) and therefore is likely to cause relatively large water quality violations in streams. Contrarily Solution D in too much of a tolerance for uncertainty is so conservative that allowable BOD loadings may extremely be depressed and thereby the expected BOD concentrations in streams may less violate their targeted values. Profiling values of the water quality constituents is of essential need to know the anticipated result of a strategic water quality management in terms of the streamwise water quality distribution and the magnitude of the degree of water quality violations. Especially for solutions with small ω-value, such a profiling is a sheer need since river water quality standards are likely to be largely violated.

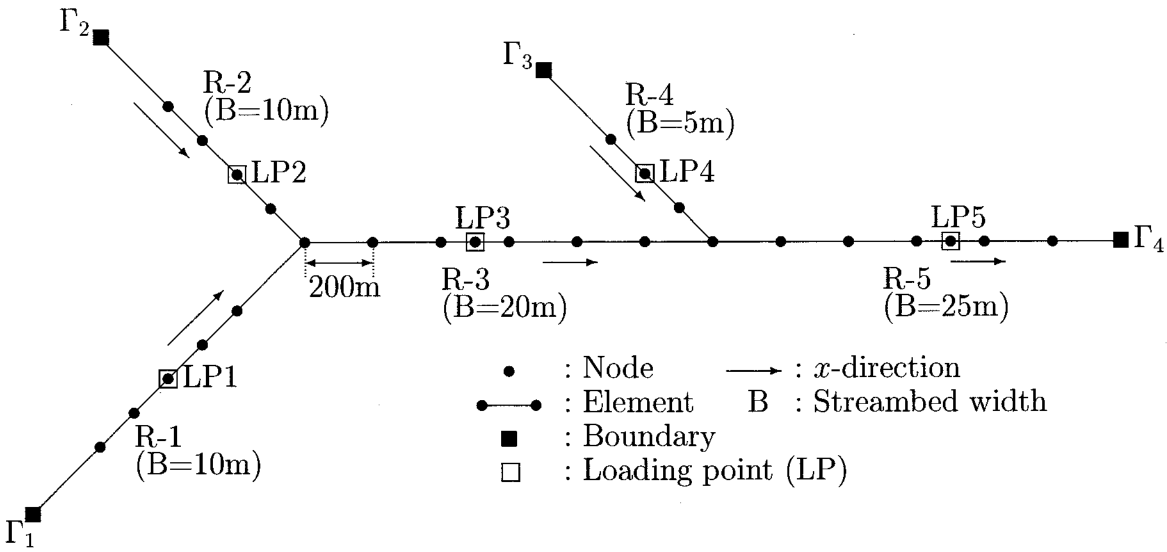

Figure 6.1: Hypothetical river network

Table 6.1: Effluent limitations and wastewater discharges

LP j	L_{js}^{Ll} (mg/L)	L_{js}^{Lu} (mg/L)	C_{js}^{Ll} (mg/L)	C_{js}^{Lu} (mg/L)	q_j (m^3/s)
1	5.0	160.0	1.0	4.0	0.25
2	5.0	160.0	1.0	4.0	0.25
3	5.0	160.0	1.0	4.0	0.5
4	5.0	160.0	1.0	4.0	0.5
5	5.0	160.0	1.0	4.0	0.5

Table 6.2: Scenarios for realizations of boundary values and stream water temperature

s	Q_1^*	Q_2^*	Q_3^*	h_4^*	L_1^*	L_2^*	L_3^*	C_1^*	C_2^*	C_3^*	T	p_s
1	10	10	5	1.5	2.5	2.5	4.0	6.5	6.5	5.5	10	0.0625
2	10	10	15	1.9	2.5	2.5	3.5	6.5	6.5	5.0	10	0.0625
3	20	20	7.5	2.4	2.0	2.0	4.0	7.0	7.0	5.5	10	0.1250
4	20	20	17.5	2.8	2.0	2.0	3.5	7.0	7.0	5.0	10	0.1250
5	30	30	10	3.3	1.5	1.5	4.0	7.5	7.5	5.5	10	0.0625
6	30	30	20	3.7	1.5	1.5	3.0	7.5	7.5	4.5	10	0.0625
7	10	10	5	1.5	2.5	2.5	4.0	6.5	6.5	5.5	15	0.0625
8	10	10	15	1.9	2.5	2.5	3.5	6.5	6.5	5.0	15	0.0625
9	20	20	7.5	2.4	2.0	2.0	4.0	7.0	7.0	5.5	15	0.1250
10	20	20	17.5	2.8	2.0	2.0	3.5	7.0	7.0	5.0	15	0.1250
11	30	30	10	3.3	1.5	1.5	4.0	7.5	7.5	5.5	15	0.0625
12	30	30	20	3.7	1.5	1.5	3.0	7.5	7.5	4.5	15	0.0625

Units: $Q_1^* \sim Q_3^*$(m^3/s); h_4^*(m); $L_1^* \sim L_3^*$ and $C_1^* \sim C_3^*$(mg/L); T(°C).

Table 6.3: Quantified objective achievements

Solution	Weights λ, ω	Expected total BOD loading (g/s)	Standard deviation of total BOD loading (g/s)	Expected sum of deviations from BOD standard (mg/L)	Expected sum of deviations from DO standard (mg/L)
A	1, 50	34.753	21.952	5.859	7.549
B	1, 150	33.949	20.719	5.796	7.549
C	15, 100	29.317	13.491	5.793	7.549
D	20, 100	10.248	0.000	5.794	7.549
E	15, 1000	29.375	13.666	5.789	7.549

Table 6.4: Optimal expected values of wastewater qualities and BOD loading

Solution		LP1	LP2	LP3	LP4	LP5	Total BOD loading(g/s)
A	BOD(mg/L)	14.170	15.573	5.000	5.000	44.634	
	DO(mg/L)	3.834	3.828	3.625	3.625	1.000	
	Loading(g/s)	3.542	3.893	2.500	2.500	22.317	34.753
B	BOD(mg/L)	13.943	15.573	5.000	5.000	43.140	
	DO(mg/L)	3.834	3.828	3.625	3.625	1.375	
	Loading(g/s)	3.486	3.893	2.500	2.500	21.570	33.949
C	BOD(mg/L)	5.000	5.000	5.000	5.000	43.634	
	DO(mg/L)	3.834	3.827	3.625	3.625	1.000	
	Loading(g/s)	1.250	1.250	2.500	2.500	21.817	29.317
D	BOD(mg/L)	5.000	5.000	5.000	5.000	5.495	
	DO(mg/L)	3.834	3.827	3.625	3.625	1.000	
	Loading(g/s)	1.250	1.250	2.500	2.500	2.748	10.248
E	BOD(mg/L)	5.000	5.000	5.000	5.000	43.750	
	DO(mg/L)	3.834	3.827	3.625	3.625	1.375	
	Loading(g/s)	1.250	1.250	2.500	2.500	21.875	29.375

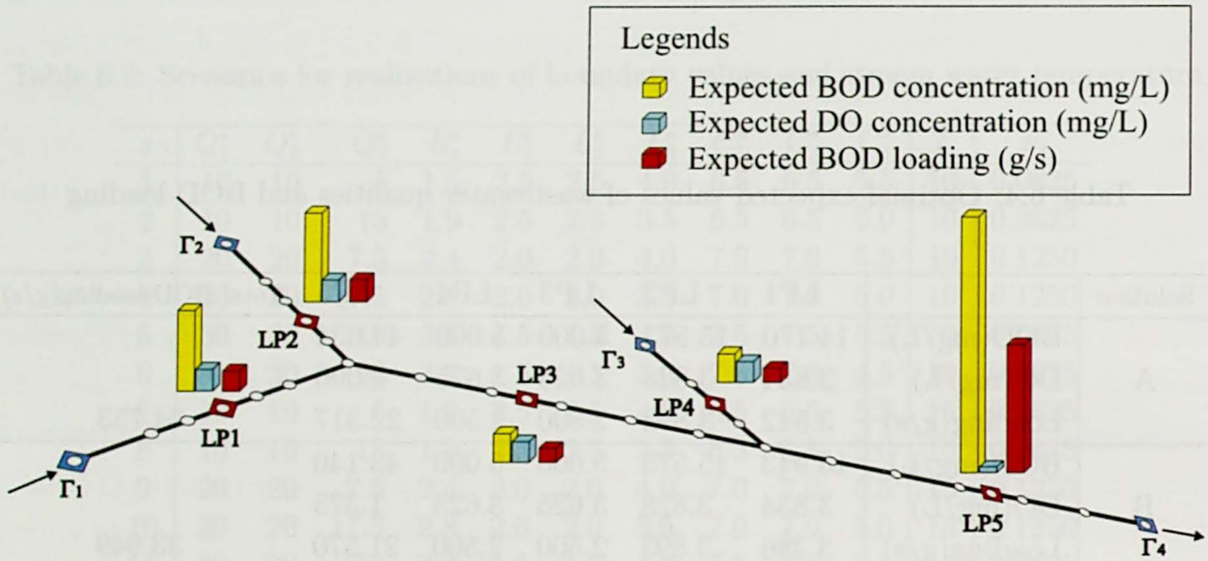

Figure 6.2: Optimal wasteload allocation in Solution A

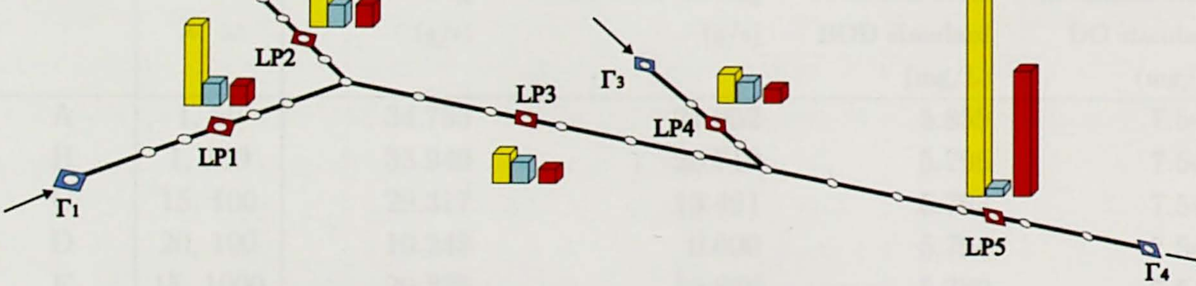

Figure 6.3: Optimal wasteload allocation in Solution B

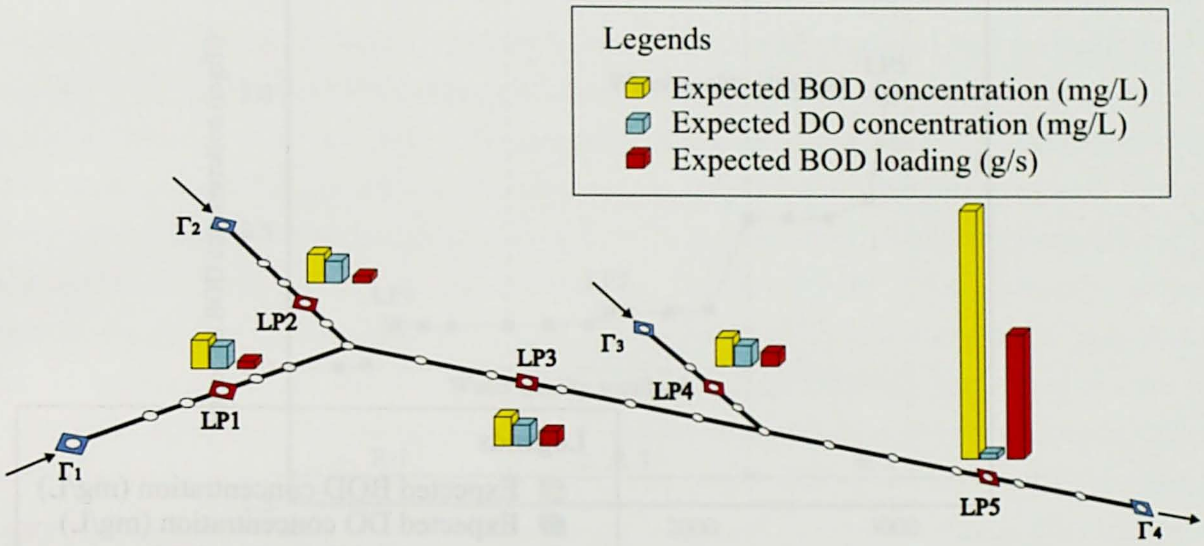

Figure 6.4: Optimal wasteload allocation in Solution C

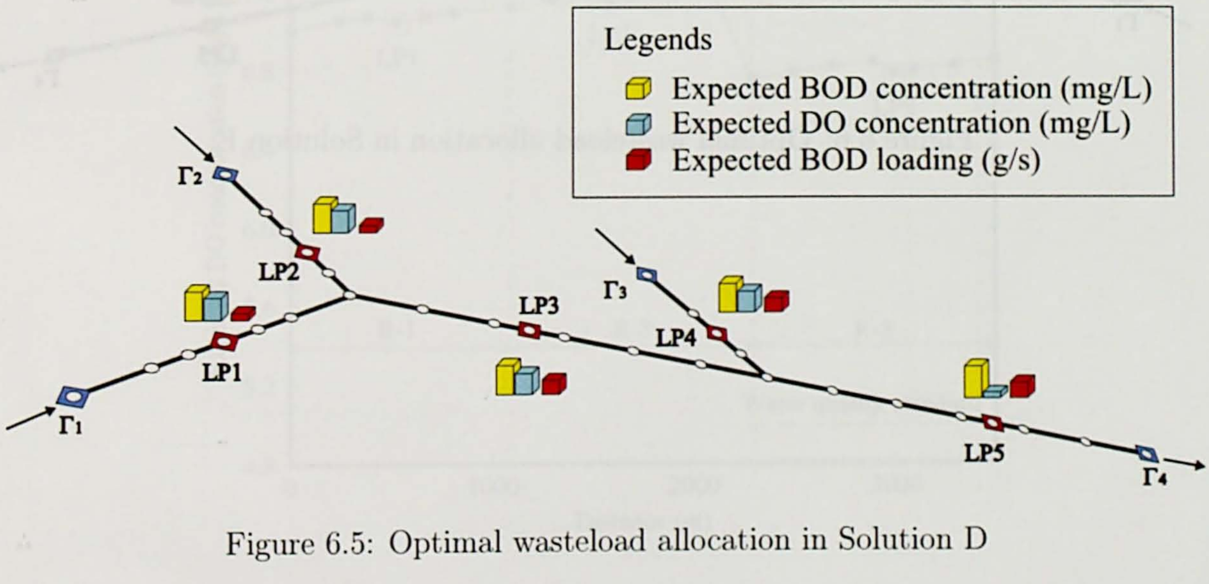

Figure 6.5: Optimal wasteload allocation in Solution D

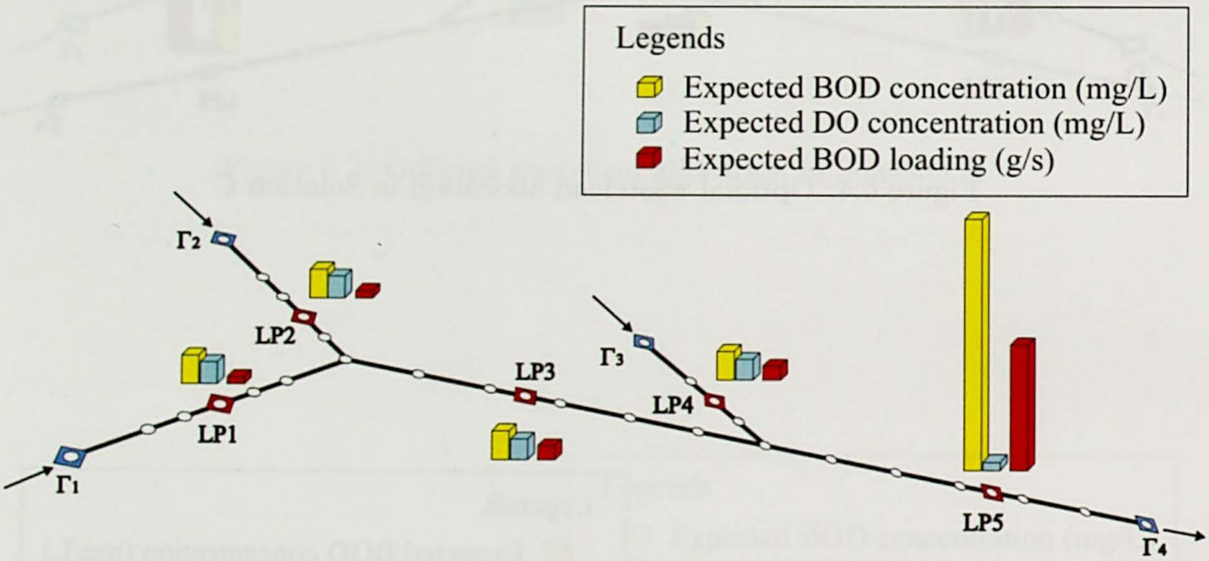

Figure 6.6: Optimal wasteload allocation in Solution E

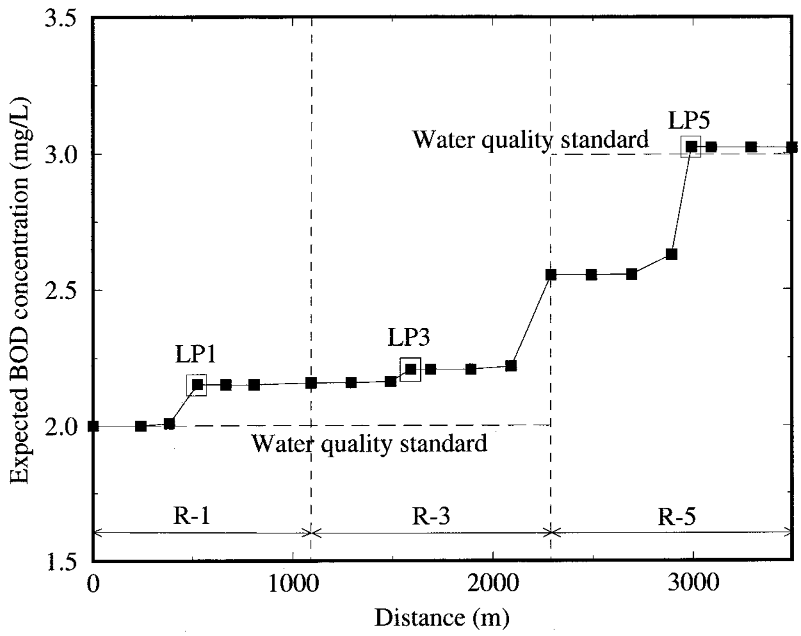

Figure 6.7: Expected BOD concentration profile of Solution A

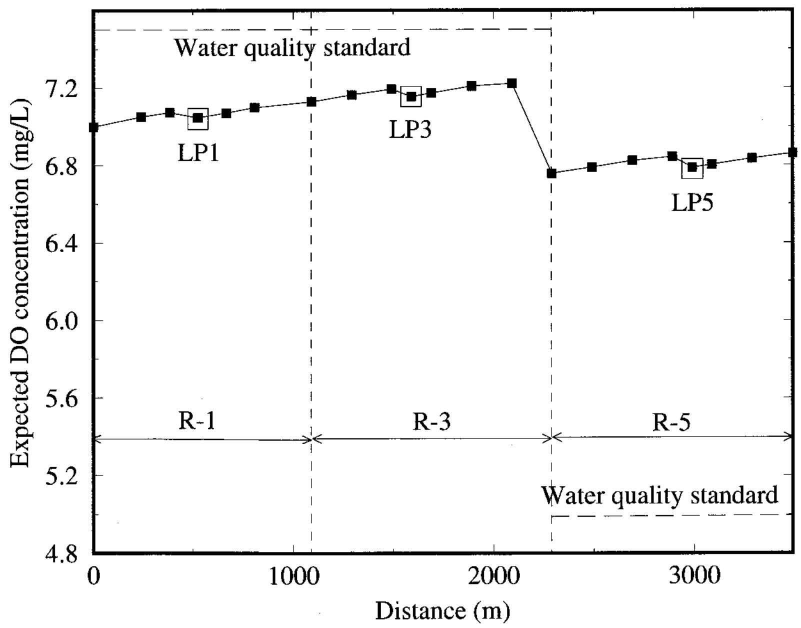

Figure 6.8: Expected DO concentration profile of Solution A

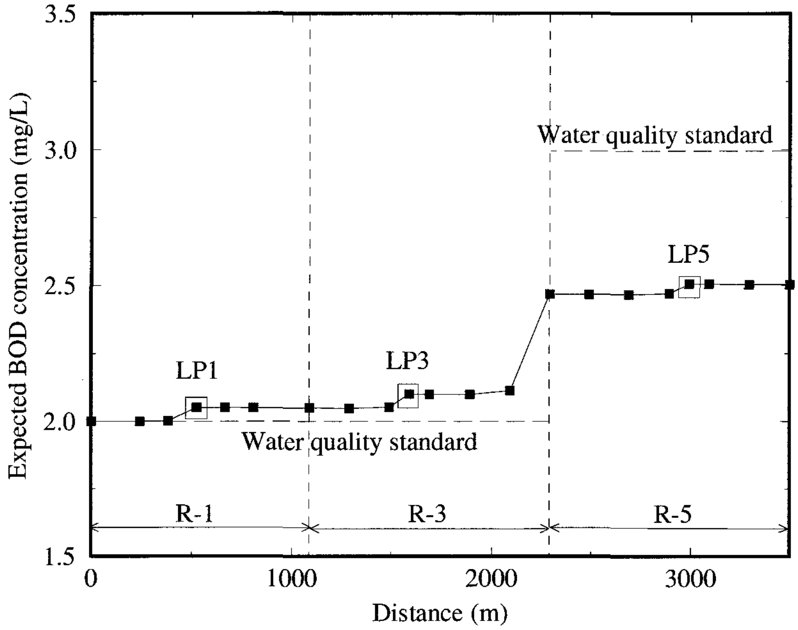

Figure 6.9: Expected BOD concentration profile of Solution D

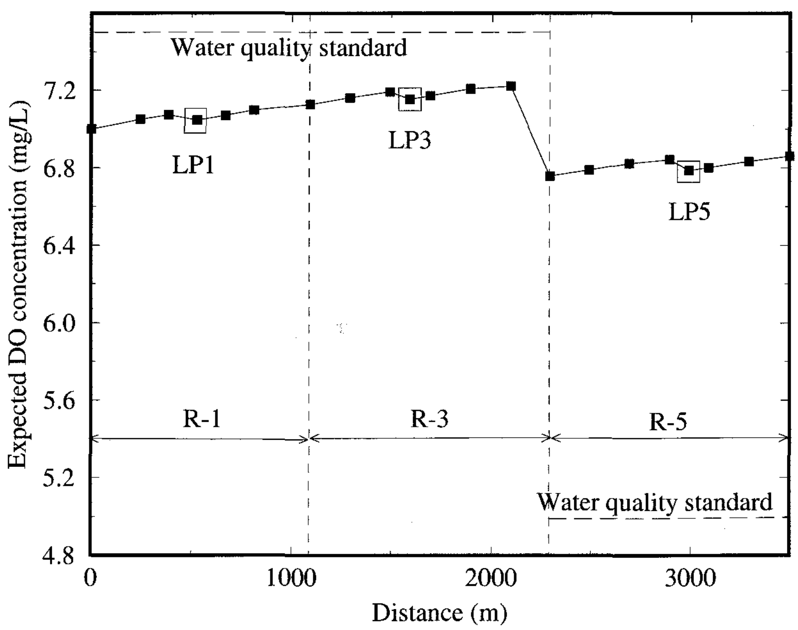

Figure 6.10: Expected DO concentration profile of Solution D

6.5 Conclusions

An advanced RO model for river water quality management has been proposed as an improvement over the primarily developed one. The model proposed can successfully be applied to the problem of controlling pollutant loading to a multitudinously delimited network of streams. BOD and DO concentrations of the incoming water from the upstream ends that delimit the network are treated not as deterministic boundary values, but as parts of the basic uncertainty parameters to take into account their stochastically varying nature. In their scenario-based descriptions, therefore, these external concentrations can be realized with the allowance of violating the water quality standards prescribed for an internal body of water. Due to this unerring improvement, the model results in embracing every conceivable uncertainty of central importance, and concomitantly being much more feasible. Reduced imposition of the constraints associated with in-stream water quality standards, confined to loading points and stream junctions, also widens a spectrum of feasible solutions, and at the same time effectively decreases computational efforts correspondingly to the resultant downsizing of the model.

CHAPTER 7

ϵ-CONSTRAINT APPROACH FOR ROBUST OPTIMIZATION OF WASTELOAD ALLOCATION IN RIVER SYSTEMS

7.1 Introduction

Modern optimization models for controlling water pollution in river systems treat uncertainty of the environment and/or vagueness of management goals [e.g., Lohani and Thanh(1979)[56], Burn and McBean(1985)[10], Chang *et al.*(1997)[14], and Sasikumar and Mujumdar(1998)[87]]. In Chapters 5 and 6, linear programming models have been developed in the framework of the robust optimization (RO) presented by Mulvey *et al.*(1995)[70], in order to solve a problem of BOD loading allocation in rivers. RO is a hybrid of stochastic and multiobjective optimization: on the introduction of scenarios, RO provides a means of trading off among an expected value, the risk of suboptimality (represented by solution robustness), and the risk of infeasibility (represented by model robustness) [Watkins and McKinney(1997)[100]].

Generally, a final decision is made on water quality management after a multiobjective problem on that matter is solved. In the works presented in the previous chapters, however, the role of the optimization model in the decision-making process is not explicitly investigated. In this study, the multiobjective decision-making process (MDMP) is supposed to consist of the following three steps after Haimes and Chankong(1979)[37], as described in Chapter 3:

1. Analysts generate noninferior solutions of a multiobjective optimization problem;
2. Obtain meaningful information to interact with a decision-maker (DM); and

3. Use information obtained in Step 2 to interact with the DM and select the final solution based on the DM's preference response.

The RO models developed in the preceding chapters can be interpreted as tools for generating noninferior solutions in Step 1 of the MDMP on river water quality management. Since the RO model produces the large number of noninferior solutions, it is needed to modify it in order to obtain only such solutions that are worth discussing in the MDMP. Thus, in the present chapter, the method for generating noninferior solutions is changed from the Lagrangian method, which is a kind of the weighting method and commonly adopted in RO models in other various subjects [e.g., Malcolm and Zenios(1994)[64], Mulvey *et al.*(1995)[70], and Watkins and McKinney(1997)[100]], to the ϵ-constraint method [Chankong and Haimes(1983)[15]]. The advantages of the ϵ-constraint method are a complete setting of values of objective functions except a prime objective one before starting the optimization, and acquisition of trade-off rates among objectives within the noninferior set.

Not to mention that more realistic modeling of management objectives is necessary. Here, a new objective is added to the RO model in order to control river water quality thoroughly.

This chapter is organized as follows. In Section 7.2, the RO model developed in Chapter 6 is briefly reviewed. As a modification of the RO model, an optimization model with four objectives in a ϵ-constraint form, ϵ-RO model, is formulated in Section 7.3. An optimization example is shown in Section 7.4 to verify the advantages of the ϵ-RO model developed. Finally, conclusions are given in Section 7.5.

7.2 RO Model for Managing River Water Quality

The RO model presented in Chapter 6 is reexpressed as follows.

$$\text{Minimize } f_1 + \alpha \hat{f}_2 + \hat{\beta} \hat{f}_3 \tag{7.1}$$

subject to

- BOD and DO transport equations that are discretized by the FEM at all scenarios

$$E_s \mathbf{L}_s + F_s \mathbf{L}_s^L = \mathbf{b}_s, \quad G_s \mathbf{C}_s + H_s \mathbf{C}_s^L = \mathbf{d}_s, \quad \forall s \tag{7.2}$$

- Effluent limitation standards at all scenarios

$$\mathbf{L}_s^{Ll} \leq \mathbf{L}_s^L \leq \mathbf{L}_s^{Lu}, \quad \mathbf{C}_s^{Ll} \leq \mathbf{C}_s^L \leq \mathbf{C}_s^{Lu}, \quad \forall s \tag{7.3}$$

- River water quality standards (WQSs) at loading points and tributary mouths at all scenarios

$$0 \leq \mathbf{L}_s^o \leq \mathbf{L}_s^{ou} + \mathbf{u}_s^o, \quad 0 \leq \mathbf{C}_s^{ol} - \mathbf{v}_s^o \leq \mathbf{C}_s^o, \quad \forall s \tag{7.4}$$

- Nonnegativity at all scenarios

$$\mathbf{L}_s, \mathbf{L}_s^L, \mathbf{L}_s^o, \mathbf{C}_s, \mathbf{C}_s^L, \mathbf{C}_s^o, \mathbf{u}_s^o, \mathbf{v}_s^o \geq 0, \quad \forall s \tag{7.5}$$

where

$$f_1 = -\sum_s p_s x_s, \quad \hat{f}_2 = \max_s \left\{ p_s \left| x_s - \sum_s p_s x_s \right| \right\},$$

$$\hat{f}_3 = \sum_s \sum_i p_s (u_{is}^o + v_{is}^o),$$

the superscripts L, o, u and l stand for injected wastewater, loading point or tributary mouth, upper limit, and lower limit, respectively, the subscript s stands for scenario, p_s = probability of a scenario s, $x_s = \sum_j q_j L_{js}^L$ = total BOD loading at a scenario s, q_j = discharge of wastewater injected into a loading point j, $\mathbf{L}_s$ and $\mathbf{C}_s$ = vectors whose components are BOD and DO concentrations of river water at the nodes, respectively, $\mathbf{L}_s^L$ and $\mathbf{C}_s^L$ = vectors whose components are BOD and DO concentrations of wastewaters that are injected into the j-th loading points, L_{js}^L and C_{js}^L, respectively, E_s and G_s = state matrices obtained from application of the FEM, F_s and H_s = matrices associated with $\mathbf{L}_s^L$ and $\mathbf{C}_s^L$, respectively, $\mathbf{u}_s^o$ and $\mathbf{v}_s^o$ = relaxation vectors whose i-th component is the variable relaxing WQSs for BOD and DO concentrations at the i-th node, u_{is}^o and v_{is}^o, respectively, and α and $\hat{\beta}$ = weights. Note that $\mathbf{L}_s^L$ and $\mathbf{C}_s^L$ are decision variable vectors, while $\mathbf{L}_s$, $\mathbf{L}_s^o$, $\mathbf{C}_s$, $\mathbf{C}_s^o$, $\mathbf{u}_s^o$ and $\mathbf{v}_s^o$ are state variable vectors, and $\mathbf{b}_s$, $\mathbf{d}_s$, $\mathbf{L}_s^{Ll}$, $\mathbf{L}_s^{Lu}$, $\mathbf{C}_s^{Ll}$, $\mathbf{C}_s^{Lu}$, $\mathbf{L}_s^{ou}$ and $\mathbf{C}_s^{ol}$ are constant vectors. The objective function, Eqn.(7.1), includes the following three terms: f_1, an expected total BOD loading with the negative sign; $\hat{f}_2$, a maximum expected absolute deviation of a total BOD loading at each scenario from an expected total BOD loading (i.e., a measure of solution robustness); and $\hat{f}_3$, a penalty for not maintaining WQSs for BOD and DO concentrations at loading points and tributary mouths (i.e., a measure of model robustness). Small values of $\hat{f}_2$ and $\hat{f}_3$ lead to solutions that are robust in solution and in model, respectively.

By introducing variables y_s^+ and y_s^- that are defined as

$$y_s^+ = \frac{1}{2} \left\{ \left| x_s - \sum_s p_s x_s \right| + x_s - \sum_s p_s x_s \right\}, \ \forall s \tag{7.6}$$

$$y_s^- = \frac{1}{2}\left\{|x_s - \sum_s p_s x_s| - (x_s - \sum_s p_s x_s)\right\}, \ \forall s \tag{7.7}$$

and w, the RO model [Eqns.(7.1)-(7.5)] is converted into the following equivalent linear programming form to which the simplex method can be applied directly.

$$\text{Minimize} \quad f_1 + \alpha f_2 + \hat{\beta}\hat{f}_3 \tag{7.8}$$

subject to

$$E_s\mathbf{L}_s + F_s\mathbf{L}_s^L = \mathbf{b}_s, \quad G_s\mathbf{C}_s + H_s\mathbf{C}_s^L = \mathbf{d}_s, \quad \forall s \tag{7.9}$$

$$\mathbf{L}_s^{Ll} \leq \mathbf{L}_s^L \leq \mathbf{L}_s^{Lu}, \quad \mathbf{C}_s^{Ll} \leq \mathbf{C}_s^L \leq \mathbf{C}_s^{Lu}, \quad \forall s \tag{7.10}$$

$$0 \leq \mathbf{L}_s^o \leq \mathbf{L}_s^{ou} + \mathbf{u}_s^o, \quad 0 \leq \mathbf{C}_s^{ol} - \mathbf{v}_s^o \leq \mathbf{C}_s^o, \quad \forall s \tag{7.11}$$

$$y_s^+ - y_s^- = x_s - \sum_s p_s x_s, \quad \forall s \tag{7.12}$$

$$p_s(y_s^+ + y_s^-) - w \leq 0, \quad \forall s \tag{7.13}$$

$$\mathbf{L}_s, \mathbf{L}_s^L, \mathbf{L}_s^o, \mathbf{C}_s, \mathbf{C}_s^L, \mathbf{C}_s^o, \mathbf{u}_s^o, \mathbf{v}_s^o, w, y_s^+, y_s^- \geq 0, \ \forall s \tag{7.14}$$

where $f_2 = w$. The RO model obtains individual solutions for each scenario and consolidate these solutions into a management alternative. In this case, an allocation of an expected BOD loading is obtained as an alternative in the MDMP by solving the RO model with positive weights α and $\hat{\beta}$.

7.3 Reformulation

7.3.1 Introducing new objective

What model analysts can do is to provide the entire range of options to a decision-maker, and not to make a decision. More realistic representation of control objectives is needed to obtain more valuable results from operating the optimization model. Thus the RO model is modified by introducing a new objective. For this, *surplus deviation* at a node is defined as an absolute difference in concentration between BOD (or DO) and the WQS for BOD (or DO), only when the WQS at the node is not violated. The concept of surplus deviation as well as violated deviation is sketched in Figure 7.1. Surplus deviation is recognized at a node when BOD concentration in a river is less than the WQS for BOD concentration, or when DO concentration in a river is greater than the WQS for DO concentration.

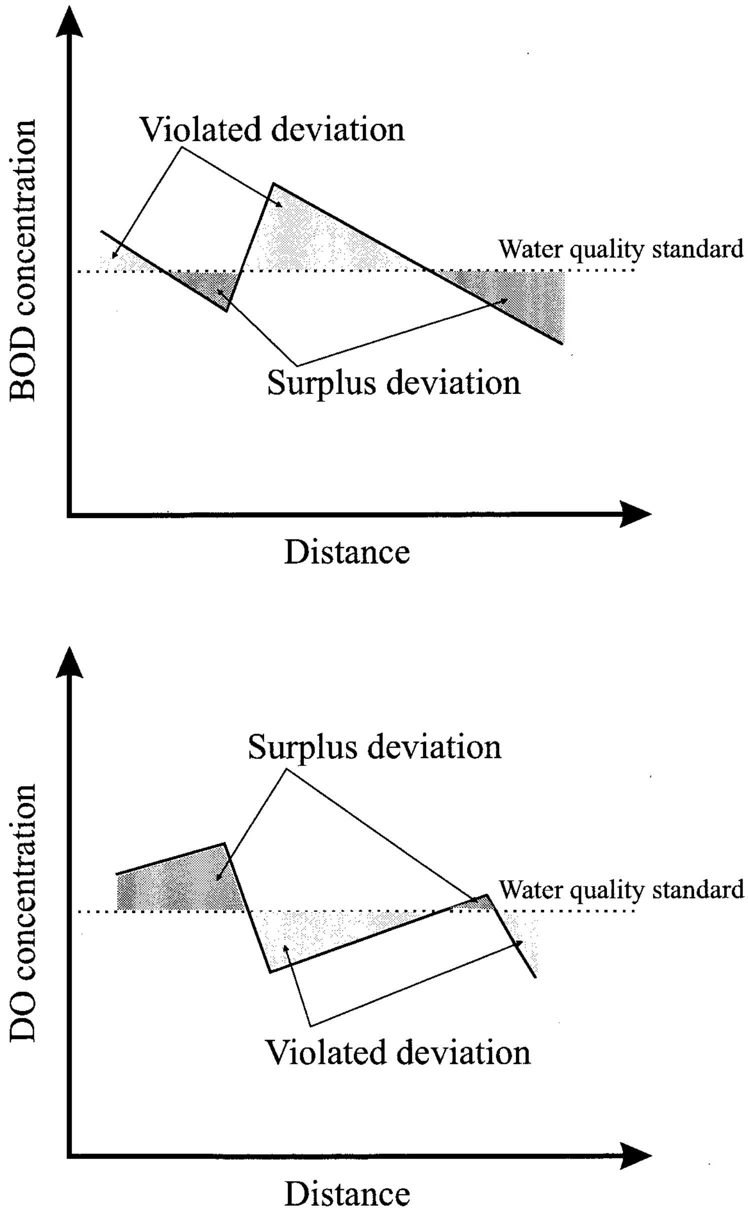

Figure 7.1: Conceptual sketch of surplus deviation and violated deviation

In the RO model [Eqns.(7.8)-(7.14)], minimizing violated deviations from WQSs is taken into account as one of the objective criterion. However, it is considered that without controlling surplus deviations, the optimization model often produces a solution where the allowable amounts of BOD loading at loading points are quite diverse, which may not be an equitable allocation. Equity among dischargers has historically been one of primary planning objectives because of its importance in developing politically acceptable and implementable plans, and various equity measures have been presented [Brill *et al.*(1976)[7] and Marsh and Schilling(1994)[65]]. Therefore, it would be better to consider the equity of allocation of BOD loading into each loading point. In the present study, minimizing surplus deviations is added to the objective criterion in the RO model, so that river water quality can be kept closer to the WQS in some solutions. As a result, the difference in allowable BOD loading among loading points is expected to be reduced to a certain degree. Furthermore, this modification can explicit the trade-off, which is probably one of the most important trade-offs to be considered, among maximizing allowable total BOD loading (note that this leads to minimization of treatment cost of effluents) and minimizing surplus deviation.

A modified RO model is initially expressed as:

$$\text{Minimize } f_1 + \alpha f_2 + \tilde{\beta}\tilde{f}_3 \tag{7.15}$$

subject to

$$E_s\mathbf{L}_s + F_s\mathbf{L}_s^L = \mathbf{b}_s, \quad G_s\mathbf{C}_s + H_s\mathbf{C}_s^L = \mathbf{d}_s, \quad \forall s \tag{7.16}$$

$$\mathbf{L}_s^{Ll} \le \mathbf{L}_s^L \le \mathbf{L}_s^{Lu}, \quad \mathbf{C}_s^{Ll} \le \mathbf{C}_s^L \le \mathbf{C}_s^{Lu}, \quad \forall s \tag{7.17}$$

$$\mathbf{L}_s^o - \mathbf{L}_s^{ou} = \mathbf{U}_s^o, \quad \mathbf{C}_s^{ol} - \mathbf{C}_s^o = \mathbf{V}_s^o, \quad \forall s \tag{7.18}$$

$$y_s^+ - y_s^- = x_s - \sum_s p_s x_s, \quad \forall s \tag{7.19}$$

$$p_s(y_s^+ + y_s^-) - w \le 0, \quad \forall s \tag{7.20}$$

$$\mathbf{L}_s, \mathbf{L}_s^L, \mathbf{L}_s^o, \mathbf{C}_s, \mathbf{C}_s^L, \mathbf{C}_s^o, w, y_s^+, y_s^- \ge 0, \quad \forall s \tag{7.21}$$

$$-\infty < \mathbf{U}_s^o, \mathbf{V}_s^o < \infty, \quad \forall s \tag{7.22}$$

where

$$\tilde{f}_3 = \sum_s \sum_i p_s(|U_{is}^o| + |V_{is}^o|)$$

$\mathbf{U}_s^o$ and $\mathbf{V}_s^o$ = deviation vectors whose i-th component is the deviation from WQSs for BOD and DO concentrations at the i-th node, U_{is}^o and V_{is}^o, respectively, and α and $\tilde{\beta}$ = weights. Note that the third objective in Eqn.(7.8), i.e., $\hat{f}_3$, is altered to $\tilde{f}_3$, and the inequality constraints related to WQSs [Eqn.(7.11)] are converted into equality constraints Eqn.(7.18). Additionally, both upper and lower limits of $\mathbf{U}_s^o$ and $\mathbf{V}_s^o$ are eliminated [Eqn.(7.22)]. This model can be rewritten in a linear programming form in the same way described in Section 7.2 by introducing variables

$$U_{is}^{o+} = \frac{1}{2}\{|U_{is}^o| + U_{is}^o\}, \quad U_{is}^{o-} = \frac{1}{2}\{|U_{is}^o| - U_{is}^o\}$$
$$V_{is}^{o+} = \frac{1}{2}\{|V_{is}^o| + V_{is}^o\}, \quad V_{is}^{o-} = \frac{1}{2}\{|V_{is}^o| - V_{is}^o\}$$

for all s. Note that if the deviation $U_{is}^o > 0$, then $U_{is}^{o+} = U_{is}^o(> 0)$ and $U_{is}^{o-} = 0$; if $U_{is}^o < 0$, then $U_{is}^{o+} = 0$ and $U_{is}^{o-} = -U_{is}^o(> 0)$; and if $U_{is}^o = 0$, then $U_{is}^{o+} = U_{is}^{o-} = 0$. These relations similarly hold among V_{is}^o, V_{is}^{o+} and V_{is}^{o-}. Therefore, U_{is}^{o+} and V_{is}^{o+} represent the violated deviations of BOD and DO concentrations from the WQSs, respectively, and U_{is}^{o-} and V_{is}^{o-} their surplus deviations. Using these new variables, the objective $\tilde{f}_3$ is divided into the following two objectives.

$$f_3 = \sum_s \sum_i p_s\left(U_{is}^{o+} + V_{is}^{o+}\right), \quad f_4 = \sum_s \sum_i p_s\left(U_{is}^{o-} + V_{is}^{o-}\right)$$

where f_3 represents an expected sum of violated deviations of both BOD and DO concentrations from the WQSs at loading points and tributary mouths o, and f_4 an expected sum of surplus deviations of both BOD and DO concentrations from the WQSs at loading points and tributary mouths o. Consequently, Eqns.(7.15)-(7.22) can be rewritten as

$$\text{Minimize} \quad f_1 + \alpha f_2 + \beta f_3 + \gamma f_4 \tag{7.23}$$

subject to

$$E_s\mathbf{L}_s + F_s\mathbf{L}_s^L = \mathbf{b}_s, \quad G_s\mathbf{C}_s + H_s\mathbf{C}_s^L = \mathbf{d}_s, \quad \forall s \tag{7.24}$$

$$\mathbf{L}_s^{Ll} \leq \mathbf{L}_s^L \leq \mathbf{L}_s^{Lu}, \quad \mathbf{C}_s^{Ll} \leq \mathbf{C}_s^L \leq \mathbf{C}_s^{Lu}, \quad \forall s \tag{7.25}$$

$$\mathbf{L}_s^o - \mathbf{L}_s^{ou} = \mathbf{U}_s^{o+} - \mathbf{U}_s^{o-}, \quad \mathbf{C}_s^{ol} - \mathbf{C}_s^o = \mathbf{V}_s^{o+} - \mathbf{V}_s^{o-}, \forall s \tag{7.26}$$

$$y_s^+ - y_s^- = x_s - \sum_s p_s x_s, \quad \forall s \tag{7.27}$$

$$p_s(y_s^+ + y_s^-) - w \leq 0, \quad \forall s \tag{7.28}$$

$$\mathbf{L}_s, \mathbf{L}_s^L, \mathbf{L}_s^o, \mathbf{C}_s, \mathbf{C}_s^L, \mathbf{C}_s^o, w, y_s^+, y_s^-, \mathbf{U}_s^{o+}, \mathbf{U}_s^{o-}, \mathbf{V}_s^{o+}, \mathbf{V}_s^{o-} \geq 0, \quad \forall s \tag{7.29}$$

where $\mathbf{U}_s^{o+}$ and $\mathbf{V}_s^{o+}$ = violated deviation vectors whose components are U_{is}^{o+} and V_{is}^{o+}, respectively, $\mathbf{U}_s^{o-}$ and $\mathbf{V}_s^{o-}$ = surplus deviation vectors whose components are U_{is}^{o-} and V_{is}^{o-}, respectively, and α, β and γ = weights. Both of the objectives f_3 and f_4 could be measures of model robustness in the RO framework. Contrary to the RO model previously developed, the model currently developed can produce noninferior solutions where both violated and surplus deviations are quantified and controlled proactively, thus providing more desirable alternatives to the DM-analyst interactive phase in the MDMP.

7.3.2 Applying ϵ-constraint method

Though the weighting method is perhaps the simplest multiple objective technique, and hence one of the most accepted by decision-makers, it has a number of serious drawbacks [Watkins and McKinney(1997)[100]]. The RO model described above [i.e., Eqns.(7.23)-(7.29)] can generate various noninferior solutions by setting positive values of weights α, β and γ in the objective function (the Lagrangian method), but the relationships between those weights and objective values are not obvious. Besides, there are cases in which changes in the weights can lead to no corresponding change in the objective values. Therefore obtaining some preferable noninferior solutions efficiently from which analysts could indicate one final solution in Step 3 of the MDMP may still be a hard task, which is not fully discussed in the RO framework developed by Mulvey *et al.*(1995)[70] though.

In contrast, the ϵ-constraint method enables analysts to derive noninferior solutions by directly determining the values of not the weights (α, β and γ) but all objective functions (namely f_2, f_3 and f_4) except a prime objective one, which is easier for a DM because of their clear meanings. Another distinct advantage of this method is that it provides trade-off rates among objectives of each noninferior solution [Chankong and Haimes(1983)[15]]. Trade-off is probably the most widely accepted and appears in most decision-making problems. It has a potential of providing a systematic assessment, e.g., comparing two objectives at a time [Haimes and Chankong(1979)[37]]. For the linear case with four objectives like this study, the following theorem holds: For some given ϵ_j, j=2, 3 and 4, let $\mathbf{x}^*$ be a solution of a ϵ-constraint problem in the decision space, and let $-\lambda_{1j}^*$ denote the optimal simplex multipliers corresponding to the binding ϵ-constraint $f_j = \epsilon_j$, j=2, 3 and 4. Then the following relation holds for each j=2, 3 and 4 in a neighborhood of (f_2^*, f_3^*, f_4^*) in the objective space.

$$\frac{\partial f_1(f_2^*, f_3^*, f_4^*)}{\partial f_j} = -\lambda_{1j}^* \tag{7.30}$$

where $f_j^* = f_j(\mathbf{x}^*)$ and f_1 is a continuously differentiable function of f_2^*, f_3^* and f_4^* [Haimes and Chankong (1979)[37]]. The left hand side of this equation represents the ratio of change of f_1 per one unit change in f_j when all other objectives remain unchanged. When the RO model is solved by the simplex method, optimal simplex multipliers $-\lambda_{1j}^*$'s are produced at the same time. Hence considering these merits, the ϵ-constraint method is employed to solve the RO problem in the present study.

By the ϵ-constraint approach, the modified RO problem [Eqns.(7.23)-(7.29)] can be reduced to an ϵ-constraint problem. In the ϵ-RO model obtained, out of the four objectives defined above, only f_1 is taken as an objective function, whereas other objectives f_2, f_3 and f_4 are replaced by inequality constraints with new parameters ϵ_2, ϵ_3 and ϵ_4. Thus the ϵ-RO model can be described as

$$\text{Minimize} \quad f_1(\mathbf{x}) \tag{7.31}$$

subject to

$$h_i(\mathbf{x}) = 0, \qquad i = 1, \cdots, m_1 \tag{7.32}$$

$$f_j(\mathbf{x}) \leq \epsilon_j, \qquad j = 2, 3 \text{ and } 4 \ (\epsilon\text{-constraints}) \tag{7.33}$$

where Eqn.(7.32) represents the set of constraints in Eqns.(7.24)-(7.29) (all inequality constraints are converted to equality constraints by introducing slack or surplus variables), m_1 = the number of constraints in Eqn.(7.32), and $\mathbf{x}$ = vector of variables. Noninferior solutions of the ϵ-RO model are created by setting values of ϵ_2, ϵ_3 and ϵ_4 before starting optimization. Note that a solution of the ϵ-RO model can be specified as a noninferior solution if and only if all the ϵ-constraints of the model are binding [Chankong and Haimes(1983)[15, Theorem 4.3]]. Besides, if the λ_{1j}^* is found strictly positive, then the corresponding ϵ-constraint j can be judged binding [Haimes *et al.*(1990)[41]]. In that case, the respective objective values f_j, j=2, 3 and 4 are equal to given values of ϵ_j, j=2, 3 and 4. This means that these objective values of the solution are completely controlled by analysts. In contrast, if at least one of the ϵ-constraints of the ϵ-RO model at a solution is found not binding, then the solution is discarded because of its inferiority.

The addition of the new objective f_4 makes the region of feasible solutions narrower. Therefore analysts may often fail to obtain a noninferior solution of the model. However, this problem could readily be overcome by referring to the trade-off information among objectives, $\lambda_{12}^*, \lambda_{13}^*$ and λ_{14}^*, related to a noninferior solution that has already obtained.

7.4 Demonstrative Example

7.4.1 Preliminaries

Sample optimizations are implemented to compare the ϵ-RO model [Eqns.(7.31)-(7.33)] with the RO model presented in the last chapter [i.e., Eqns.(7.8)-(7.14)]. The ϵ-constraint method is considered to be superior to the Lagrangian method due to some merits mentioned in the last subsection. Therefore, in order to highlight the effect of introducing the new objective f_4 to the RO model, the RO model previously developed is also solved by the ϵ-constraint method, without any changes of the model's meaning, using the following formulation.

RO model developed in Chapter 6 can be written in ϵ-constraint form

$$\text{Minimize} \quad f_1(\mathbf{x}) \tag{7.34}$$

subject to

$$g_i(\mathbf{x}) = 0, \quad i = 1, \cdots, m_2 \tag{7.35}$$

$$f_2(\mathbf{x}) \leq \epsilon_2, \quad \hat{f}_3(\mathbf{x}) \leq \hat{\epsilon}_3 \tag{7.36}$$

where Eqn.(7.35) is a compact form of the constraints in Eqns.(7.9)-(7.14), m_2 = the number of constraints in Eqn.(7.35), and $\mathbf{x}$ = vector of variables.

Both the ϵ-RO model [Eqns.(7.31)-(7.33)] and the RO model [Eqns.(7.34)-(7.36)] (hereafter in this chapter, the set of Eqns.(7.34)-(7.36) is referred to as 'the RO model') are applied to manage water quality in a hypothetical river network composed of reaches R-1 through R-5, as shown in Figure 7.2. The network is fragmented into 29 line elements with 30 nodes. The same effluent limitations, WQSs, five wastewater discharges at loading points (LPs) (Table 6.1) and twelve scenarios (Table 6.2) as in Chapter 6 are considered. Fairly 'bad' conditions are presumed that WQSs for BOD and DO concentrations on upstream boundaries Γ_1, Γ_2 and Γ_3 are violated under most scenarios.

In order to figure out the hydraulic ingredients in both ϵ-RO and RO models in advance, steady-state open channel flow simulation is performed for each and every scenarios.

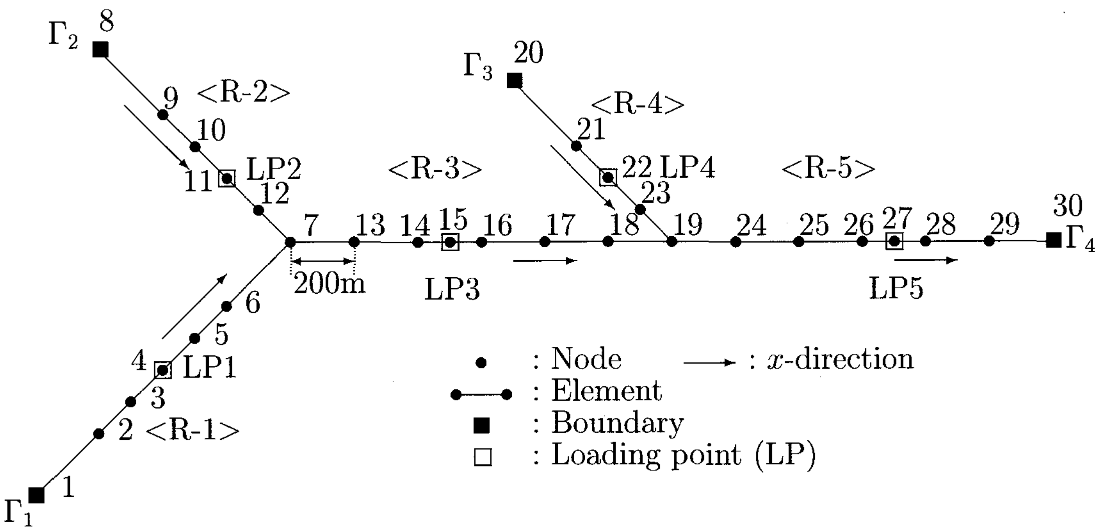

Figure 7.2: Hypothetical river network

Further, before operating the optimization models, minimum values of f_1, f_2, f_3 and f_4 in the ϵ-RO model are obtained by solving the following problems for j = 1, 2, 3 and 4, respectively

$$\text{Minimize} \quad f_j(\mathbf{x}) \tag{7.37}$$

subject to

$$h_i(\mathbf{x}) = 0, \quad i = 1, \cdots, m_1 \tag{7.38}$$

Note that Eqn.(7.38) is the same as Eqn.(7.32). Then minimum extreme values are found: $f_1 = -320$(g/s), $f_2 = 0$(g/s), $f_3 = 3.15$(mg/L) and $f_4 = 3.43$(mg/L). The determined values of ϵ_j's should be greater than these computed values to ensure the feasibility of the ϵ-RO model.

7.4.2 Results and discussion

The ϵ-RO model, as well as the RO model, is solved to produce noninferior solutions. There are certainly much choices in the parameter values of ϵ_j's. However, since minimizing f_2, f_3 (or $\hat{f}_3$) and f_4 is a part of objectives in this management, rather small values of ϵ_j's should be set in order to generate meaningful noninferior solutions. Furthermore, if

Table 7.1: Parameter values for $\epsilon_j (j = 2, 3 \text{ and } 4)$ and $\hat{\epsilon}_3$

Solution	ϵ_2(g/s)	ϵ_3 or $\hat{\epsilon}_3$(mg/L)	ϵ_4(mg/L)
A	2.0	3.6	3.7
B	2.0	3.6	3.8
C	2.0	3.6	4.1
D	2.0	3.6	–[a]

[a]: Value does not exist.

relatively large values are adopted for the set of ϵ_j's, obtained solution may not be judged noninferior. In this demonstrative example, therefore, parameter values listed in Table 7.1 are set.

Three noninferior solutions of the ϵ-RO model, Solutions A, B and C, and one noninferior solution of the RO model, Solution D, are contrasted. For the first three different solutions, different values of ϵ_4 are specified. The respective values of ϵ_2 and ϵ_3 (or $\hat{\epsilon}_3$) are taken as the same for all four solutions.

Note that, from the computational result, $\hat{f}_3$-value of Solution D is found strictly equal to the expected sum of violated deviations at loading points and tributary mouths. Namely $\hat{f}_3$ of the RO model is identical to f_3 of the ϵ-RO model in this case. Therefore the influence caused only by ϵ_4 can be examined by varying the value of ϵ_4 in the ϵ-RO model like 3.7, 3.8 and 4.1, as shown in Table 7.1.

Tables 7.2 and 7.3 show objective achievements and trade-off rates for all the solutions, respectively. Various advantages of employing the ϵ-constraint method can be seen from these tables. It can be confirmed that the objective values of f_2, f_3 and f_4 are perfectly fixed at the given values of ϵ_2, ϵ_3 (or $\hat{\epsilon}_3$) and ϵ_4. Besides, the values of ϵ_j and $\lambda^*_{1j}, j = 2, 3$ and 4 could be helpful for analysts to systematically derive meaningful alternatives considering the relation in Eqn.(7.30), and avoid meaningless computations. For instance, λ^*_{14}=0.027 in Solution C means that f_1 won't decrease so much even if analysts set another bigger value of ϵ_4 with the fixed level of ϵ_2 and ϵ_3.

Moreover, Table 7.2 shows that the expected sum of surplus deviations at all nodes decreases with the decreasing value of $\epsilon_4 (= f_4)$. This implies that some important alternatives can be provided for a DM who wants to decrease surplus deviations by using the ϵ-RO model. The expected sum of violated deviations for the river network is nearly the same in all four solutions due to the same given value of ϵ_3.

Table 7.2: Objective achievements

Solution	f_1 (g/s)	f_2 (g/s)	f_3 or $\hat{f}_3$ (mg/L)	f_4 (mg/L)	Violated deviations along the river (mg/L)	Surplus deviations along the river (mg/L)
A	−34.712	2.0	3.6	3.7	15.011	16.269
B	−41.274	2.0	3.6	3.8	15.037	16.638
C	−51.728	2.0	3.6	4.1	15.009	17.745
D	−51.729	2.0	3.6	−[a]	15.009	18.184

[a]: Value does not exist.

Distributions of expected BOD and DO concentrations in the river network in Solutions A through D are shown in Figure 7.3 and Figure 7.4, respectively, with WQSs for both water quality indices. The BOD and DO concentrations in each solution vary especially along the reach R-5, though the relation $\epsilon_3 = f_3$ strictly holds. Figure 7.3 shows that the surplus deviation of expected BOD concentration from the WQS occurs at nodes 19, 24, 25 and 26. For all of the four nodes, the surplus deviation has a minimum in Solution A, and a maximum in Solution D. Again, it is confirmed that the surplus deviation can be successfully reduced by decreasing the value of $\epsilon_4 (= f_4)$ at each of those nodes. With regard to expected DO concentration, shown in Figure 7.4, the surplus deviation is found at nodes 19 through 30. This deviation is likely to decrease slightly by pushing down the value of $\epsilon_4 (= f_4)$.

Table 7.4 gives the noninferior solutions; expected BOD and DO concentrations of wastewater injected at the LPs, and expected BOD loadings (i.e., allocated BOD loadings). Also Table 7.4 includes two important attributes of each solution; the total expected BOD loading into the whole river system, and difference between maximum and minimum expected BOD loadings in the solution. From the viewpoint of reducing the difference, i.e., laying emphasis on equity, it can be stated that all the solutions obtained by the ϵ-RO model (Solutions A, B and C) are superior to the solution by the RO model (Solution D). Figure 7.5 - 7.8 show optimal wasteload allocation in each solution, respectively.

Table 7.3: Trade-off rates

Solution	λ^*_{12} (–)	λ^*_{13} or $\hat{\lambda}^{*b}_{13}$ (m^3/s)	λ^*_{14} (m^3/s)
A	6.035	58.410	80.543
B	0.865	44.701	52.733
C	3.706	35.401	0.027
D	3.706	35.401	–[a]

[a]: Value does not exist. [b]: Trade-off rate between f_1 and $\hat{f}_3$: $\hat{\lambda}^*_{13} = -\frac{\partial f_1(f^*_2, \hat{f}^*_3)}{\partial \hat{f}_3}$.

Table 7.4: Optimal expected values of wastewater qualities, BOD loading and difference

Solution		LP1	LP2	LP3	LP4	LP5	Total[a] (g/s)	Difference[b] (g/s)
A	BOD(mg/L)	15.547	15.553	30.408	5.000	18.466		
	DO(mg/L)	3.250	3.250	1.000	2.092	1.000		
	Loading(g/s)	3.887	3.888	**15.204**	**2.500**	9.233	34.712	12.704
B	BOD(mg/L)	15.547	15.553	21.364	5.000	40.634		
	DO(mg/L)	3.313	3.411	1.000	2.441	1.000		
	Loading(g/s)	3.887	3.888	10.682	**2.500**	**20.317**	41.274	17.817
C	BOD(mg/L)	14.161	15.557	5.000	5.000	78.598		
	DO(mg/L)	3.497	3.693	3.264	2.441	1.000		
	Loading(g/s)	3.540	3.889	**2.500**	**2.500**	**39.299**	51.728	36.799
D	BOD(mg/L)	14.161	15.557	5.000	5.000	78.599		
	DO(mg/L)	3.734	3.829	3.625	3.625	1.000		
	Loading(g/s)	3.540	3.889	**2.500**	**2.500**	**39.300**	51.729	36.800

[a]: Total BOD loading.
[b]: Difference between maximum and minimum expected BOD loadings (typed in boldfaced letters).

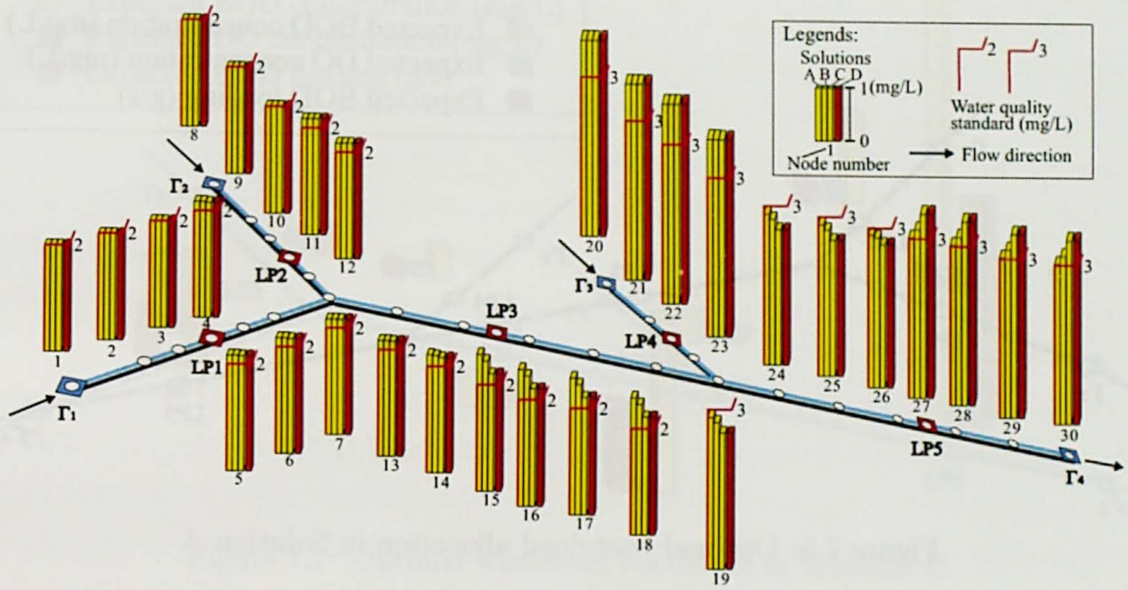

Figure 7.3: Expected BOD concentrations in river system in Solutions A through D

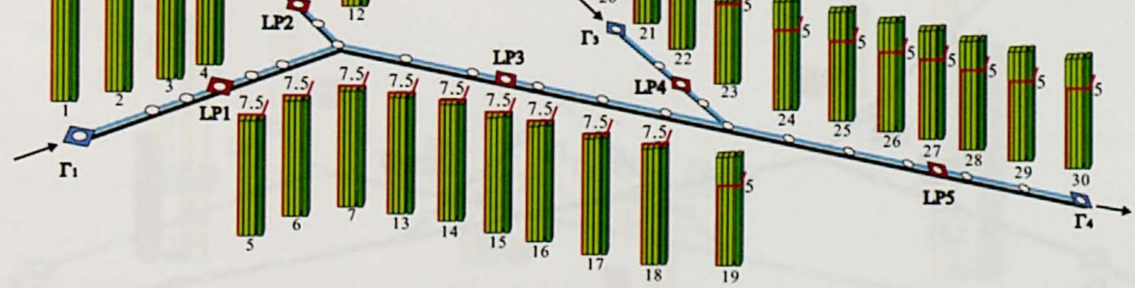

Figure 7.4: Expected DO concentrations in river system in Solutions A through D

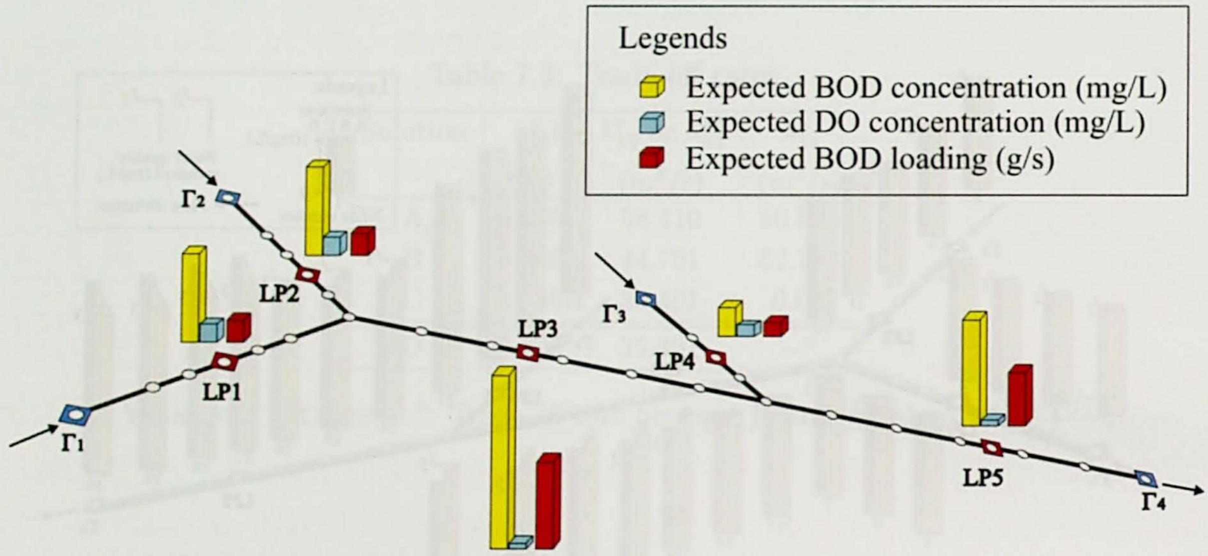

Figure 7.5: Optimal wasteload allocation in Solution A

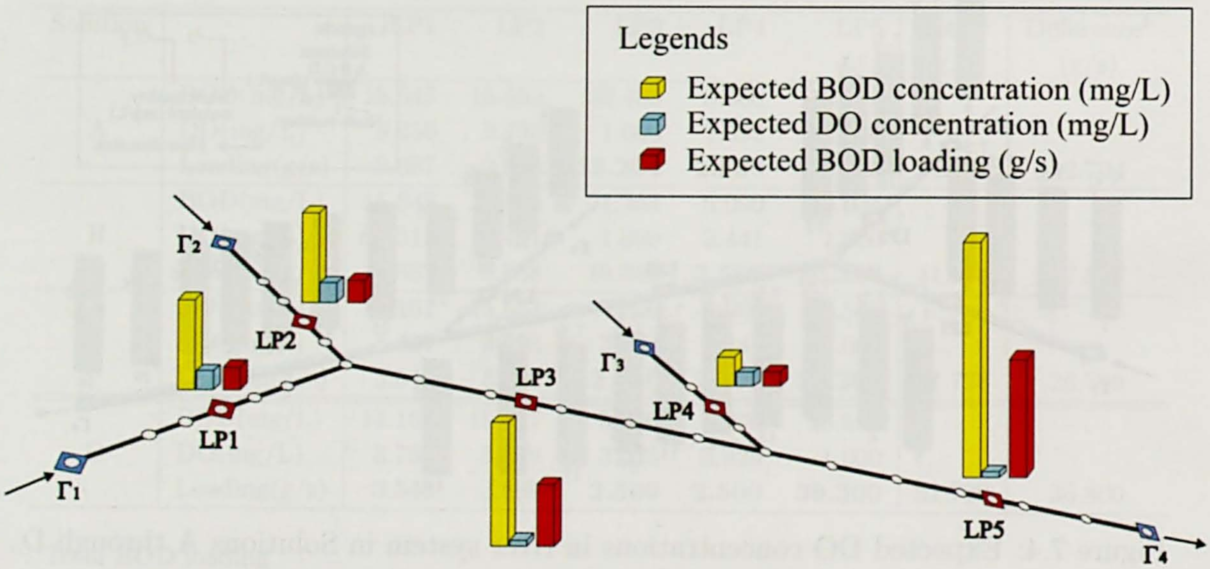

Figure 7.6: Optimal wasteload allocation in Solution B

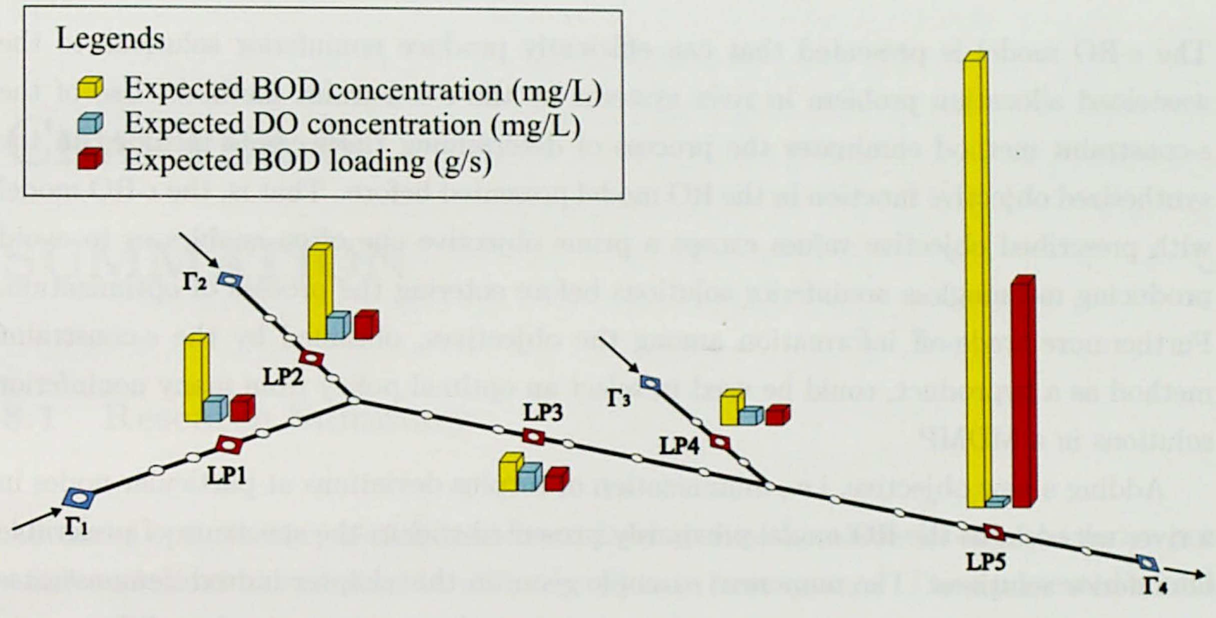

Figure 7.7: Optimal wasteload allocation in Solution C

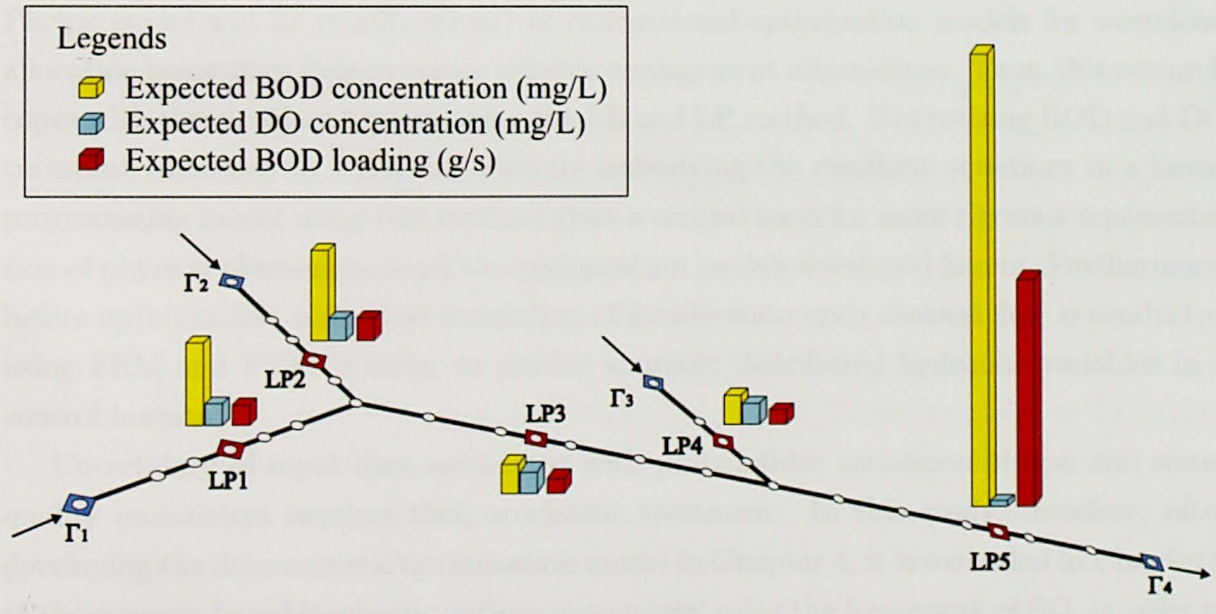

Figure 7.8: Optimal wasteload allocation in Solution D

7.5 Conclusions

The ϵ-RO model is presented that can efficiently produce noninferior solutions to the wasteload allocation problem in river systems by the ϵ-constraint method. Use of the ϵ-constraint method eliminates the process of determining the weights included in the synthesized objective function in the RO model presented before. That is, the ϵ-RO model with prescribed objective values except a prime objective one often enables us to avoid producing meaningless noninferior solutions before entering the process of optimization. Furthermore trade-off information among the objectives, obtained by the ϵ-constraint method as a byproduct, could be used to select an optimal policy from many noninferior solutions in a MDMP.

Adding a new objective, i.e., minimization of surplus deviations at particular nodes in a river network, to the RO model previously presented widens the spectrum of preferable noninferior solutions. The numerical example given in this chapter indeed demonstrates that the ϵ-RO model can produce some noninferior solutions that give less difference in BOD loading among LPs and therefore can achieve relatively equitable wasteload allocation over the whole network of streams.

Chapter 8

SUMMATION

8.1 Research Summary

Advanced optimization models have been developed in this thesis in order to derive persuasive solutions to wasteload allocation problem in river systems. These presented models can quantitatively deal with the difficult points that often annoy water quality managers, such as how to harmonize competing objectives of economic growth and water quality conservation, and how to obtain rational management plans in uncertain natural environment.

Employing common but primitive expression on transport of pollutants (e.g., Streeter-Phelps model and its modifications) in conventional optimization models for wasteload allocation sometimes fails to create reliable management alternatives. Thus, this research copes with this problem by employing the FE and LP method. Discretizing BOD and DO transport equations by FEM and directly embodying the resultant equations in a linear programming model using this method gives a central basis for more rigorous representation of physical phenomena in all the optimization models developed herein. Furthermore, before optimization, numerical simulation of steady-state open channel flow is conducted using FEM and FVM in order to predict spatially distributed hydraulic variables in a control horizon.

Uncertainty of input data associated with probabilistic variations of flow and water quality parameters requires their stochastic treatment. In this study, therefore, after developing the deterministic optimization model in Chapter 4, it is extended in Chapter 5 to the scenario-based stochastic optimization model using the framework of RO, in order to generate robust solutions in the sense of optimality (solution robust) and feasibility (model robust) under uncertain environment. A solution robust plan is theoretically less sensitive to the randomness of input data. In other words, it remains optimal or nearly optimal during the period of controlling water quality. A model robust solution assures small-scale or no violations of water quality standards in the river at all scenarios, which corresponds to less magnitude of relaxation of constraints in the 'original' model formulation. In Chapter 6, modification of the way of scenario generation, elimination of the concept of monitoring point and reduction of unimportant constraints from the optimization model are made. Consequently, the RO model becomes applicable to network-type rivers with several upstream and downstream boundaries, and computational efforts are decreased.

The studies done in Chapters 5 and 6 pay little attention to the importance of efficiency for creating meaningful management alternatives in the whole multiobjective decision-making process where optimization stage is of central significance. Since the RO models developed in these chapters have multiple objectives conflicting each other, numerous alternatives, i.e., noninferior solutions, can be obtained by vector optimization theory. In order to efficiently select a final management policy from these solutions, alternatives that are only worthy of discussion in the remaining procedure in the decision-making process should be derived. In Chapter 7, considering the role of the optimization model as a generator of alternatives in the decision-making process, the ϵ-constraint method is adopted to solve the multiobjective optimization problem including four management goals. This method also obtains trade-off rates among objectives, which can be useful information to screen the obtained alternatives.

8.2 Conclusions

Competing various demands of wastewater dischargers and those of water users lead to unsatisfactory policies of water quality remediation. Aiming at supporting decision-making in such complex river systems, this study establishes optimization models where goals of dischargers (i.e., maximization of total allowable BOD loadings) and demands of water users (i.e., conservation of water quality environment) are represented by objective functions and constraints related to dynamics of BOD and DO, water quality standards and effluent limitations.

Three kinds of models, i.e., the deterministic optimization model, RO model and ϵ-RO model, are presented in this thesis. Since the size of the deterministic optimization model is much smaller than the sizes of the RO and ϵ-RO model, the former model is less computationally laborious. Using the deterministic model may therefore be preferable under relatively less uncertain environment. On the contrary, both the RO and ϵ-RO model embraces stochastic nature of river flow and water quality environment by the scenario-based approach to control random effect on optimality and feasibility in the solutions. Use of these models have thus their own benefits.

These stochastic optimization models reveal the importance of exploring management alternatives where in-stream water quality standards are relaxed. From computational point of view, the relaxation of these standards, certainly to a rational degree, is important because it increases feasibility of the optimization models. Even deterministic models can have the same influence, though the effect may be smaller than that in stochastic optimization models. However, more important fact is that requiring too strict observance of water quality standards sometimes overlooks the management plans that deserve to be discussed. Since the relaxation of the standards generally results in increasing allowable BOD loading (or decreasing treatment costs of wastewater), and in addition, the crisp standards are originally given from exterior in no relation to the model formulation, relaxing the water quality standards to a rational degree could have significant value for producing strategic policies. Therefore in the RO model, violation of the standards is allowed by introducing relaxation vectors related to the standards, with the addition of a penalty term to the objective function. Furthermore, in the ϵ-RO model, the idea that the water quality standards give not limiting values of water quality but give target values of that is employed. That is, surplus deviations as well as violated deviations from water quality standards can be adjusted in creating management strategies of water quality in the ϵ-RO model.

It is thus concluded that the developed optimization models in this thesis could be used to provide rational, persuasive and implementable management alternatives for wasteload allocation in actual river systems effectively, compared to conventional management models.

8.3 Future Works

Future works will be directed toward a modification or an improvement of the ϵ-RO model developed. They are summarized below.

The accuracy of predicting flow and water quality environment by a probabilistic distribution of variables in the scenario-based approach significantly affects derived noninferior solutions. Due to several uncertain parameters considered in each scenario, synthesizing each stochastic nature into a probabilistic distribution of realization is a hard task. Therefore it may be needed to examine whether assumed scenarios in the ϵ-RO model are adequate or not, in order to obtain a more reliable result of optimization.

Employing many scenarios could increase the credibility of the solution generated by the ϵ-RO model. This leads to an increase of the model size, and therefore of computational effort. To overcome this problem, the following two approaches might work. One is to decompose the ϵ-RO model into a main model and several small sub-models [Vladimirou and Zenios(1997a)[97]]. This method may require the reformulation of the ϵ-RO model and a change of algorithm to operate it. The other is to apply the idea of multiple-realization technique [Ranjithan *et al.*(1993)[78] and Takyi and Lence(1999)[90]] to the ϵ-RO model. In this method, only some critical realizations to an optimal solution are selected from many (e.g., several thousand) ones by pre-processing. Then constraints under the critical realizations are only considered in an optimization model, and therefore the problem size of the original model could be reduced.

Verifying the applicability of the ϵ-RO model to real rivers is necessary for its practical use. In this case, scenarios should be created from observed data in the river of interest. The method employed by Watkins and McKinney(1999)[101] where historical hydrologic data are used could be referred in the stage of scenario generation.

Since water pollution caused by nonpoint sources such as agricultural and domestic ones is relatively difficult to detect, a small number of researches has been made that deals with controlling the nonpoint source pollution. However such distributed pollutants should also be considered in the ϵ-RO model as controllable or uncontrollable variables to effectively manage river water quality, because of their serious impact on the water environment.

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