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# Topics from Competitive Game Theory

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Some of the recent works by the present author are given in the two parts $\boxed{A}$ and $\boxed{B}$. A lot of interesting open problems are mentioned.

## $\boxed{A}$ 2- AND 3- PLAYER GAMES OF SCORE SHOWDOWN

**1. 3-Player Games of Score Showdown.** Let $X_{ij}$ $(i=1,2,3; j=1,2)$ be the j-th r.v. observed by Player i. Assume that $\{X_{ij}\}$ are i.i.d. with $U_{[0,1]}$ distribution. Each player i first observes $X_{i1}=x_{i1}$ and chooses A/R. If A(R) is chosen, the $x_{i1}$ is accepted ( rejected and the second r,v $X_{i2}$ is observed). Player i's score is

$$S_i(X_{i1},X_{i2}) = \begin{cases} X_{i1} \\ \varphi(X_{i1},X_{i2}) \end{cases}, \text{ if } X_{i1}=x_{i1} \text{ is } \begin{cases} \text{accepted.} \\ \text{rejected \& } X_{i2} \text{ is observed.} \end{cases}$$

We consider the cases

$$\varphi(X_{i1},X_{i2}) = X_{i2},\ X_{i2}I(X_{i2}\geq X_{i1}),\ (X_{i1}+X_{i2})I(X_{i1}+X_{i2}\geq 1),\ \tfrac{1}{2}(X_{i1}+X_{i2})$$

(Keep-or-Exchange, Risky-Exchange, Showcase-Showdown,CompetingAverage,resp)

Player who gets the highest score is the winner. Each player wants Pr( he wins)→max

Also we consider the two versions of the games;

Simultanious-move version —— Players' choices are made indep. and are not known to his rivals.

Sequential- move version —— Players move sequentially, i.e.,I at first, II at second,and III at third. After I's move, II and III are informed of $X_{11}=x_1$ and I's choice of A/R. After II's move, III is informed of $X_{21}=x_2$ and II's choice of A/R. All players are intelligent, and each player should prepare for that any subsequent player must employ their optimal strategies.

In the present article, we use $\Gamma$= game, C=common, EQ=equilibrium, S=strategy, V= value, D= draw. Players 1,2,3 are sometimes written by I,II, III, resp.

**2. Solutions to the Games,** It is shown that games of $\Gamma^{(2)}$SS and $\Gamma^{(2)}$RE have the same solution (REF[3]).Some other results are given below ( Ref[1,2,3])

• Solution to $\Gamma^{(2)}$KE (simult.-move): CEQS is "Choose A(R) if his first observ.is $>(<)$ $a^* = \frac{1}{2}(\sqrt{5}-1) \doteqdot 0.618$, i.e., a unique root in $(0,1)$ of the equation $a^2+a=1$." CEQV= $\frac{1}{2}$. In $\Gamma^{(3)}$KE, the $a^*$ the equation and CEQV change to $a^* \doteqdot 0.691$, $2a^4 = 1-a+a^2-a^3$ and $\frac{1}{3}$ resp.

• Solution to $\Gamma^{(2)}$RE (simult.-move).; CEQS is "Choose A(R), if his first observ. is $>(<)$ $a^* \doteqdot 0.544$, i.e., a unique root in $(0,1)$ of the equation $a^3+a^2+a=1$." $P(D) = \frac{1}{4}a^{*4} \doteqdot 0.022$, $P(W_1) = P(W_2) = \frac{1}{2}(1-P(D)) \doteqdot 0.489$. In $\Gamma^{(3)}$RE. $a^*$ the equation and the other result change to $a^* \doteqdot 0.656$, $2a^4+a^5 = 1-a+a^2-a^3$, $P(D) = \frac{1}{8}a^{*6} \doteqdot 0.010$, $P(W_1) = P(W_2) = P(W_3) = \frac{1}{3}(1-P(D)) \doteqdot 0.330$.

• Solution to $\Gamma^{(2)}$RE ( seq.-move ); I's opt.str. is "Choose A (R) $X_{11} = x_1$, if $x_1 >(<) x_0$ where $x_0(\doteqdot 0.570)$ is a unique root in $(\sqrt{2}-1, 1)$ of $\bar{x}^4 = 2x(1-3x^2)$" II's opt.str. is

"Choose A(R) $X_{21} = x_2$ if $x_2 >(<) \begin{cases} x_1 \\ y_0(x_1) \end{cases}$, in state $\begin{cases} (x_2 \mid x_1, A) \\ (x_2 \mid x_1, R) \end{cases}$,
where $y_0(x_1) \equiv (\sqrt{2}-1)I(x_1 \leq \sqrt{2}-1) + (\bar{x}_1^2/(2x_1))I(x_1 > \sqrt{2}-1)$." We obtain $P(D)$, $P(W_1)$, $P(W_2) \doteqdot 0.011, 0.477, 0.512$, resp.

• Solution to $\Gamma^{(2)}$CA (seq.-move ): I's opt. str. is "Choose A(R) $X_{11} = x_1$ if $x_1 > (<) x_0$, where $x_0(\doteqdot 0.549)$ is a unique root in $(\frac{1}{2}, 1)$ of $7/(2x) - 4\sqrt{2x} = 6-(5/2)x+x^2$." II's opt. str. is "Choose A(R) $X_{21} = x_2$ if $x_2 >(<) \begin{cases} x_1 \\ y_0(x_1) \end{cases}$, in state $\begin{cases} (x_2 \mid x_1, A) \\ (x_2 \mid x_1, R) \end{cases}$,

where $y_0(x_1) \equiv (\sqrt{2\bar{x}_1} - \bar{x}_1)I(x_1 < \frac{1}{2}) + (\sqrt{2x_1} - x_1)I(x_1 \geq \frac{1}{2})$. We obtain $P(W_1) = 1 - P(W_2) \doteqdot 0.490$.

• Solution to $\Gamma^{(3)}$KE (seq.-move): It is too complicate to write here. We found that $P(W_1), P(W_2), P(W_3) \doteqdot 0.32309, 0.33270, 0.34421$, resp. The sum is 1.

3. Information Types in 2-Player Games. We consider the two information types, under which players decide their choices of A/R.

$I^{11-11}$ means that I chooses $X_{11} = x_1$, II chooses $X_{21} = x_2$ and each player informs of his observed value to his opponent.

$I^{10-11}$ means that I observes $X_{11} = x_1$, II observes $X_{21} = x_2$ and I informs his $x_1$ to II, and II doesn't inform his $x_2$ to I.

It is clear that $I^{10-01}$ is the information type discussed in Section 2.

Some results are given as follows ( Ref[4]).

• Solution to $\Gamma^{(2)}$RE under $I^{11-11}$ ; In state $x_1 - x_2$ there exists a unique saddle point at R-A, R-R, A-R, if $x_1 \vee x_2 \vee (\sqrt{2}-1) = x_2, \sqrt{2}-1, x_1$, resp.

We obtain $P(D)=\frac{1}{4}(\sqrt{2}-1)^4 \doteqdot 0.007$, $P(W_1)=P(W_2)=\frac{1}{2}(1-P(D)) \doteqdot 0.496$.

• Solution to $\Gamma^{(2)}$KE under $I^{(0-1)}$ : I's opt. str. is "Choose A ( R) $X_{11}=x_1$, if $x_1 > (<)\ a^*=\sqrt{3/8} \doteqdot 0.6124$." II's opt. str.in state $x_1-x_2$ is given by

| Condition | II's opt. Choice |
|---|---|
| $x_1 < a^*$ | $A(R)$, if $x_2 > (<) \frac{1}{2}$ |
| $a^* < x_1 < x_2$ | $A$ |
| $x_1 > a^* \vee x_2$ | $R$ |

We obtain $P(W_1)=1-P(W_2)=\frac{1}{3}+\frac{1}{4}a^* \doteqdot 0.4864$.

4. 2-Player game of Continue-or-Stop. Player i observes ( $X_{i1}, X_{i2}, \cdots X_{in}$), sequentially one-by-one, and facing each $X_{ij}$ chooses Cont/Stop. If Stop is chosen $X_{ij}$ is accepted by player i and his play ends. If Cont. is chosen, $X_{ij}$ is rejected and the next $X_{i,j+1}$ is observed and the game continues. Assume that $\{X_{ij}\}$ are i.i.d. with $U_{[0,1]}$ distribution. Score is

$$S_i(X_{i1},\cdots,X_{in})=\begin{cases} X_{i1} \\ X_{i,j+1} \end{cases}, \text{ if } \begin{cases} \text{stop at } X_{i1} \\ X_{i1}, X_{i2}, \cdots, X_{ij} \text{ are rejected and stop at } X_{i,j+1} \end{cases}$$

Consider the situation where player i has $n-1$ decision thresholds $(1>) a_{i1} > a_{i2} > \cdots > a_{i,n-1}$ $(>0)$, so that $i$ chooses Stop (Cont. ) if $X_{ij} > (<)\ a_{ij}$. Player who gets the higher score than his opponent is the winner. Each player aims Pr ( he wins ) → max.

• Solution when n = 3 ( Ref[5]); The common optimal thresholds are $(b_1^0, b_2^0) \doteqdot (0.743, 0.657)$, where $(b_1^0, b_2^0)$ is a unique root in $(0,1)^2$ of

$$\begin{cases} b_2^2+b_2-(b_1^{-1}-1+b_1)=0 \\ b_1^2+(1+b_2)^{-1}b_1-1=0 \end{cases}$$

$P(W_1)=P(W_2)=\frac{1}{2}$.

5. Remark. There are many open problems of interest around the topic— games of score showdown. Three among them are given below.

(1). Let $(X_{ij}, Y_{ij})$, $i=1,2$, $j=1,2$, are i.i.d. with joint p.d.f.

$$f(x,y)=1+\gamma(1-2x)(1-2y), \quad \forall (x,y)\in[0,1]^2,$$

with $|\gamma|\le 1$. In the game $\Gamma^{(2)}$KE (simult.-move ), the score is

$$S_i(X_{i1}Y_{i1}, X_{i2}Y_{i2})=\begin{cases} (X_{i1}, Y_{i1}) \\ (X_{i2}, Y_{i2}) \end{cases}, \quad \text{if } (X_{i1}, Y_{i1}) \text{ is } \begin{cases} \text{accepted} \\ \text{rejected} \end{cases}.$$

Define that ( x, y ) is higher than [lower than, intermediate to] ( x′, y′ ), if (x, y ) $\geqslant$ [ <, otherwise] ( x′, y′ ). Player with score ( x, y ) gets 1 [−1, 0 ]. Solve this zero-sum game.

(2). Sequential-move games $\Gamma^{(3)}$RE and $\Gamma^{(3)}$CA remain unsolved.

(3). 3-player games under various information types are of interest. $\Gamma^{(3)}$KE, $\Gamma^{(3)}$RE and $\Gamma^{(3)}$CA, under $I^{111\text{-}111\text{-}111}$, $I^{100\text{-}110\text{-}111}$, $I^{110\text{-}011\text{-}101}$, etc. remain unsolved. The last one $I^{110\text{-}011\text{-}101}$, for example, means that each player observes his own $X_{i1} = x_{i1}$ and, in addition, I knows $x_{21}$, II knows $x_{31}$ and III knows $x_{11}$.

## REFERENCES

[1,2,3] M.Sakaguchi, SCMJ ,60(2004), 411-421; 61(2005), 335-344,; 61(2005), 145-151, resp.

[4,5] ———, Game Th. Appl. 10(2005), 148-160 ; To appear in 11(2006), resp.

## $\boxed{B}$ MULTISTAGE OPTIMAL STOPPING GAMES

1. Each Player has his Priority. Player I, II, and III observe $X_1, X_2, \cdots, X_n$ with i.i.d. $U_{[0,1]}$ distribution sequentially one-by-one. They have their previously given priorities $\langle p_1, p_2, p_3 \rangle$. Facing $X_j$ each player chooses R/A, independently of his rivals.

If only one player chooses A, he gets $X_j$ with his priority, dropping out from the game thereafter, and the remaining two players continue their two-player game with the "revised" priorities. If two players choose A, one player selected according to the "revised" priorities gets $X_j$, drops out from the game thereafter, and the remaining two players continue their 2-player game with the "revised" priorities. If the choices are A-A-A player i gets $X_j$ with prob. $p_i$ dropping out from the game, and the remaining two players continue their 2-player game with the revised" priorities. If the choices are R-R-R then $X_j$ is rejected, the next $X_{j+1}$ is observed and the subsequent 3-player game continues. Each player aims to maximize the ENV he can get ( N in ENV means net, i.e. no-observation-cost and no-discounting ).

Let $W_n$ ($V_n$) be CEQV, for 3-player ( 2-player ) equal-priority game. Then the Opt. Eq. is

$$(W_n, \ W_n, \ W_n) = E\left[\text{eq. val. } M_n(X)\right] \qquad (n \geqslant 1, \ W_0 = V_0 = 0)$$

$$M_n(x) = \begin{cases} \xrightarrow{\text{R by III}} M_{n,R}(x) = \begin{array}{c|c|c} & \text{R by II} & \text{A by II} \\ \hline \text{R by I} & W,\ W,\ W & V,\ x,\ V \\ \text{A by I} & x,\ V,\ V & \frac{1}{2}(x+V),\ \frac{1}{2}(x+V),\ V \end{array} \\ \xrightarrow{\text{A by III}} M_{n,A}(x) = \begin{array}{|c|c|} \hline V,\ V,\ x & V,\ \frac{1}{2}(x+V),\ \frac{1}{2}(x+V), \\ \hline \frac{1}{2}(x+V),\ V,\ \frac{1}{2}(x+V), & \frac{1}{3}(x+2V),\ \frac{1}{3}(x+2V),\ \frac{1}{3}(x+2V), \\ \hline \end{array} \end{cases}$$

[$\overline{W}(\overline{V})$ is $W_{n-1}(\overline{V}_{n-1})$]

$$(V_n, V_n) = E\left\{ \text{val.} \begin{array}{c|c|c} & R & A \\ \hline R & V_{n-1},\ V_{n-1} & U_{n-1},\ x \\ A & x,\ U_{n-1} & \frac{1}{2}(x+U_{n-1})\ \frac{1}{2}(x+U_{n-1}) \end{array} \right\}$$

where $U_n = \frac{1}{2}(1+U_{n-1}^2)$, $(n \geq 1, U_0 = 0)$.

Common opt. str. for each player is derived, and CEQ $\overline{V}$ is computed as

$U_n = 1/2,\ 0.7417\ \ 0.8364,\ 0.8791,$ for $n = 1, 4, 8, 12$, resp.

$V_n = 1/4,\ 0.6468,\ 0.7782,\ 0.8361,$ ibid

$W_n = 1/6\ \ 0.5506,\ 0.7196,\ 0.7936$ ibid

The cases $\langle p_1, p_2 \rangle = \langle 1, 0 \rangle$ and $\langle p_1, p_2, p_3 \rangle = \langle \frac{1}{2}, \frac{1}{2}, 0 \rangle, \langle 1, 0, 0 \rangle$ are also solved (Ref[J~4])

Player with low priority stands at disadvantage, since he remains late in the game, and faces less offers.

The cases where player's aim is Pr. ( he gets a r.v. better than opponent ) $\longrightarrow$ max. are solved in Ref[1, 2]. In the 2-player No-Information case, for example, let us

Define state $(i, y)$ to mean that (1) both players remain in the game, and (2) players currently face the r.v. $Y_i = y$. Let $V(i, y)$ be the value of the game in state $(i, y)$, for the $n$-problem. Note that $n$ is fixed throughout, players should choose A-A in state $(n, y)$ and hence draw of the game cannot occur.

Then t he Opt. Eq. in state ( i,y ) is

$$V(i, y) = \text{val.} \begin{array}{c} \\ R \\ A \end{array} \begin{array}{c} \begin{array}{cc} R & A \end{array} \\ \left[ \begin{array}{cc} \mu_{i+1} & 1 - g(i, y) \\ g(i, y) & (p - \bar{p}) g(i, y) + \bar{p} \end{array} \right] \end{array}$$

$1 \leq i \leq n-1$, $V(n, y) = p \in [1/2, 1]$, $\forall y \in \{1, 2, \cdots, n\}$.

where $\mu_{i+1} = (i+1)^{-1} \sum_{y'=1}^{i+1} V(i+1, y')$, and

$$g(i, y) = \Pr\{i\text{-th has absolute rank } y, \text{ in state } (i, y)\}$$
$$= \prod_{j=i}^{n} \left(1 - \frac{y}{i+j}\right) = \binom{i}{y} \Big/ \binom{n}{y} = (i)_y / (n)_y$$

The solution is: Opt. str. pair in state $(i, y)$ is
R-R, R-A, A-A, if $0<g(i,y)<\bar{\mu}_{i+1}$, $\bar{\mu}_{i+1}<g(i,y)<\frac{1}{2}$, $\frac{1}{2}<g(i,y)<1$, resp.

The winning prob. for $n=10$ are computed downward in $i$ until reaching $\mu_1 = V(1,1)$. We obtain
$\Pr(\text{I wins}) \doteqdot 0.578,\ 0.619$, for $p=3/4, 1$, resp.

3. Commitee's Selection I and II observe $(X_j, Y_j)\ j=1,\cdots,n$, i.i.d. with $U_{[0,1]\times[0,1]}$ distribution sequentially one-by-one, and each player chooses R/A. $X_j$ ($Y_j$) is I's (II's) evaluation of j-th applicant's ability ( Ref.[9]). The Opt. Eq. is

$$(u_n, v_n) = E[\text{eq.val}\, M_n(X,Y)],\quad M_n(x,y) = \begin{array}{c c} & \begin{array}{cc} R & A \end{array} \\ \begin{array}{c} R \\ A \end{array} & \begin{pmatrix} u_{n-1}, v_{n-1} & pu_{n-1}+\bar{p}x, pv_{n-1}+\bar{p}y \\ px+\bar{p}u_{n-1}, py+\bar{p}v_{n-1} & x, y \end{pmatrix} \end{array}$$

The solution is : In state ( $X_j = x, Y_j = y$ ) :

I chooses A (R), if $x \geq (<)\ u_{n-1}$, indep. of $y$,
II chooses A (R), if $y \geq (<)\ v_{n-1}$, indep. of $x$,

where
$$u_n = \frac{1}{2}\{pu_{n-1}^2 + \bar{p}(2u_{n-1}-1)v_{n-1} + 1\},\quad v_n = \text{same with } p \to \bar{p} \text{ and } u \leftrightarrow v.$$

It is shown that $u_n \uparrow u_\infty$, $v_n \uparrow v_\infty$, and $(u_\infty, v_\infty)$ is a unique root in $(0,1)^2$ of
$$u = \frac{\sqrt{1-\bar{p}v}}{\sqrt{1-\bar{p}v}+\sqrt{\bar{p}\bar{v}}},\quad v = \frac{\sqrt{1-pu}}{\sqrt{1-pu}+\sqrt{p\bar{u}}}$$

Convergence is quick:

| | | | | | |
|---|---|---|---|---|---|
| $u_{10}=$ | 0.6592, | 0.6867, | 0.7530, | 0.8611, | for $p=0.5, 0.6, 0.8, 1.0$, resp. |
| $v_{10}=$ | 0.6592, | 0.6331, | 0.5788, | 1/2, | ibid |
| $u_\infty=$ | 2/3, | 0.6946, | 0.7663, | 1, | ibid. |
| $v_\infty=$ | 2/3, | 0.6408, | 0.5899, | 1/2, | ibid |

Various interesting open problems arise. 1 If $X_j$ ( $Y_j$ ) is the ability of management ( foreign language ), then $X_j$ and $Y_j$ are not independent. 2 The case where players aim ENV of ( $X_j \mid Y_j \geq a$ ) $\longrightarrow$ max, ( Ref.[11]). 3 3-player game where $(X_j, Y_j, Z_j)$ is observed.

## 3. High-Hand-Wins Poker.

We first consider a simple n-round poker. Each of two players I and II receives a hand $x$ and $y$, respectively, in $[0,1]$, according to a uniform distribution, and chooses one of two alternatives Reject or Accept. If choice-pair is R–R, the game proceeds to the next round and both players are dealt new hands $x$ and $y$. If the choice-pair is A–A showdown occurs and the game ends with I's reward $sgn(x-y)$. If players choose different choices, then arbitration comes in, and forces them to take the same choices as I's (II's) with probability $p$ ($\bar{p}$). This zero-sum game is played in n-rounds, and player I(II) aims to maximize(minimize) the expected reward to I.

Let $\phi_n(x)(\psi_n(y))$ be the probability that player I (II) chooses A on the hand $x$ ($y$). Also let $v_n$ be the value (for I) of the n-round game. Then we have

$$v_n = \max_{\phi_n(\cdot)} \min_{\psi_n(\cdot)} E_{x,y}[(\bar{\phi}_n(x), \phi_n(x))M_n(x,y)(\bar{\psi}_n(y), \psi_n(y))^T]$$

where

$$M_n(x,y) = \begin{array}{c c} & \begin{array}{cc} R & A \end{array} \\ \begin{array}{c} R \\ A \end{array} & \begin{pmatrix} v_{n-1} & \bar{p}sgn(x-y) + pv_{n-1} \\ psgn(x-y) + \bar{p}v_{n-1} & sgn(x-y) \end{pmatrix} \end{array}$$

The solution is

$$\phi_n^*(x) = I(x > a_n), \quad \psi_n^*(y) = I(y > \bar{a}_n), \quad v_n = 2a_{n+1} - 1$$

where $\{a_n\}$ is determined by $a_{n+1} = a_n + \frac{1}{2}(p\bar{a}_n^2 - \bar{p}\, a_n^2)$ $(n \geq 1,\ a_1 = \frac{1}{2})$.

We obtain $a_n \uparrow a_\infty = \sqrt{p}/(\sqrt{p} + \sqrt{\bar{p}})$, $v_n \uparrow 2a_\infty - 1 = (\sqrt{p} - \sqrt{\bar{p}})/(\sqrt{p} + \sqrt{\bar{p}})$.

The bilateral-move version, when p = $1/2$ has an interesting solution. Let $w_n$ be the value of the n-stage game. $w_n <$ 0, since I must move first and inevitably gives some information about his true hand. We find that $w_n \downarrow w_\infty = -(1-\sqrt{g})/(1+\sqrt{g}) \doteqdot -0.1197$ where g $= \frac{1}{2}(\sqrt{5}-1) \doteqdot 0.618$, the golden bisection number.
Also we find that disadvantage disappears when $p = 0.6$, i.e. $w_n = 0, \forall n \geq 1$. (Ref.[10]).

3-player High-Hand-Wins poker, under simple-majority rule and with bet $B \geq 0$, is also an interesting open problem.

**4. Odd-Man-Wins and Odd-Man-Out.** In the three-player two-choice games there often appear Odd-Man and Even-Men. What is the reasonable partition among players of each $X_j$ ?

Let $v_n$ ( $w_n$ ) be CEQV of n-stage Odd-Man-Wins ( Odd-Man-Out ). Then the payoff matrices $M_n$( X ) are

| | | R by III | A by III |
|---|---|---|---|
| R by I | R by II | $v_{n-1}, v_{n-1}, v_{n-1}$ | $0,\ 0,\ x$ |
| | A by II | $0,\ x,\ 0$ | $x,\ 0,\ 0$ |
| A by I | R by II | $x,\ 0,\ 0$ | $0,\ x,\ 0$ |
| | A by II | $0,\ 0,\ x$ | $x/3,\ x/3,\ x/3$ |

$n \geq 1$, $v_1 = \frac{1}{6}$,

| | | R by III | A by III |
|---|---|---|---|
| R by I | R by II | $w_{n-1}, w_{n-1}, w_{n-1}$ | $x/2,\ x/2,\ 0$ |
| | A by II | $x/2,\ 0,\ x/2$ | $0,\ x/2,\ x/2$ |
| A by I | R by II | $0,\ x/2,\ x/2$ | $x/2,\ 0,\ x/2$ |
| | A by II | $x/2,\ x/2,\ 0$ | $x/3,\ x/3,\ x/3$ |

$n \geq 1$, $w_1 = \frac{1}{6}$,

for Odd-Man-Wins, Odd-Man-Out, resp.

Each player must think about : (1) He wants to become the odd-man (an even-man), when the game is Odd-Man-Wins (Odd-Man-Out), especially when he faces a very large $X_j$, and (2) Since $X_j$ is a random variable, he can expect a larger one may come up in the future.

It is shown that ( Ref.[6] ), $v_n \uparrow v_\infty \doteqdot 0.205$ and $w_n \downarrow w_\infty \doteqdot 0.160$. So,multi-stage play yields each player a merit of size $v_\infty - \frac{1}{6} \doteqdot 0.039$ and a demerit of size $\frac{1}{6} - w_\infty \doteqdot 0.0064$.

The extension to the many-player two-choice simple-majority games is an interesting open problem.

The case where the odd-man has priority p, and the even-men has priority $\bar{p}/2$ is solved in Ref.[7]. When p = $1/3$, CEQV = $\frac{1}{3}\mu_n$, where $\{\mu_n\}$ is the Moser's sequence. When p = 0 ( 1 ), the game reduces to Odd-Man-Out ( Odd-Man-Wins ).

## REFERENCES

[1, 2, ··· ,7] SCMJ 53(2001) 257-268, 125-134, 56 (2002) 457-473, 317-325, 58(2003) 183-189, 60(2004), 315-325, 403-410..

[8, 9, 10] GTA 7(2001) 156-169, 8(2002) 95-106, 9(2003) 141-149.

[11] Bruss and Lecam ( edt ), Lecture Notes 35 (2000), Inst, Math. Stat., 111-125.

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