The Origin of Nonlinear Phenomena in TCR-SVC Associated With Parametric Excitation of Intrinsic Oscillation and External Excitation

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***Abstract*—This paper focuses on anomalous nonlinear phenomena in a thyristor controlled reactor-static var compensator (TCR-SVC) system, called switching time bifurcation, and clarifies the relationship between the occurrence of nonlinear phenomenon and the intrinsic characteristics of the circuit equation with switching function. The occurrence of nonlinear phenomenon in a TCR-SVC system cannot be predicted when the dynamics of the circuit related to the switching action is neglected. Therefore, this paper considers the dynamics related to the switching operations in the analysis of nonlinear phenomena. The parametric excitation of circuit is discussed in relation to the homogeneous expression of the circuit equation. This paper indicates that the homogeneous equation of the TCR-SVC system results in Hill's equation, and it can be approximated as Mathieu's equation. The occurrence of nonlinear phenomena in the system is evaluated using the characteristics of Mathieu's equation. The anomalous nonlinear phenomena occur when the natural frequency of Mathieu's equation coincides with the frequency of the ac voltage in a TCR-SVC system. The parametric excitation is also confirmed through the characteristics of Hill's equation. This study clarified the interaction between the switching dynamics of the circuit and its external excitation, and clued the occurrence of nonlinear phenomena in the circuit. The proposed procedure can also be applied to the analysis for other switching converter circuits with periodic excitation source.**

***Index Terms*—Homogeneous equation, intrinsic oscillation, Mathieu's equation, nonlinear phenomenon, switching function, switching time bifurcation, TCR-SVC.**

I. Introduction

A THYRISTOR controlled reactor-static var compensator (TCR-SVC) is a power electronics apparatus for power systems, which uses thyristors as the switching devices [1], [2]. The thyristors control the amount of lagging reactive current flowing through the reactor. Then, the reactive power output of TCR-SVC is regulated to maintain the ac system voltage. Thus, TCR-SVC maintains power quality to the disturbances caused by faults or by load changes.

The switching operation of thyristors generates harmonics in periodical steady state, which is related to fundamental utility frequency. References [3], [4] studied the interaction between all the harmonics in TCR-SVC based on the Fourier series expansion of switching function and on the harmonic admittance matrix of the circuit. Reference [4] assessed the stability of TCR-SVC circuit by the eigenvalues of the exponential matrix derived from the state transition matrix for piece width linear circuit. Furthermore, [5] proposed to analyze the distorted waveform in Walsh domain. The work achieved computational efficiency by accurately approximating the switching function with smaller numbers of Walsh coefficients than conventional Fourier and Hartley coefficients. Reference [6] discussed the stability of switching circuit with Lie Algebraic, which is based on the Lyapnov function for the system. References [7], [8] analyzed the border collision bifurcation in dc-dc converter with the explicit map and the symbolic analysis method, respectively. However, a TCR-SVC is different from dc-dc converter in having external periodical excitation ac voltage, and these analysis methods are not suitably applicable.

A TCR-SVC in a power circuit is a nonsmooth dynamical system consisting of linear and continuous power circuit components, and thyristor switches. A thyristor is a semi-controllable device; the turn-on operation is driven by a gate pulse, however, the turn-off is determined by the conditions of device voltage and current. Therefore, the voltage of a power system is required for a thyristor power conversion circuit as an external periodic excitation or a forced commutation circuit. The occurrence of nonlinear phenomena related to switching operations of thyristor has been previously reported [9], [10]. The nonlinear phenomena are a nonsmooth change in the conduction angle to a smooth change in the firing angle. References [11]–[13] demonstrated the occurrence of switching time bifurcation in a TCR-SVC. The non-normality in power system operation has been discussed on the egienvalues of the Jacobian of differential equations at the equilibrium; such as, saddle node bifurcation [14] and Hopf bifurcation [15]. Also, the mechanics for the inter area electric power system oscillation is shown in [16], and subharmonic oscillations are discussed by Melnikov functions in [17]. These approaches are adequate for conventional power system, which can be expressed as a smooth dynamical system. References [12], [13] indicated that the occurrence of switching time bifurcation cannot be predicted from the Jacobian of the fixed point of the system accompanied by switching operations in the TCR-SVC. They also presented coexisting solutions to a given control parameter, and discussed the relationship between switching time bifurcations and their domains of attraction. References [18], [19] reported other nonlinear phenomena occurring in a TCR-SVC, e.g., half-wave asymmetrical and sub-

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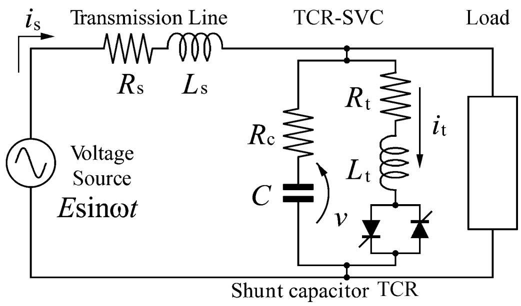

Fig. 1. Schematic diagram of single phase TCR-SVC.

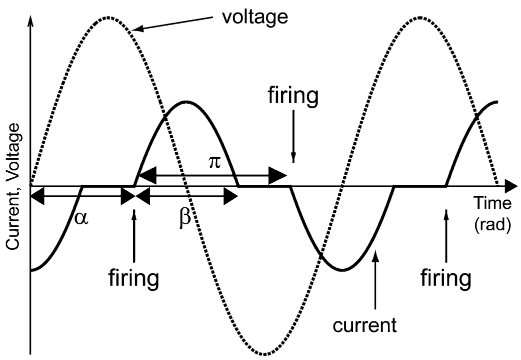

Fig. 2. Ideal voltage and current waveforms of TCR.

TABLE I
CIRCUIT PARAMETERS OF THE STUDIED TCR-SVC SYSTEM

Symbol	Value
ω	60 Hz $\times 2\pi$
L_s	39.32 mH
R_s	44.47 mΩ
L_t	26.12 mH
R_t	29.54 mΩ
C	4.4564 μF
R_c	1.743 Ω
μ	3.33×10^2

harmonic oscillations. These are also classified as one type of switching time bifurcation, because they both have nonsmooth changes in their conduction angles to the smooth changes in the firing angle. The occurrence of unpredictable switching time bifurcation is undesirable for TCR-SVC operation, but it can be managed if bifurcation is predictable. Therefore, it is necessary to clarify the origin of nonlinear phenomena in a TCR-SVC.

This paper focuses on the homogeneous expression of the circuit equation for a TCR-SVC, and discusses the relationship between its characteristics and the occurrence of nonlinear phenomenon. The circuit equation of a TCR-SVC can be expressed by a simple differential equation with variable parameters when the switching operation of the thyristor is expressed by a switching function. The switching function of a TCR-SVC can be expressed as a periodic function with ac fundamental frequency for a periodic steady state operation. Thus, this paper reveals that the homogeneous equation can be derived as Hill's equation by neglecting the damping component in the circuit. In addition, Hill's equation is approximated by Mathieu's equation by extracting the dc and fundamental ac frequency components from the switching function. This paper figures out that there exists the possibility of predicting the occurrence of nonlinear phenomena based on the stability analysis of both Mathieu's equation and Hill's equation. That is, any nonlinear phenomenon occurring in the TCR-SVC is attributable to the interaction between parametric excitation of intrinsic oscillations and the external excitation by the ac system. The numerical and experimental results elucidate the sufficient condition for the occurrence of nonlinear phenomenon in TCR-SVC.

II. BASIC OPERATION OF TCR-SVC AND SYSTEM EQUATION

First, this section describes the circuit configuration and the basic operation of a TCR-SVC system. Then, the circuit equation for a TCR-SVC is established for the conducting and blocking state of thyristor switch and the solution of the circuit equation is derived for a half-wave symmetrical fundamental periodic steady state. Finally, the occurrence of anomalous nonlinear phenomena is estimated based on the derived solutions.

A. *Circuit Configuration of TCR-SVC*

A TCR-SVC system is generally configured as a three-phase system. Fig. 1 depicts one phase component of the TCR-SVC. This study analyzes the behavior of the TCR-SVC in a single-phase configuration to simplify the analysis. This simplification is valid for the phenomenon that is free from inter-phase interaction.

A long transmission line induces a voltage drop between the power station and the load in a power system. The line is represented by lumped inductance L_s and resistance R_s. The drop is due to the current flowing through the transmission line when electric power is transmitted. The power station is treated as an infinite bus and is expressed by the voltage source $E \sin \omega t$. A TCR-SVC consists of a shunt capacitor and a thyristor controlled reactor (TCR). The shunt capacitor consists of a capacitance C and an equivalent series resistance R_c, and supplies leading reactive power to the power system. The TCR comprises an inductance L_t with winding resistance R_t and a back-to-back connected thyristor. It regulates the lagging reactive power output in accordance with current flowing through the reactor, which is controlled by the firing angle α of thyristor as shown in Fig. 2. This system is oscillatory due to its second-order resonant circuit configuration, and the connected load has a damping effect on oscillation. Then, a light or no load condition induces nonlinear phenomena associated with the oscillation and switching of the thyristor. The circuit parameters of the studied system are given in Table I. This study assumes the no load condition to have the most distorted oscillatory waveform, which is subjected to [9]–[12] to observe notably the anomalous nonlinear phenomena in TCR-SVC. The circuit parameters used in the study is chosen to have the same quality factor $\mu \equiv \omega L_s/R_s = \omega L_t/R_t = 1/\omega C R_c = 3.33 \times 10^2$ as the respective circuit component in order to simplify the analysis and to facilitate the derivation of a solution. This assumption of the same quality factor to the respective component in the circuit will not affect on the following analytical result. Because, the damping term in the differential equation of the circuit model will be neglected in the transformation of formula. Also, the high value of quality factor will not affect essentially on the analytical result.

A thyristor, as a switching device, plays an important role in determining the system state. The turn-on operation and conducting state of the thyristor are described as follows. A thyristor holds a blocking state until a firing pulse is injected to the gate at a forward biased condition, and then it turns on. The conducting state is held by the forward current. It turns off when the thyristor current decreases to zero and reverse bias voltage is applied. The voltage and current waveform in TCR for the ideal operating condition is shown in Fig. 2. The solid and dashed lines indicate the TCR current and voltage, respectively.

B. Circuit Equations for TCR-SVC

The time of firing pulse injection is defined by the firing angle α in reference to the phase angle of the applied ac voltage. A periodic firing pulse is injected symmetrically at every half cycle of the ac voltage at periodic steady state. The conduction period of the thyristor, which is expressed by an angle β, varies with the change in the firing angle α. That is, the turn-off of the thyristor depends on the voltage and current in the circuit at the firing, and the TCR current is regulated by the firing angle. Fig. 2 denotes the relationships between the angles, the ac voltage and TCR current waveforms. The specified domain of the firing angle is between $\pi/2$ and π. Because the conduction angle reaches π for a firing angle less than $\pi/2$, and a thyristor cannot turn on for the firing angle larger than π due to reverse bias voltage condition. The conduction angle for a given firing angle is derived based on the circuit equation as follows.

The dynamics of the system are dependent on the behavior in the state variables of transmission line current i_s, TCR current i_t, and shunt capacitor voltage v. The circuit equation of the system is expressed as (1) for the conducting condition of thyristor $(\alpha \leq t \leq \alpha + \beta)$

$$\begin{cases} \frac{di_{s(\mathrm{on})}}{dt} = -\frac{R_s+R_c}{\omega L_s} i_{s(\mathrm{on})} + \frac{R_c}{\omega L_s} i_{t(\mathrm{on})} - \frac{1}{\omega L_s} v_{(\mathrm{on})} + \frac{E}{\omega L_s}\sin t \\ \frac{di_{t(\mathrm{on})}}{dt} = \frac{R_c}{\omega L_t} i_{s(\mathrm{on})} - \frac{R_t+R_c}{\omega L_t} i_{t(\mathrm{on})} + \frac{1}{\omega L_t} v_{(\mathrm{on})} \\ \frac{dv_{(\mathrm{on})}}{dt} = \frac{1}{\omega C} i_{s(\mathrm{on})} - \frac{1}{\omega C} i_{t(\mathrm{on})} \end{cases} . \tag{1}$$

Here, time t is in canonical form with the angular frequency ω.

This circuit equation becomes (2) when the thyristor is in blocking condition $(\alpha + \beta < t \leq \alpha + \pi)$

$$\begin{cases} \frac{di_{s(\mathrm{off})}}{dt} = -\frac{R_s+R_c}{\omega L_s} i_{s(\mathrm{off})} - \frac{1}{\omega L_s} v_{(\mathrm{off})} + \frac{E}{\omega L_s}\sin t \\ i_{t(\mathrm{off})} = 0 \\ \frac{dv_{(\mathrm{off})}}{dt} = \frac{1}{\omega C} i_{s(\mathrm{off})} \end{cases} . \tag{2}$$

The state variables in (1) and (2) are associated with the following boundary conditions at the instant of turn-off to satisfy the condition of continuity

$$\begin{cases} i_{s(\mathrm{on})}(\alpha+\beta) = i_{s(\mathrm{off})}(\alpha+\beta) \\ i_{t(\mathrm{on})}(\alpha+\beta) = 0 \\ v_{(\mathrm{on})}(\alpha+\beta) = v_{(\mathrm{off})}(\alpha+\beta) \end{cases} . \tag{3}$$

In addition to the boundary condition at turn-off given in (3), the periodic steady state condition with half-wave symmetry[1] is represented by the following boundary condition at the instant of turn-on

$$\begin{cases} i_{s(\mathrm{on})}(\alpha) = -i_{s(\mathrm{off})}(\alpha+\pi) \\ i_{t(\mathrm{on})}(\alpha) = 0 \\ v_{(\mathrm{on})}(\alpha) = -v_{(\mathrm{off})}(\alpha+\pi) \end{cases} . \tag{4}$$

[1]The ac system, in which TCR-SVC is connected, has symmetric property in the waveform due to the sinusoidal voltage waveform. Then, the system in periodic steady state with fundamental frequency of the ac system shows half-wave symmetry in their voltage and current waveform.

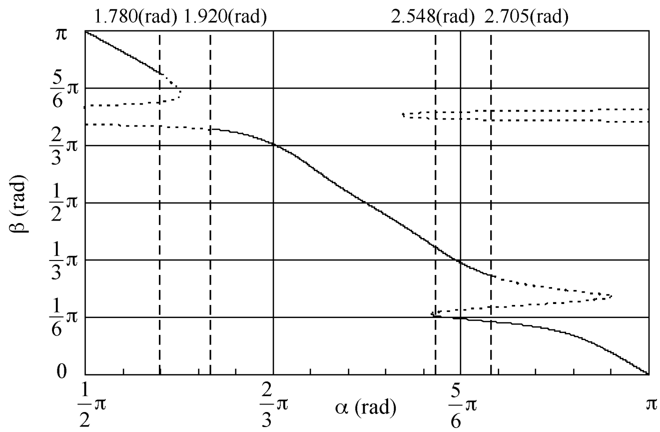

Fig. 3. Relationship between conduction angle β and firing angle α in half-wave symmetrical periodic steady state.

Equations (1) and (2) were solved to obtain the periodic steady state solution with the boundary conditions given by (3) and (4). The calculated relationship between firing angle α and conduction angle β is given in Fig. 3. The solid line indicates the solution, which does exist when the turn-on and turn-off conditions of the thyristor are satisfied. That is, the turn-off time point given by β is the first time point after turn-on, where thyristor current $i_{t(\mathrm{on})}$ decreases to zero. The dashed line in the figure is also the solution of (1) and (2), which satisfies the boundary conditions of (3) and (4); however, it does not satisfy the thyristor operating conditions. That is, there is an extra point in time where the thyristor current reaches zero during the conduction period β, and it turns off before the conduction period ends. Thus, the solution obtained as a dashed line cannot be actually observed in the circuit. Fig. 3 also indicates that the firing angle has the region in which there is no half-wave symmetrical periodic steady state solution $(1.780 \leq \alpha \leq 1.920)$, and the region in which coexisting multiple solutions exist $(2.548 \leq \alpha \leq 2.705)$.

The occurrence of switching time bifurcation can be explained based on the numerical solutions for (1) and (2). The TCR current waveform distorts with the oscillatory behavior of the TCR-SVC. This is due to the second-order resonance circuit of the shunt capacitor and the inductance in the TCR and the transmission line. Then, the current waveform is obtained as shown in Fig. 4(a). The dint at the center of the current waveform in Fig. 4(a) is apart from zero, but it approaches to zero according to increase of the firing angle α. The thyristor turns off when the dint reaches to zero as shown in Fig. 4(b), but the successive waveform, drawn by the dashed line, cannot be achieved in the circuit due to the extinction of thyristor. Thus, the conduction period suddenly becomes small value. The adverse phenomenon occurs in the increment of conduction period to the decrease of firing angle α. That is, there is nonsmooth change in the thyristor conduction period to the smooth change of the firing angle. These phenomena are called

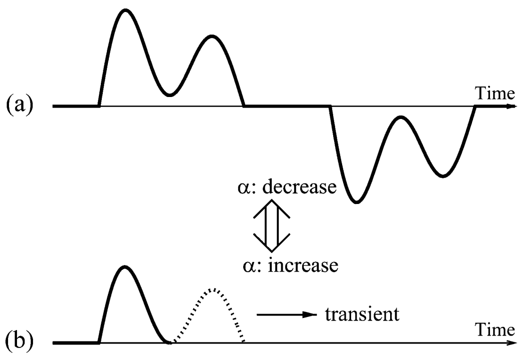

Fig. 4. Current waveform of TCR (a) before and (b) after switching time bifurcation.

switching time bifurcation [11], [12]. The circuit state converges to a periodic steady state after the onset of switching time bifurcation, provided that the system shows a feasible periodic solution to the given firing angle. The occurrence of switching time bifurcation depends on the region of firing angle, which has coexisting solutions of conduction angle. The solution set for periodic steady state changes for the firing angle α variation around 2.548 and 2.705 shown in Fig. 3, where switching time bifurcation were observed. A half-wave symmetric solution cannot be obtained for $1.780 \leq \alpha \leq 1.920$. The half wave asymmetric and subharmonic oscillations were found in this region. The following sections discuss the relationship between the occurrence of nonlinear phenomena and the solution of the system.

III. Homogeneous Expression of Circuit Equation and the Relationship to Occurrence of Nonlinear Phenomena

A. Hill's Equation Related to TCR-SVC

This section focuses on the intrinsic characteristics of the circuit equations for the TCR-SVC. The circuit equations of the TCR-SVC are respectively given by (1) and (2) for the conducting and blocking conditions of the thyristor. They have consistent form except for the term related to the TCR current i_t. Subsequently, the two equations can be consolidated into (5) by applying a switching function q

$$\frac{d^2v}{dt^2} + \frac{1}{\mu}\left(1 + \frac{R_c}{R_s} + q\frac{R_c}{R_t}\right)\frac{dv}{dt} + \frac{1}{\omega^2 C}\left(\frac{1}{L_s} + q\frac{1}{L_t}\right)v = \frac{E}{\omega^2 L_s C}\sin t. \quad (5)$$

Here, the switching function q takes the unit value (1) for the conducting state of thyristor $(\alpha \leq t \leq \alpha + \beta)$, and null value (0) for the blocking state $(\alpha + \beta < t \leq \alpha + \pi)$. The function q directly depends on the system state, which is determined by the control parameter of the firing angle α. Therefore, (5) can be regarded as a second-order differential equation with a variable parameter q.

The TCR-SVC is interconnected to a utility grid which is expressed by an ac voltage source in the right-hand side of (5). That is, the ac voltage source becomes a forced external excitation term in the mathematical expression of the dynamical system. Therefore, the homogeneous expression of the circuit equation governs the intrinsic frequency of the system by considering the switching of the thyristor. The feasibility of nonlinear phenomena in the TCR-SVC is investigated based on the relationship between the solutions in the homogeneous form and the dynamics in the non-homogeneous form. The homogeneous form of (5) is given by (6) with switching function q

$$\frac{d^2v}{dt^2} + \frac{1}{\mu}\left(1 + \frac{R_c}{R_s} + q\frac{R_c}{R_t}\right)\frac{dv}{dt} + \frac{1}{\omega^2 C}\left(\frac{1}{L_s} + q\frac{1}{L_t}\right)v = 0. \quad (6)$$

The second term in the left-hand side of (6) gives a damping term. The criteria for the occurrence of nonlinear phenomena can be notably observed by neglecting the damping term in the equation, as in (7)

$$\frac{d^2v}{dt^2} + \frac{1}{\omega^2 C}\left(\frac{1}{L_s} + q\frac{1}{L_t}\right)v = 0. \quad (7)$$

The switching function q is a periodic function with period π when the operation of the TCR-SVC is in the half-wave symmetrical periodic steady state. Then, (7) is a homogeneous second-order differential equation with a periodic parameter q. It is obviously a type of "Hill's equation" [20], [21].

B. Approximation by Mathieu's Equation and Stability Evaluation

Mathieu's equation, given as (8), is a type of Hill's equation that has a sinusoidal periodic parameter

$$\frac{d^2x}{dt^2} + (\delta - 2\varepsilon\cos 2t) = 0. \quad (8)$$

The solutions of Mathieu's equation and their characteristics have been studied with relation to the parameters δ and ε in (8) [21]–[23]. The solutions of Mathieu's equation are classified into 3 groups: periodic solutions, quasi-periodic solutions, and solutions that diverge exponentially. The parameters in the stable region give periodic solutions or quasi-periodic solutions. On the other hand, the parameters in the unstable region make the solutions diverge over time. The domain of parameters space (δ, ε) in Mathieu's equation is sectioned into the stable and unstable regions, as shown in Fig. 5. The stable regions are filled with white and the unstable regions are filled with gray. The dotted and solid lines in the figure indicate the borders between the stable and unstable regions. The solid line gives the parameter sets for the borders when Mathieu's equation has a periodic solution with period π as a general solution. The dotted line gives the parameter sets for borders when Mathieu's equation has a periodic solution with period 2π. The dashed line, plotted in the form of a parabolic curve gives the parameter sets for δ and ε, which is obtained from (7) by the approximation of switching function q as following.

The switching function q is expanded to the Fourier series for the half-wave symmetrical periodic steady state. Mathieu's equation is approximated with the dc and second-order components of Fourier coefficient as (9). Since, the switching function is unipolar and pulsates twice an ac voltage cycle

$$q^*(t) = \frac{a_0}{2} + a_2\cos 2t. \quad (9)$$

Here, a_n is the nth order of Fourier coefficient, and $n = 2$ corresponds to the fundamental frequency component of the

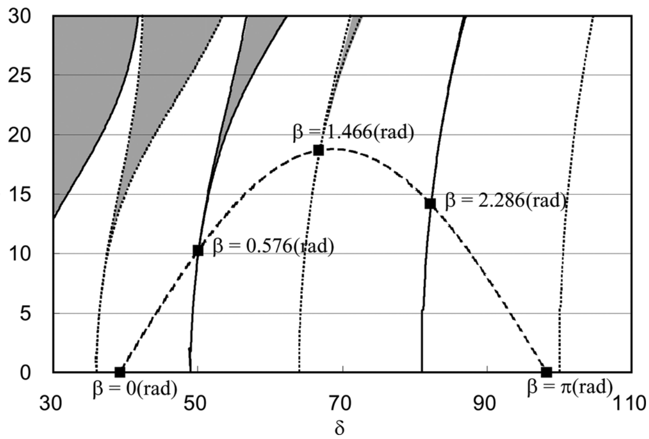

Fig. 5. Discrimination of stable and unstable parameter region for Mathieu's equation and corresponding parameters of TCR-SVC operation.

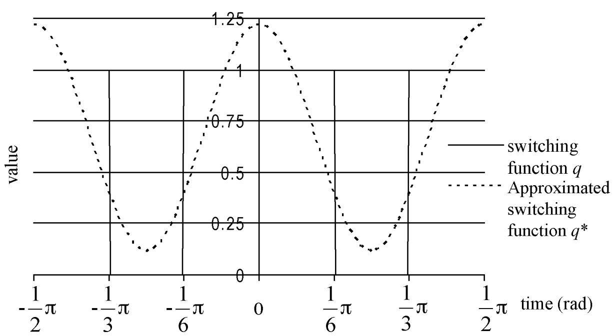

Fig. 6. Example of switching function and approximated switching function in Mathieu's equation ($\beta = (2/3)\pi$).

switching function attributed to the symmetry of the ac waveform.

Fig. 6 shows the relationship between the original switching function q and the approximated switching function q^* by considering the case of $\beta = (2/3)\pi$ as an example. Therefore, the parameters in (8) are derived from (10) using Fourier coefficients

$$\begin{cases} \delta = \frac{1}{\omega^2 C}\left(\frac{1}{L_s} + \frac{a_0}{2L_t}\right) \\ \varepsilon = \left|\frac{-a_2}{2\omega^2 C L_t}\right| \end{cases}. \tag{10}$$

Based on the above formulation, we can relate the origin of nonlinear phenomena in TCR-SVC with the stability of solutions in Mathieu's equation. There are parameter sets for approximated switching function q^*, which are located in the unstable region of Mathieu's equation. These correspond to the conduction angle around $\beta =$0.576, 1.466, and 2.286. The conduction angles around β =0.576 and 2.286, respectively, located in the region of firing angle $1.780 \leq \alpha \leq 1.920$ and $2.548 \leq \alpha \leq 2.705$ as shown in Fig. 3. The occurrence of nonlinear phenomena is observed in these firing angle regions. The parameter sets for these conduction angles are sandwiched by the borders given with the solid line in Fig. 5, in which borders Mathieu's equation has a periodic solution with period 2π. The parameters for the conduction angle $\beta = 1.466$ are also located in the unstable region, which is encompassed by the parameter sets of the borders given by the dotted line, in which the period of periodic solution is π. However, nonlinear phenomena do not appear over this unstable region and the symmetrical half-wave periodic solution is obtained, as shown in Fig. 3.

There are choosy emergences of nonlinear phenomena in the unstable region of Mathieu's equation. The incidence are confined to the unstable region surrounded by the borders for the periodic solution with period 2π. Then, it can be attributed to the resonance of the TCR-SVC circuit. That is, the TCR-SVC is amenable to the external excitation of an ac voltage source with period 2π when the natural frequency of the homogeneous expression for the system coincides with period 2π and does not keep the stability. However, the TCR-SVC is insensitive to the external excitation with period 2π and keeps the stability, even if the homogeneous equation is unstable when the region is enclosed by the stability boundary with period π.

C. Discrimination of Nonlinear Phenomena Occurrence Based on Characteristic Equation of Hill's Equation

The last subsection discussed that the region of parameters, which cause nonlinear phenomena in the TCR-SVC, coincides with the unstable regions of Mathieu's equation. However, their emergences are confined to the unstable region surrounded by the boundary for the periodic solution with period 2π. This section discusses the relationship among the parameter region, stable solution, and the occurrence of nonlinear phenomena. The discussion is based on the characteristics of Hill's equation, which carries out the analysis with accurate behavior of the switching function.

Hill's equation, (7), is a second-order ordinary differential equation which has a general solution as a linear combination of two linearly independent fundamental solutions. The two fundamental solutions $\phi_1(t)$ and $\phi_2(t)$ are chosen to satisfy the initial conditions given in

$$\begin{cases} \phi_1(0) = 1, & \frac{d\phi_1(0)}{dt} = 0 \\ \phi_2(0) = 0, & \frac{d\phi_2(0)}{dt} = 1 \end{cases}. \tag{11}$$

The characteristic equation of Hill's equation is given by (12)

$$\rho^2 - \left\{\phi_1(\pi) + \frac{d\phi_2(\pi)}{dt}\right\}\rho + 1 = 0. \tag{12}$$

The fundamental solutions, which correspond to the accurate switching function q, are derived as follows. The state variable v corresponds to the solution of the following differential equation for the conducting period of the thyristor $(0 \leq t \leq \beta)$

$$\frac{d^2v}{dt^2} + \frac{1}{\omega^2 C}\left(\frac{1}{L_s} + \frac{1}{L_t}\right)v = 0. \tag{13}$$

The two fundamental solutions, which satisfy the initial conditions (11), can be obtained by (14)

$$\begin{cases} \phi_{1\mathrm{ON}}(t) = \cos k_1 t \\ \phi_{2\mathrm{ON}}(t) = \frac{1}{k_1}\sin k_1 t \end{cases}. \tag{14}$$

Here, $k_1 = \sqrt{(1/\omega^2 C)(1/L_s + 1/L_t)}$.

The state variable v corresponds to the solution of the following differential equation for the blocking period of the thyristor $(\beta < t < \pi)$

$$\frac{d^2v}{dt^2} + \frac{1}{\omega^2 C}\frac{1}{L_s}v = 0. \tag{15}$$

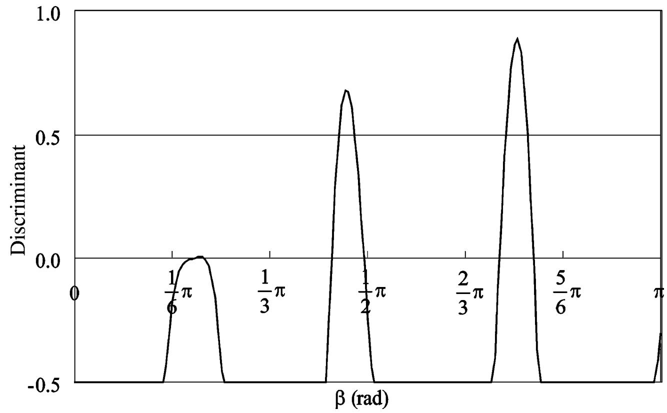

Fig. 7. Variation of discriminant for characteristic equation of Hill's equation to parameter β.

The state variable v and its time derivative dv/dt are continuous at $t=\beta$, which are the boundary conditions between (13) and (15). Then, the two fundamental solutions of (15) are obtained

$$\begin{cases} \phi_{1\mathrm{OFF}}(t)=\cos k_1\beta \cos k_2(t-\beta)-\frac{k_1}{k_2}\sin k_1\beta \sin k_2(t-\beta) \\ \phi_{2\mathrm{OFF}}(t)=\cos k_1\beta \cos k_2(t-\beta)-\frac{k_2}{k_1}\sin k_1\beta \sin k_2(t-\beta) \end{cases}. \tag{16}$$

Here, $k_2 = \sqrt{1/\omega^2 CL_s}$.

Therefore, $\phi_1(\pi)$ and $d\phi_2(\pi)/dt$ are calculated from (16). The characteristic equation of Hill's equation is obtained as follows:

$$\rho^2 - \left[2\cos k_1\beta \cos k_2(\pi-\beta) - \left(\frac{k_2}{k_1}+\frac{k_1}{k_2}\right)\times \sin k_1\beta \sin k_2(t-\beta)\right]\rho + 1 = 0. \tag{17}$$

The stability of the solutions of Hill's equation can be estimated based on the discriminant D of characteristic equation (17). Fig. 7 shows the discriminant for the parameter of conduction angle β. The solution of Hill's equation diverges when the discriminant becomes $D > 0$.

The characteristic equation has solutions with multiple roots when the discriminant becomes $D = 0$. Hill's equation has a periodic function with period π as a general solution following Floquet's theory. Whereby the solution of the characteristic equation with multiple roots becomes 1 at $\beta =$1.375 and 1.550. On the other hand, Hill's equation has the periodic function with period 2π as a general solution when the multiple root becomes -1 at $\beta =$0.635, 0.698, 2.276, and 2.459.

The ac voltage source connected to the TCR-SVC corresponds to the external forced excitation in the dynamical form. Thus, Hill's equation, which is the homogeneous equation of the TCR-SVC with an accurate switching function, has indefinite solution, when the period of external excitation coincide with the periodic solution with period 2π. This corresponds to the region of parameter β, in which a nonlinear phenomenon occurs as shown in Fig. 3.

The above discussions are based on the characteristics of Hill's equation, which is the homogeneous equation of the TCR-SVC without the damping term. The effect of the damping term on the stability should be assessed because it may converge and stabilize the solution, even if the solution of Hill's equation diverges. We confirmed that the homogeneous equation (6) has parameter regions of $2.327 < \beta < 2.410$ and $1.407 < \beta < 1.508$, where the solution diverges even if the effect of damping term is considered. These respectively coincide with the unstable region of $2.276 < \beta < 2.459$ and $1.377 < \beta < 1.550$ in Hill's equation (7). Although, the parameter region of $0.635 < \beta < 0.698$ yields an unstable solution when neglecting the damping effect in the homogeneous equation as (7), but it produces a stable solution when damping in the circuit is considered by (6).

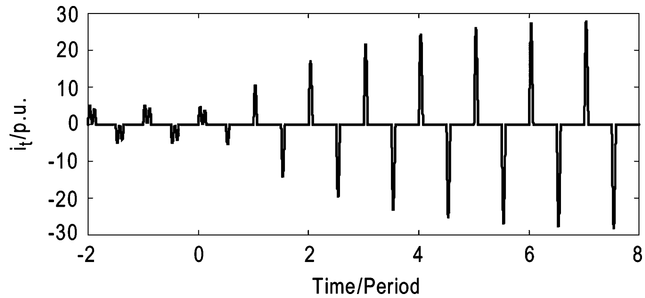

Fig. 8. Numerical simulation result of TCR-SVC (The occurrence of switching time bifurcation, α changes from 2.705 to 2.722 at $t = 0$).

The numerical analysis based on the circuit equation of TCR-SVC confirmed the occurrence of nonlinear phenomena around this parameter region. Fig. 8 shows the occurrence of switching time bifurcation in the time domain analysis of TCR-SVC, when the firing angle α is changed from 2.705 to 2.722 at $t = 0$. The anomalous phenomenon can be estimated from the jump of solution set for periodic steady state shown in Fig. 3. This phenomenon also coincide with the unstable solution of Hill's equation to the parameter region of $0.635 < \beta < 0.698$, which becomes stable when damping term in the homogeneous form in (6) is considered. Therefore, the result indicated that there is a resonance between the natural resonant frequency of the circuit and an external forced excitation, even if damping exists in the circuit.

IV. Conclusion

This paper discussed the relationship between the occurrence of nonlinear phenomena in a TCR-SVC, and the parametric excitation of intrinsic oscillation in the circuit with external excitation by ac system.

The homogeneous expression of the TCR-SVC model equation can be estimated by Hill's equation with expressing the switching function by a periodic parameter and neglecting the damping term. Furthermore, it can be approximated as Mathieu's equation by extracting the dc and fundamental frequency components of the switching function. It became clear that the regions where nonlinear phenomena occur in the TCR-SVC coincide with the regions where Mathieu's equation gives unstable solutions. The solution of Hill's equation with the accurate switching function affirmed the resonance of TCR-SVC excited by the ac voltage. The resonance was also validated with the discriminant for the characteristic equation.

The proposed analysis makes it possible to analyze system behavior associated with the switching dynamics and also the parametric resonance due to external forced excitation. The basics of the analysis procedure used in this paper are for the classical dynamical system. Therefore, the analysis is expected to apply other types of periodically excited electrical switching system. It can be applied to the analysis of nonlinear phenomena for other types of nonautonomous switching converter circuit which is interconnected to ac power system. Moreover, it can be applied to analyze motor drive system, micro electronic machine, etc.

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