<table>
<thead>
<tr>
<th>Title</th>
<th>Reflection Groups, Combinatorics and Multi-Derivations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>TERAO, Hiroaki</td>
</tr>
<tr>
<td>Citation</td>
<td>数理解析研究所講究録 (1987), 634: 25-39</td>
</tr>
<tr>
<td>Issue Date</td>
<td>1987-12</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/100092">http://hdl.handle.net/2433/100092</a></td>
</tr>
<tr>
<td>Right</td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
</tr>
</tbody>
</table>

Kyoto University
Reflection Groups, Combinatorics and Multi-Derivations

BY Hiroaki TERA

0. Introduction.

Our object to study is a finite subset of non-zero elements of a vector space. Although it might look a too simple and too naive object, this object appears in various mathematics and seems interesting enough by itself.

First this object itself is a matroid which is extensively studied as a main theme in combinatorics. It is also studied under the different name, but the equivalent concept, as a geometric lattice[5][13][70][71].

Next if one considers the dual object in the dual vector space, one gets a finite family of hyperplanes, often called an arrangement of hyperplanes. When the coefficient field is a topological field, one can ask topological questions concerning the union of these hyperplanes or the complement of them. For example the homology and the homotopy groups of the complement space are studied in [4][8][48][53].

Thirdly assume that the vector space has a metric. Consider a group generated by all orthogonal reflections through a hyperplane of
the arrangement in the dual vector space. (In some sense this is a
group theoretic interpretation of our object.) In general this group
is not finite. In case that it is finite and the coefficient field
is the real number field, the group is called a Coxeter group, which
plays an important role in the theory of Lie groups. If the
coefficient field is the complex number field, it is called unitary
reflection groups, which are classified in [61] for finite groups.

An algebrao-geometric study of our object is the study of the
product of all the elements in the symmetric algebra of the vector
space. It can also be interpreted as a defining equation for the
arrangement in the dual vector space. It is thus the study of a
product of linear forms or of a divisor consisting of hyperplanes.

This note is a general survey on these four aspects of the study
of a finite set of non-zero elements of a vector space from a
specific standpoint: we study a certain class of derivations of the
symmetric algebra of the vector space and show that it relates the
four (combinatorial, topological, group theoretic and
algebrao-geometric) aspects. Our standpoint is the theory of
logarithmic multi-derivations or multi-derivations along the divisor
consisting of the hyperplanes of the arrangement. We will explain
how the theory of logarithmic multi-derivations connect the different
aspects of our object by using examples. One of the central examples
is the polynomial called the characteristic polynomial studied in the
matroid theory. It permits other interpretations [66][48][82][68]
which we will explain.

CONTENTS

1. Combinatorics
2. Reflection Groups
3. Topological properties
4. Multi-derivations
2. Aomoto, K.: Poicaré series of the holonomy Lie algebra
   attached to configuration of lines (preprint).
3. Arnol'd, V. I.: Wavefront evolution and equivariant Morse
4. Arnol'd, V. I.: The cohomology ring of the colored braid group.
5. Baclawski, K.: Whitney numbers of geometric lattices. Adv. in
8. Brieskorn, E.: Sur les groupes de tresses(d'après V.I.Arnold),
   Séminaire Bourbaki 24e année, 1971/72, n° 401. Lecture Notes in
9. Bruce, J. W.: Vector fields on discriminants and bifurcation
11. Bruce, J. W.: Generic functions on semi-algebraic sets. Quart
12. Bruce, J. W. and Roberts R. M.: Critical points of functions on
    analytic varieties. 1986 (preprint).


42. Kohno, T.: On the holonomy Lie algebra and the nilpotent tower


49. Orlik, P.: Basic derivations for unitary reflection groups. (to appear Proc. of the intern. sympo. on Singularities at Iowa)


53. Orlik, P. and Solomon, L.: Arrangements in unitary and
orthogonal geometry over finite fields. J. of combinatorial

54. Orlik, P. and Solomon, L.: Combinatorics and topology of
complements of hyperplanes. Inventiones math. 56, 167-189
(1980).

55. Orlik, P. and Solomon, L.: Complexes for reflection groups.

56. Orlik, P. and Solomon, L.: A character formula for the unitary

57. Orlik, P., Solomon, L. and Terao, H.: Arrangements of
hyperplanes and differential forms. Combinatorics and algebra.
86m:32018]

58. Orlik, P., Solomon, L. and Terao, H.: Arrangements of
hyperplanes (to appear).

59. Orlik, P., Solomon, L. and Terao, H.: On Coxeter arrangement and
the Coxeter number. Complex Analytic Singularities. Advanced
Studies in Pure Math. 8. Tokyo-Amsterdam: Kinokuniya and

60. Randell, R.: The fundamental group of the complement of a union


70. Solomon, L.: A fixed point formula for the classical groups


Hiroaki TERAO

Department of Mathematics
International Christian University
Mitaka, Tokyo
181, JAPAN