<table>
<thead>
<tr>
<th>Title</th>
<th>Microfunctions for boundary values problems (Developments of Algebraic Analysis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>SCHAPIRA, Pierre</td>
</tr>
<tr>
<td>Citation</td>
<td>数理解析研究所講究録</td>
</tr>
<tr>
<td>Issue Date</td>
<td>1988-01</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/100154">http://hdl.handle.net/2433/100154</a></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
</tr>
</tbody>
</table>

Kyoto University
Microfunctions for boundary values problems

Pierre SCHAPIRA (Univ. Paris-Nord)

1) Let $X$ be a real manifold of class $C^2$, and let $D^+(X)$ denote the derived category of complexes of sheaves on $X$ with cohomology bounded from below. Let $\pi: T^*X \to X$ denote the cotangent bundle to $X$.

In [K-S] we associate to $F \in \text{Ob}(D^+(X))$ a closed subset $SS(F)$ of $T^*X$, the micro-support of $F$, and we also introduce the bifunctor $\mu_{\text{hom}}$ from $D^b(X)^O \times D^+(X)$ to $D^+(T^*X)$ which generalizes the functor $\mu^F_M$ of Sato's microlocalization along $M$. One has:

\[(1.1) \supp(\mu_{\text{hom}}(F,G)) \subseteq SS(F) \cap SS(G).\]

2) Let $\Omega$ be an open subset of $X$, $\overline{\Omega}$ its closure, $\partial \Omega = \overline{\Omega} \setminus \Omega$. We say that $\Omega$ is lct in $X$ if:

\[(2.1) \quad \forall x \in \partial \Omega, (R\Gamma_\Omega(Z_X))_x = 0, (R\Gamma_\Omega(Z_X))_x = Z.\]

Let $^\#$ denote the functor $R\text{Hom}(\cdot, Z_X)$. Then if $\Omega$ is lct, one has:

\[(2.2) \quad (Z_\Omega)^\# = Z_{\overline{\Omega}}, (Z_{\overline{\Omega}})^\# = Z_\overline{\Omega}.\]

Now let $M$ be a closed submanifold of $X$. Assume:

\[(2.3) \quad \Omega \text{ is lct in } X \text{ and } \overline{\Omega} \cap M.\]
Then the morphism $Z_\Omega \to Z_M$ defines by duality:

$$2.4 \quad b: Z_M \to Z_\Omega \otimes \omega_{M/X}[d]$$

where $\omega_{M/X}$ is the relative orientation sheaf and $d$ is the codimension of $M$. We call $b$ the "boundary value" morphism.

3) Assume now $M$ is a real analytic manifold of dimension $n$ and $X$ is the complexification of $M$. Applying the functor $\mu\text{hom}(\cdot, \mathcal{O}_X)$ to (2.4) we get:

**Proposition 3.1** Let $f$ be a holomorphic function on $\Omega$. Then $b(f)$ is a well-defined hyperfunction on $M$, and

$$SS(b(f)) \subset T^*_M \cap SS(Z_\Omega).$$

4) Let $\omega$ be an open subset of $M$, $j$ the inclusion map $\omega \to X$. We set:

$$4.1 \quad C_{\omega}|_X = \mu\text{hom}(Z_{\omega}, \mathcal{O}_X) \otimes \omega_{M/X}[n]$$

There is a well-defined morphism:

$$4.2 \quad \alpha: j_\#j^{-1}B_M \to \pi_*\mathcal{H}^0(C_{\omega}|_X)$$

If $M$ is a coherent $\mathcal{D}_X$-module, there is a similarly defined morphism:
(4.3) $\alpha: E^j_{\text{Ext}^1_D(M, j_*j^{-1}E_M)} + \pi_*^j H^j(D_X^{\mathbb{R}}(M, C^\omega|X))$

Definition 4.1 Let $u$ be a section of $j_*j^{-1}E_M$ (resp. of $E^j_{\text{Ext}^1_D(M, j_*j^{-1}E_M)}$). One sets: $SS^\omega_u(u) = \text{supp}(\alpha(u))$

(resp.: $SS^M_{\omega,j}(u) = \text{supp}(\alpha(u))$).

There are similar definitions for microfunctions (using the morphism $C_M \rightarrow C^\omega|X$).

5) Let $N$ be an analytic submanifold of $M$, $Y$ the complexification of $N$ in $X$.

Assume:

(5.1) $Y$ is non characteristic for $M$
(5.2) $\omega$ is $\Lambda^ct$ in $M$ and $\tilde{\omega} \supset N$.

Using (2.4) one easily obtains a morphism

(5.3) $b: R^j \text{Hom}_D^X(M, j_*j^{-1}E_M) \rightarrow R^j \text{Hom}_D^Y(M_Y, E_N)$

This morphism extends the construction of [Sl] and [Ko],
(cf. also [6]).

Theorem 5.1 Let $u \in \text{Ext}^1_D^j(M, E_M(\omega))$. Then:

1) $SS^j_{N,Y}(b(u)) \subset \int_{\omega^{-1}} SS^M_{\omega,j}(u)$

ii) If $j = 0$, codim$_M N = 1$, this inclusion is an equality.
Here \( \mathcal{F} \) and \( \bar{\omega} \) denote as usual the map from \( Y \times T^*X \) to \( T^*Y \) and \( T^*X \), respectively. Remark that if \( (C_\omega|_X)^{T^*_X} \) is in degree 0 (e.g.: \( \omega = M \) or else \( \partial \omega \) is an analytic hypersurface).

Then \( SS^{M,0}_\omega(u) \cap T^*_M X = SS_\omega(u) \). In [S-Z] we study a notion of \( \omega \)-regularity for \( M \) which extends that of [S2] and [Ka] and which ensures that

\[
SS_\omega(u) = SS(u|_{\omega}).
\]

The results of this paper are detailed in [S3].

Bibliography


