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# Microfunctions for boundary values problems

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1) Let $X$ be a real manifold of class $C^2$, and let $D^+(X)$ denote the derived category of complexes of sheaves on $X$ with cohomology bounded from below. Let $\pi: T^*X \to X$ denote the cotangent bundle to $X$.
In [K-S] we associate to $F \in Ob(D^+(X))$ a closed subset $SS(F)$ of $T^*X$, the micro-support of $F$, and we also introduce the bifunctor $\mu hom$ from $D^b(X)^o \times D^+(X)$ to $D^+(T^*X)$ which generalizes the functor $\mu_M$ of Sato's microlocalization along $M$. One has:

(1.1) $\mathrm{supp}(\mu hom(F,G)) \subset SS(F) \cap SS(G)$.

2) Let $\Omega$ be an open subset of $X$, $\bar{\Omega}$ its closure, $\partial\Omega = \bar{\Omega} \setminus \Omega$. We say that $\Omega$ is $\ell ct$ in $X$ if:
(2.1) $\forall x \in \partial\Omega, (R\Gamma_{\bar{\Omega}}(\mathbb{Z}_X))_x = 0, (R\Gamma_{\Omega}(\mathbb{Z}_X))_x = \mathbb{Z}$.
Let $^*$ denote the functor $R\underline{H}om(\cdot, \mathbb{Z}_X)$. Then if $\Omega$ is $\ell ct$, one has:

(2.2) $(\mathbb{Z}_\Omega)^* = \mathbb{Z}_{\bar{\Omega}}, (\mathbb{Z}_{\bar{\Omega}})^* = \mathbb{Z}_\Omega$.

Now let $M$ be a closed submanifold of $X$. Assume:

(2.3) $\Omega$ is $\ell ct$ in $X$ and $\bar{\Omega} \supset M$.

Then the morphism $\mathbb{Z}_{\overline{\Omega}} \to \mathbb{Z}_M$ defines by duality:

(2.4) $b: \mathbb{Z}_M \to \mathbb{Z}_\Omega \otimes \omega_{M/X}[d]$

where $\omega_{M/X}$ is the relative orientation sheaf and $d$ is the codimension of $M$. We call $b$ the "boundary value" morphism.

3) Assume now $M$ is a real analytic manifold of dimension $n$ and $X$ is the complexification of $M$. Applying the functor $\mu hom(\cdot, \mathcal{O}_X)$ to (2.4) we get:

<u>Proposition 3.1</u> Let $f$ be a holomorphic function on $\Omega$. Then $b(f)$ is a well-defined hyperfunction on $M$, and

$$SS(b(f)) \subset T^*_M X \cap SS(\underline{\mathbb{Z}}_\Omega) .$$

4) Let $\omega$ be an open subset of $M$, $j$ the inclusion map $\omega \to X$. We set:

(4.1) $C_{\omega|X} = \mu hom(\underline{\mathbb{Z}}_\omega , \mathcal{O}_X) \otimes \omega_{M/X} [n]$

There is a well-defined morphism:

(4.2) $\alpha: \; j_* j^{-1} \mathcal{B}_M \to \pi_* H^0(C_{\omega|X})$

If $\mathcal{M}$ is a coherent $\mathcal{D}_X$-module, there is a similarly defined morphism:

(4.3) $\alpha$: $\mathcal{E}xt^j_{\mathcal{D}_X}(\mathcal{M}, j_*j^{-1}\mathcal{B}_M) \to \pi_*H^j(R\underline{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\omega|X})$

<u>Defunction 4.1</u> Let $u$ be a section of $j_*j^{-1}\mathcal{B}_M$ (resp. of $\mathcal{E}xt^j_{\mathcal{D}_X}(\mathcal{M}, j_*j^{-1}\mathcal{B}_M)$) . One sets: $SS_\omega(u) = \mathrm{supp}(\alpha(u))$

(resp.: $SS^{\mathcal{M},j}_\omega(u) = \mathrm{supp}(\alpha(u))$) .

There are similar definitions for microfunctions (using the morphism $\mathcal{C}_M \to \mathcal{C}_{\omega|X}$) .

5) Let $N$ be an analytic submanifold of $M$, $Y$ the complexification of $N$ in $X$ .

Assume:

(5.1) $Y$ is non characteristic for $\mathcal{M}$

(5.2) $\omega$ is $\ell ct$ in $M$ and $\bar{\omega} \supset N$ .

Using (2.4) one easily obtains a morphism

(5.3) b: $R\underline{H}om_{\mathcal{D}_X}(\mathcal{M}, j_*j^{-1}\mathcal{B}_M) \to R\underline{H}om_{\mathcal{D}_Y}(\mathcal{M}_Y, \mathcal{B}_N)$

This morphism extends the construction of [Sl] and [Ko], (cf. also [Ô]).

<u>Theorem 5.1</u> Let $u \in \mathrm{Ext}^j_{\mathcal{D}_X}(\mathcal{M}, \mathcal{B}_M(\omega))$ . Then:

i) $SS^{\mathcal{M}_Y,j}_N(b(u)) \subset \rho\bar{\omega}^{-1}SS^{\mathcal{M},j}_\omega(u)$

ii) If $j = 0$ , $\mathrm{codim}_M N = 1$ , this inclusion is an equality.

Here $f$ and $\bar{\omega}$ denote as usual the map from $Y \times_X T^*X$ to $T^*Y$ and $T^*X$, respectively. Remark that if $(C_{\omega|X})_{T^*_M X}$ is in degree 0 (e.g.: $\omega = M$ or else $\partial\omega$ is an analytic hypersurface). Then $SS^{M,0}_{\omega}(u) \cap T^*_M X = SS_{\omega}(u)$. In [S-Z] we study a notion of $\omega$-regulariry for $M$ which extends that of [S2] and [Ka] and which ensures that

$$SS_{\omega}(u) = \overline{SS(u|_{\omega})} .$$

The results of this paper are detailed in [S3].

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