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Kyoto University
On Some Classes of 2-microhyperbolic systems

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§1. Introduction

Hereafter M denotes a real analytic manifold with a complexification X. We study a system of microdifferential equations \( \mathcal{M} \) defined in a neighborhood of \( \rho_0 \in T^*_M X \). We assume that the characteristic variety of \( \mathcal{M} \) is written as \( \text{Ch}(\mathcal{M}) = \{ \rho \in T^*_M X; p(\rho) = 0 \} \) by a homogeneous holomorphic function \( p \) defined in a neighborhood of \( \rho_0 \) satisfying the following conditions.

1. \( p \) is real valued on \( T^*_M X \).
2. \( \Sigma = \{ \rho \in T^*_M X; p(\rho) = 0, dp(\rho) = 0 \} \) is a regular involutory submanifold of \( T^*_M X \) of codimension \( d \) through \( \rho_0 \).
3. \( \text{Hess}(p)(\rho) \) has rank \( d \) with positivity 1.

The problem is to study the structure of \( \text{Hom}_\mathcal{E}_X (\mathcal{M}, \mathcal{E}_M) \big| \Sigma \), the sheaf of microfunction solutions of \( \mathcal{M} \) on \( \Sigma \).

§2 Canonical form
To express the canonical form, we take an open subset $M_0$ in $\mathbb{R}^{n-d} \times K_x$ and a complex neighborhood $X_0$ of $M_0$ in $C_{w}^{n-d} \times C_z^d$. Then $(w,z;\theta dw+\xi dz) \ [\text{resp. } (t,x;\sqrt{-1}(\tau dt+\xi dx))]$ denotes a point of $T^*X$ [resp. $T^*_{M_0}X$] with $\theta \in C_{n-d}$ and $\xi \in C^d$ [resp. $\tau \in \mathbb{R}^{n-d}$ and $\xi \in \mathbb{R}^d$].

By finding a suitable quantized contact transformation, the problem is reduced to study the system $\mathcal{M}_0$ defined in a neighborhood of $\rho_0=(t=0, x=0; \sqrt{-1}tdt_{n-d})$ whose characteristic variety is written as

$$\text{Ch}(\mathcal{M})=\{(w, z; \theta, \xi) \in T^*X; \xi_1^2 - \sum_{2 \leq i, j \leq d} a_{ij}(w, z; \theta, \xi) x_i^j = 0\}.$$ Here $a_{ij}$'s are homogeneous holomorphic functions of order 0 defined in a neighborhood of $\rho_0$ and satisfy the condition

$$(a_{ij})_{2 \leq i, j \leq d} \text{ is positive definite on } \Sigma_0 = \{(t, x; \sqrt{-1}(\tau, \xi)) \in T^*_{M_0}X_0; \xi = 0\}.$$ 

§3. Bisymplectic Structure due to Y. Laurent

To state the main theorem in an invariant form, we introduce the bisymplectic structure due to Y. Laurent[L].

Let $\Lambda$ be a complexification of $\Sigma$ in $T^*X$. By definition $\tilde{\Sigma}$ is the union of all bicharacteristic leaves of $\Lambda$ issued from $\Sigma$. In case $\Sigma=\Sigma_0$, we may identify

$$\tilde{\Sigma}_0 \sim C_z^d \times \sqrt{-1}T^*R^{n-d} (t, \sqrt{-1}tdt) .$$

Then we can take a coordinate of $T^*\tilde{\Sigma}_0$ as $(t, x; \sqrt{-1}tdt; \sqrt{-1}x^*dx)$ with $x^* \in \mathbb{R}^d$.

We define a map

$$p: T^*\tilde{\Sigma} \rightarrow \Sigma \rightarrow T^*M_0X$$

and the canonical 1-form of $T^*\tilde{\Sigma}$ by $\omega_{\tilde{\Sigma}} = p^*\omega_M$. Here $\omega_M$ is a canonical 1-form of $T^*M_0X$. We put $\Omega_{\tilde{\Sigma}} = d\omega_{\tilde{\Sigma}}$.
In case $\Sigma=\Sigma_0$, $\omega_{\Sigma}$ is written by coordinates as

$$\omega_{\Sigma} = \sum_j t_j dt_j.$$  

We set

$$T_{rel}T^*_{\Sigma \Sigma} = \ker(TT^*_{\Sigma \Sigma} \to T^*_{\Sigma \Sigma}) \hookrightarrow TT^*_{\Sigma \Sigma}.$$  

Here the morphism above in the definition of $T_{rel}T^*_{\Sigma \Sigma}$ is defined naturally by $\Omega_{\Sigma \Sigma}$. We dualize the exact sequence

$$0 \rightarrow T_{rel}T^*_{\Sigma \Sigma} \rightarrow TT^*_{\Sigma \Sigma}$$  

and obtain

$$0 \rightarrow T^*_{rel}T_{\Sigma \Sigma} \rightarrow T^*_{\Sigma \Sigma}.$$  

We can take a section of $T^*_{rel}T^*_{\Sigma \Sigma}$ canonically, which is denoted by $\omega^r_{\Sigma}$ and called the relative canonical 1-form of $T^*_{\Sigma \Sigma}$. We also define the relative 2-form $Q^r_{\Sigma \Sigma} = d\omega_{\Sigma}$.

In case $\Sigma = \Sigma_0$,

$$\omega^r_{\Sigma} = \sum_j x^*_j dx_j.$$  

Associated with $\Omega^r_{\Sigma \Sigma}$ we can define an isomorphism

$$H^r_{\Sigma \Sigma}: T^*_{rel}T^*_{\Sigma \Sigma} \sim \rightarrow T_{rel}T^*_{\Sigma \Sigma}.$$  

For a function $f$ defined on an open subset of $T^*_{\Sigma \Sigma}$, we set

$$H^r_f = H^r_{\Sigma \Sigma}(\overrightarrow{df})$$  

where $\overrightarrow{df}$ is the image of $df$ by $T^*_{rel}T^*_{\Sigma \Sigma} \to T^*_{\Sigma \Sigma}$.

In case $\Sigma = \Sigma_0$, it is written by coordinates as

$$H^r_f = \sum_j \langle \partial f/\partial x_j^*, \partial/\partial x_j - \partial f/\partial x_j, \partial/\partial x_j^* \rangle.$$  

§4. 2-microfunctions
M. Kashiwara constructed the sheaf $\mathcal{E}_Σ^2$ of 2-microfunctions on $T^*_Σ\Sigma$ long time ago in Nice. We can study the properties of microfunctions defined on $Σ$ precisely by $\mathcal{E}_Σ^2$. Explicitly, there exists the sheaf $\mathcal{E}_Σ^\sim$ of microfunctions along $\sim Σ$ on $Σ$ and there exist the exact sequences

$$0 \rightarrow \mathcal{E}_Σ^\sim|Σ \rightarrow \mathcal{E}_Σ^2|Σ \rightarrow \pi_Σ^*(\mathcal{E}_Σ^2|T^*_Σ\Sigma) \rightarrow 0$$

and

$$0 \rightarrow \mathcal{E}_Σ^2|Σ \rightarrow \mathcal{E}_Σ^\sim|Σ \rightarrow \pi^*(\mathcal{E}_Σ^2|Σ) \rightarrow 0$$

Here we put $\mathcal{E}_Σ^\sim = \mathcal{E}_Σ^2|Σ$. Moreover we have the canonical spectral map

$$\text{Sp}_Σ^2 : \pi_Σ^{-1} \mathcal{E}_Σ^2 \rightarrow \mathcal{E}_Σ^2.$$ 

We put for $u \in \mathcal{E}_Σ^2|Σ$,

$$\text{SS}_Σ^2(u) = \text{supp}(\text{Sp}_Σ^2(u)).$$

See Kashiwara-Laurent[K-L] for details about $\mathcal{E}_Σ^2$.

§5 Main Theorems

We set for a point $ρ ∈ Σ$ and $τ ∈ T^*_Σ\Sigma|ρ$

$$g = \langle \text{Hess}(p)(ρ)H(q), H(q) \rangle$$

where $H : T^*_Σ\Sigma \rightarrow T^*_Σ T^*_Σ X$ is an Hamiltonian isomorphism.

In case $M = \mathbb{R}_0$,

$$g = 2x_1^2 - \sum a_{ij}(t, x; ξ = 0, τ)x_i^*x_j^*.$$ 

$2 ≤ i, j ≤ d$

Here we give

Theorem 1. Let $u$ be a section of $\text{Hom}_X(\mathcal{E}_Σ^2, \mathcal{E}_Σ^2)|Σ$ defined in a neighborhood of $ρ_0$. Then $\text{SS}_Σ^2(u) \setminus Σ \subset \{ g = 0 \}$. Moreover $\text{SS}_Σ^2(u) \setminus Σ$ is invariant under $H^g$. 

- 4 -
By Theorem 1 we can deduce a microlocal version of Holmgren's
Theorem. We set
\[ \gamma = \pi_{\Sigma} \left( \exp( sH^\Gamma_E ) \left( (\rho_0, \tau) ; f(\rho_0, \tau) = 0, s \geq 0 \right) \right). \]
Here \( \exp( sH^\Gamma_E ) (q) \) denotes the flow of \( H^\Gamma_E \) issued from \( q \). Then \( \gamma \) is
a boundary of a cone in the bicharacteristic leave \( \Gamma \) of \( \Sigma \) through \( \rho_0 \).
We take one of half-cones: \( \gamma_+ \). We give

**Theorem 2.** Let \( u \) be a section of \( \mathcal{Hom}_X^E (U, \mathbb{V}_0) \) defined in a
neighborhood of \( \rho_0 \). Then
\[ \text{supp}(u) \cap (\gamma_+ \setminus (\rho_0)) = \emptyset \]
implies that \( \rho_0 \notin \text{supp}(u) \).

Here we remark that \( \gamma_+ \) does not contain the inside of the cone.
Thus Theorem 2 generalizes the result of P. Laubin[Lb].

§6. Sketch of the proof.
6.1. As is mentioned in §2, it is enough to study the case \( \mathbb{W} = \mathbb{W}_0 \).
Thus hereafter we put \( \mathbb{W} = \mathbb{W}_0 \), \( X = X_0 \), \( M = M_0 \), \( \Sigma = \Sigma_0 \) and
\[ \Lambda = \Lambda_0 = \{(w, z; 0dw + \xi dz) \in T^*X ; \xi = 0 \}. \]

6.2. 2-microlocal canonical form —Simple equations with a
conditions on the lower terms
6.2.1. In case \( \mathbb{W} \) is reduced to a single equation: \( Pu=0 \) with \( p=\sigma(P) \)
satisfying the conditions (1), (2) and (3), we can transform the
equation into a simple canonical form 2-microlocally if we assume \( \mathbb{W} \)
has Regular Singularities along $\Lambda$ in the sense of Kashiwara-Oshima [K-O].

6.2.2.

We embed $\Lambda$ into $\Lambda \times \Lambda$ through the injection $T^*X \rightarrow T^*X \times (X \times X) \rightarrow T^*(X \times X)$. By definition, $\tilde{\Lambda}$ denotes the union of all bicharacteristic leaves of $\Lambda \times \Lambda$ passing through $\Lambda$. We take a coordinate of $T^*_{\Lambda \times \Lambda}$ as $(w, z; \theta dw; z^* dz)$ with $z^* \in C^d$. On $T^*_{\Lambda \times \Lambda}$, Y. Laurent [L] defined the sheaf $\mathcal{E}^2_{\Lambda}$ of 2-microdifferential operators of infinite order.

Definition 3 (Y. Laurent[L]) Let $\Omega$ be an open subset of $T^*_{\Lambda \times \Lambda}$. Then $\sum_{i,j} p_{ij}(w, z; \theta; z^*) \in \mathcal{E}^2_{\Lambda} (\Omega)$ if and only if the following conditions (4) and (5) are satisfied.

(4) $p_{ij}$ is holomorphic on $\Omega$ and homogeneous of order $j$ with respect to $(\theta, z^*)$ and of order $i$ with respect to $z^*$.

(5) For any compact subset $K$ of $\Omega$ and for any positive number $\varepsilon$ there exists a positive number $C_{\varepsilon, K}$ and for any compact subset $K$ of $\Omega$ there exists a positive number $C_{K}$ such that

$$\sup_{K} |p_{i, i+k}| \leq \begin{cases} C_{\varepsilon, K} \varepsilon^{i+k} / i! \cdot k! & (i, k \geq 0) \\ C_{\varepsilon, K} \varepsilon^i / i! & (i \geq 0, k < 0) \\ C_{\varepsilon, K} \varepsilon^k C_{K}^{-i} / (-i)! / k! & (k \geq 0, i < 0) \\ C_{K}^{-i-k} (-i)! / (-k)! & (i, k < 0). \end{cases}$$

We define the sheaf $\mathcal{E}^2_{\Lambda}$ of 2-microdifferential operators of finite order as follows.
**Definition 4** For \( P = \sum \varepsilon_{ij}^2 \alpha \), \( \varepsilon_{ij}^2 \) if and only if there exists \( j_0 \) such that

\[
P_{ij} = 0 \quad (j > j_0)
\]

and there exists \( \lambda(j) \) for any \( j \in \mathbb{Z} \) such that

\[
P_{ij} = 0 \quad (i < \lambda(j)).
\]

For any \( P \in \mathfrak{g}_A^2 \), the principal symbol of \( P \) is defined by

\[
\sigma_A(P) = P_{i_0 j_0}
\]

where \( j_0 = \sup(j; \text{ for some } i, P_{ij} = 0) \) and \( i_0 = \inf(i; P_{ij} = 0) \).

In the same way, we can construct the bisymplectic structure \((\Omega_A, \Omega_A^\Gamma)\). By coordinates, these are written as

\[
\Omega_A = \sum_j d\theta_j dw_j \quad \text{and} \quad \Omega_A^\Gamma = \sum_j dz_j^* dz_j.
\]

If a map \( \varphi: U \longrightarrow V \) between open subsets of \( T^*_A \) satisfies

\[
\varphi^* (\Omega_A|_V) = \Omega_A^\Gamma|_U,
\]

then we can induce an isomorphism

\[
\varphi^*: T^*_\text{rel} T^*_A|_V \times U \longrightarrow T^*_\text{rel} T^*_A|_U.
\]

Moreover if

\[
\varphi^* (\Omega_A^\Gamma|_V) = \Omega_A^\Gamma|_U
\]

and \( \varphi \) preserves the bihomogeniety structure of \( T^*_A \):

\[
(w, z; \theta; z^*) \longrightarrow (w, z; z^*; \theta)
\]

and

\[
(w, z; \theta; z^*) \longrightarrow (w, z; \theta; \lambda z^*) \quad (\lambda \in \mathbb{C}^\times),
\]

then \( \varphi \) is called a homogeneous bicanonical transformation. Associated with \( \varphi \), we can construct a ring isomorphism

\[
\Phi: \varepsilon_A^2 |_V \longrightarrow \varepsilon_A^2 |_U.
\]

See Y. Laurent[L] for details about 2-microdifferential
operators.

6.2.3.

By finding a suitable quantized bicanonical transformation, we can transform the equation \( Pu = 0 \) into

\[
RP_0 u = 0
\]
defined in a neighborhood of \( \tau_0 = (t=0, x=0; \sqrt{-1}t_{n-d}; \sqrt{-1}t_{d}) \). Here \( R \) is invertible at \( \tau_0 \) and

\[
\sigma_A(P_0) = z_1^*.
\]

We remark that

\[
S(P) = \{(j, i); P_{ij} \neq 0 \} \subset \{(i \geq j, j \leq 1)\}.
\]

Next we find an invertible 2-microdifferential operator of infinite order \( Q \) satisfying

\[
QP_0 = D_1Q.
\]

Then we can easily prove Theorem 1. See \([T_3]\) for details.

6.3. 2-microhyperbolicity — general case

6.3.1.

In general case, we prove Theorem 1 by employing the theory of microlocal analysis of sheaves due to Kashiwara-Schapira\([K-S_2]\).

6.3.2.

Let \( X \) be an \( C^\infty \) manifold and let \( M \) be a closed submanifold of \( X \) in this section 6.3.2.

\( D^+(X) \) denotes the derived category of bounded below complexes of sheaves of modules on \( X \). For \( \mathcal{F} \in \text{Ob}(D^+(X)) \), \( SS(\mathcal{F}) \) denotes the microsupport of \( \mathcal{F} \), which is a conic closed subset in \( T^*X \).

For \( \mathcal{F} \in \text{Ob}(D^+(X)) \), \( \mu_M(\mathcal{F}) \) denotes Sato's microlocalization of \( \mathcal{F} \).
along $M$, which is an object of $D^+(T^*_MX)$.

For a closed subset of $Z$, $C_M(Z)$ denotes the normal cone of $Z$ along $M$, which is a closed subset of $T^*_MX$.

We quote an important formula from Kashiwara-Schapira[K-S$_2$] as follows.

**Theorem 5** For $F \in \text{Ob}(D^+(X))$, we have

$$SS(\mu_M(F)) \subset C \quad (SS(F)).$$

$$T^*_MX$$

Here we consider the right side as a subset of $T^*T^*_MX$ through

$$(-H): T^*_MX \xrightarrow{\sim} T^*T^*_MX.$$

($H$ is the Hamiltonian isomorphism.)

6.3.3.

We set $N=(R^{n-d}\times C^d)_t \cap X$ in $X$. Then we have

$$\mathcal{E}_N^2 = \mu_{\Sigma^*U_C}(O_X)[n].$$

Thus we can show by the theory of Kashiwara-Schapira[K-S$_2$] that

$$SS(R\mathcal{Hom}_E^X(\mathcal{F}, \mathcal{E}_N^2)) \subset C \quad \subset \quad (C \quad (\text{Ch}(\mathcal{F}) \quad )).$$

$$T^*_MX \quad \subset \quad T^*_MX$$

By estimating the right side, we can show

$$SS(R\mathcal{Hom}^E_{X}(\mathcal{F}, \mathcal{E}_N^2)|_{T^*_XM \times \Sigma^*}) \subset ((\rho, \tau) \in T^*(T^*_MX \times \Sigma); g(\rho) = 0, \tau(H^g(\rho)) = 0)$$

where $\rho \in T^*_MX \times \Sigma$ and $\tau \in T^*T^*_MX|_{\rho}$. Then we can easily prove Theorem 1 by Proposition 4.1.2 of [K-S$_2$].

§7. Some Remarks
7.0. We gather results for some classes of systems of microdifferential equations in §7.

7.1. Case I

Let $M$ be a real analytic manifold with a complexification $X$. Let $\mathcal{M}$ be a coherent $\mathcal{E}_X$-module defined in a neighborhood of $\rho_0 \in \tilde{T}^* M$ whose characteristic variety is written in a neighborhood of $\rho_0$ as

$$\text{ch}(\mathcal{M}) = \{ \rho \in T^* X; p_1 = \cdots = p_{d-2} = 0, p_{d-1}, p_d = 0 \}$$

by homogeneous holomorphic functions $p_1, \ldots, p_{d-1}$ and $p_d$ satisfying the following conditions.

(6) $p_1, \ldots, p_{d-1}$ and $p_d$ are real valued on $T^*_M X$.

(7) $dp_1, \ldots, dp_{d-1}$ and $dp_d$ and $\omega$ (canonical 1-form of $T^* X$) are linearly independent at $\rho_0$.

Let $\Lambda_1 = \{ \rho \in T^*_M X; p_1 = \cdots = p_{d-1} = 0 \}$, $\Lambda_2 = \{ \rho \in T^*_M X; p_1 = \cdots = p_{d-2} = p_d = 0 \}$ and $\Lambda = \Lambda_1 \cap \Lambda_2$. Then we assume

(8) $\Lambda_1$, $\Lambda_2$ and $\Lambda$ is regular involutory submanifolds in $T^*_M X$ through $\rho_0$.

We set $\Sigma_i = T^*_M X \cap \Lambda_i$ ($i = 1, 2$) and $\Sigma = \Sigma_1 \cap \Sigma_2$. Then the result is

Theorem 6.

Let $u$ be a section of $\text{Hom}_{\mathcal{E}_X}(\mathcal{M}, \mathcal{E}_M)$ defined in a neighborhood of $\rho_0$ and let $\Gamma$ be the bicharacteristic leaf of $\Sigma$ through $\rho_0$. Then there exist a family of bicharacteristic leaves of $\Sigma_1$ on $\Gamma$: $\{ \gamma_s^{(1)} \}$ and that of $\Sigma_2$ on $\Gamma$: $\{ \gamma_s^{(2)} \}$ such that

$$\text{supp}(u) = \bigcup_s \gamma_s^{(1)} \cup \bigcup_s \gamma_s^{(2)} \cup \{ \text{some of connected}$$
components of \{ \mathfrak{R} \bigcup_s \gamma^{(1)}_s \cup \bigcup_s \gamma^{(2)}_s \} \).

(sketch of the proof)

By finding a suitable quantized contact transformation, the problem is reduced to studying a coherent \( \mathcal{E}_X \) module \( \mathfrak{r}_0 \) defined in a neighborhood of \( \rho_0 = (0, \sqrt{-d} x_n) \in \sqrt{-d}^* \mathfrak{r}_0 \) whose characteristic variety is written as

\[
\text{Ch}(\mathfrak{r}) = \{ (z, \xi dz) \in \mathfrak{T}^* X_0 ; \xi_1 = \cdots = \xi_{d-2} = 0, \xi_{d-1} \cdot \xi_d = 0 \}.
\]

Here \( \mathfrak{r}_0 \) is an open subset of \( \mathbb{R}^n_x \) and \( X_0 \) is a complex neighborhood of \( \mathfrak{r}_0 \) in \( \mathbb{C}^n_x \). Then \((z, \xi dz)\) [resp. \((x, \sqrt{-1} \xi dx)\)] denotes a point of \( \mathfrak{T}^* X_0 \) [resp. \( \mathfrak{T}^*_x \mathfrak{r}_0 X_0 \)] with \( \xi \in \mathbb{C}^n \) [resp. \( \xi \in \mathbb{R}^n \)]. We set

\[
\Sigma_0 = \{ (x, \sqrt{-1} \xi dx) \in \sqrt{-d}^* \mathfrak{r}_0 ; \xi_1 = \cdots = \xi_d = 0 \}
\]

and take a coordinate of \( \mathfrak{T}^*_x \Sigma_0 \) as \((x, \sqrt{-1} \xi^*; \sqrt{-1} x^* \) with \( \xi^* = (\xi_{d+1}, \cdots, \xi_n) \) and \( x^* = (x^*_1, \cdots, x^*_d) \). Then for a section \( u \) of \( \mathfrak{Hom}_X(\mathfrak{r}_0, \mathfrak{E}_M) \) defined in a neighborhood of \( \rho_0 \), we have

\[
\text{SS}^2_{\Sigma_0} (u) \setminus \Sigma_0 \subset \{ x^*_1 = \cdots = x^*_d = 0 \} \cup \{ x^*_1 = \cdots = x^*_{d-1} = 0, x^*_d = 0 \}.
\]

We set

\[
\Gamma_1 = \{ (x, \sqrt{-1} \xi^*; \sqrt{-1} x^* \} \in \mathfrak{T}^*_x \Sigma_0 \setminus \Sigma_0 ; \xi^*_1 = \cdots = x^*_d = 0 \}
\]

and

\[
\Gamma_2 = \{ (x, \sqrt{-1} \xi^*; \sqrt{-1} x^* \} \in \mathfrak{T}^*_x \Sigma_0 \setminus \Sigma_0 ; \xi^*_1 = \cdots = x^*_{d-2} = x^*_d = 0 \}.
\]

Then \( \text{SS}^2_{\Sigma_0} (u) \big| \Gamma_1 \) [resp. \( \text{SS}^2_{\Sigma_0} (u) \big| \Gamma_2 \)] is invariant under the integrable system \( (\partial/\partial x^*_1, \cdots, \partial/\partial x^*_{d-1}) \) [resp. \( (\partial/\partial x^*_1, \cdots, \partial/\partial x^*_{d-2}, \partial/\partial x^*_d) \)]. This fact is shown in the same way as in \$6.3.\n
(q.e.d.)
7.2. (Case II)

Let $M$ be a real analytic manifold with a complexification $X$. Let $\mathcal{M}$ be a coherent $\mathcal{E}_X$ module defined in a neighborhood of $\rho_0 \in T^*_M X$ whose characteristic variety is written in a neighborhood of $\rho_0$ as

$$\text{Ch}(\mathcal{M}) = \{ \rho \in T^*_M X ; \ p = 0 \}$$

by a homogeneous holomorphic function $p$ satisfying the following conditions.

$$\begin{align*}
(12) & \quad p \text{ is real valued on } T^*_M X. \\
(13) & \quad \Sigma = \{ \rho \in T^*_M X ; \ p(\rho) = 0, \ dp(\rho) = 0 \} \text{ is a regular involutory submanifold of codimension 2 in } T^*_M X \text{ through } \rho_0. \\
(14) & \quad \text{Hess}(p)(\rho) \text{ has rank 1 if } \rho \in \Sigma. \\
\end{align*}$$

We set for a point $\rho \in \Sigma$ and $\tau \in T^*_\Sigma \tilde{\Sigma} | \rho$

$$\begin{align*}
(15) & \quad g = \langle \text{Hess}(p)(\rho) H(\tau), H(\tau) \rangle \\
\text{where } H : T^*_\Sigma \tilde{\Sigma} \xrightarrow{\sim} T^*_\Sigma T^*_M X \text{ is Hamiltonian isomorphism. Then we have}
\end{align*}$$

Proposition 7.

The function $g$ is divided into

$$g = g_1 \cdot g_2^2$$

with $g_1 \not\equiv 0$ on $T^*_\Sigma \Sigma \setminus \Sigma$.

By the decomposition above, we have

Theorem 8.

Let $u$ be a section of $\text{Hom}_{\mathcal{E}_X} (\mathcal{M}, \mathcal{E}_M)$ defined in a neighborhood of $\rho_0$. Then

$$SS^2_\Sigma(u) \setminus \Sigma \subset \{ \varepsilon = 0 \} .$$
Moreover $SS^2_\Sigma(u)\setminus\Sigma$ is invariant under $H^r_{g_2}$.

(skeleton of the proof)

By finding a suitable quantized contact transformation, the problem is reduced to studying the system $\mathfrak{m}_0$ defined in a neighborhood of $\rho_0=(0,\sqrt{-1}dx)\in\sqrt{-1}T^*M_0$ whose characteristic variety is written as

\[ \text{Ch}(\mathfrak{m}_0) = (z,\xi dz) \in T^*X_0; \xi_1^2 - a(z,\xi')\xi_2^3 = 0. \]

Here $M_0$ is an open subset of $\mathbb{R}^n_x$ and $X_0$ is a complex neighborhood of $M_0$ in $X_0$. Then $(z,\xi dz)$ [resp. $(x,\sqrt{-1}\xi dx)$] denotes a point of $T^*X_0$ [resp. $T^*_M X_0 \sim \sqrt{-1}T^*M_0$]. Moreover $a(z,\xi')$ is a homogeneous holomorphic function of order $(-1)$ with $\xi'=(\xi_2,\ldots,\xi_n)$.

In this case,

\[ \Sigma=\{(x,\sqrt{-1}\xi dx)\in\sqrt{-1}T^*M_0; \xi_1=\xi_2=0 \}. \]

When we take a coordinate of $T^*\Sigma\setminus\Sigma$ as $(x,\sqrt{-1}\xi'';\sqrt{-1}(x_1',x_2'))$ with $\xi''=(\xi_3,\ldots,\xi_n)$, we can take $g_2$ as $x_1'$. Then in the same way as in §6.3 we have

\[ SS(R\text{Hom}_{\mathfrak{m}_0}(\mathfrak{m}_0,\mathfrak{g}_2^2)\mid T^*\Sigma\setminus\Sigma) \]

\[ \subset \{(\rho,\tau)\in T^*(T^*\Sigma\setminus\Sigma); x_1'(\rho)=0, \text{and } \tau(H^r(x_1')) \}. \]

Here $\rho\in T^*\Sigma\setminus\Sigma$ and $\tau\in T^*(T^*\Sigma\setminus\Sigma)$. In the same way as in §6.3, $SS^2_\Sigma(u)$ is invariant under $\partial/\partial x_1$ for any section of $\text{Hom}_{\mathfrak{g}_2}^r(\mathfrak{m},\mathfrak{g}_M)$.

\[ \text{(q.e.d.)} \]

We can easily show that $\Sigma$ is foliated by the projection of the integral curves of $H^r_{g_2}$ in $\{g_2=0\}$. More precisely, for $\rho\in\Sigma$
\[ \gamma(\rho) = \pi_{\Sigma}(\exp(h_{g_2}^R)(\rho, \tau); g_2(\rho, \tau) = 0, s \in \mathbb{R}) \]

is a smooth curve in the bicharacteristic leaf \( \Gamma \) of \( \Sigma \). Here we give

**Theorem 9.**

For any section \( u \) of \( \mathcal{E}_X \langle \mathbb{R}, \mathcal{E}_M \rangle |_{\Sigma} \) defined in a neighborhood of \( \rho_0 \), \( \text{supp}(u) \cap \Sigma \) propagates along the family of integral curves \( \gamma(\rho); \rho \in \Sigma \).

**Remark 10.** Theorem 9 itself can be proved by the microlocal version of Holmgren's theorem due to J.M. Bony[B].

**Reference**


On some classes of 2-microhyperbolic systems

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M 3 級解折的 3 槓体, X \in \mathbb{R}^{3} 幅稀近傍と \Gamma 3 \equiv 2

は、T^*_M X の点 \Gamma の近傍で定義された system of microdifferential
equations が 2, \forall \exists 特性的模体が 0 の近傍で著者次正則
函数 p と用い

\phi \in M_2 = \left\{ \phi \in T^*_M X : p(\phi) = 0 \right\}

と書くもお正しらしい。但し、p は以下の条件を満たすもの

(1) p は T^*_M X 上単数値
(2) \exists T^*_M X, p(\phi) = 0, d_p(\phi) = 0 \in T^*_M X 中余 2 元

\phi 正則値の部分の模体 2 と近辺を \Gamma 2 \equiv 3

(3) \exists a \in \mathbb{R} \text{ と } \text{rank}_d^2 \equiv 2 \text{ positive と }

\phi \in \Gamma 2

問題は、M 2 級変数解 a \in \mathbb{R}^2 の構造である。\mathbb{R}^2 上 2 と2
超局所化して精密に \Gamma 2 \equiv 3

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