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Kyoto University
On Design Verification
between Different Levels of Abstraction
Using Regular Temporal Logic
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1 Introduction

The progress of VLSI technology makes it a pressing need to establish methods for verifying the correctness of logic design. In order to verify whether a designed system satisfies a specification for it, formal verification methods have been developed.

In logic design, hierarchical design methodology is adopted to manage complex logic systems. Our main concern is to develop a formal verification method applicable to hierarchical design.

We consider formal verification of sequential machines in this paper. As a language for describing specification, we adopt infinitary regular temporal logic (\(\infty RTL\))[1] which is an extension of \(\epsilon\)-free RTL proposed by Hiraishi et al. [2]. While traditional temporal logic or computation tree logic (CTL) cannot characterize finite state machines[3,4], \(\infty RTL\) is powerful enough to express regular sets and \(\omega\) regular sets.

In hierarchical design, specifications and implementations are often given at different levels of abstractions. For example, a specification at register transfer level (a higher level) and an implementation at gate level (a lower level) can be given. In order to verify whether the lower-level implementation satisfies the higher-level specification, we must determine some formal relation and bridge the gap between the two levels.

In this paper, we propose a formal framework based on \(\infty RTL\) to explicitly describe relations between two different levels. We regard the
relation as a part of an implementation and show a verification method for a lower-level implementation (i.e., a lower-level sequential machine and a relation) and a higher-level specification described in $\infty$RTL.

This paper is organized as follows: Chapter 2 introduces $\infty$RTL. Chapter 3 discusses a formal framework for describing relations between different levels and shows a design verification method considering two different levels. Chapter 4 summarizes this paper.

2 Regular Temporal Logic

The empty word $\epsilon$, $\Sigma^*$ and $\Sigma^+$ are defined as in the usual way. An omega word over an alphabet $\Sigma$ is an infinite-length sequence of symbols from $\Sigma$. $\Sigma^\omega$ is the set of all omega words over $\Sigma$. $\Sigma^\infty \overset{\text{def}}{=} \Sigma^* \cup \Sigma^\omega$.

The class of infinitary regular sets is the union of regular sets[5] and $\omega$ regular sets[6].

For $\sigma \in \Sigma^\infty - \{\epsilon\}$, $|\sigma|$ denotes the length of $\sigma$, i.e., the number of symbols in $\sigma$ (If $\sigma$ is in $\Sigma^\omega$, then we denote $|\sigma| = \omega$). $\sigma(i)$ denotes the $i$th symbol of $\sigma$. In the case that $|\sigma| \geq i$, $\sigma^i$ denotes the suffix sub-sequence of $\sigma$ starting from $\sigma(i)$.

2.1 Definition of Regular Temporal Logic

Definition 1 Syntax

An $\infty$RTL formula is simply called an RTL formula. RTL formulas are defined inductively as follows. Let $AP$ be a set of atomic propositions. If $p \in AP$, and $\eta$ and $\xi$ are RTL formulas, then so are $(p)$, $(\neg \eta)$, $(\eta \lor \xi)$, $(\bigcirc \eta)$, $(\eta : \xi)$ and $(\square \eta)$.

Definition 2 Model and semantics

$M = (\Sigma, I)$ is a linear model, where $\Sigma$ is a set of states and $I : \Sigma \rightarrow 2^{AP}$ is an interpretation function.
Let $\sigma \in \Sigma^\infty - \{\epsilon\}$. $M, \sigma \models \eta$ denotes that an RTL formula $\eta$ holds along the sequence $\sigma$ with respect to a linear model $M$. If there is no confusion, $M$ is omitted like $\sigma \models \eta$. Let $p$ be an atomic proposition, $\eta$ and $\xi$ be RTL formulas. The relation $\models$ is defined inductively as follows:

1. $\sigma \models p$ iff $p \in I(\sigma(1))$.
2. $\sigma \models (\neg \eta)$ iff $\sigma \not\models \eta$.
3. $\sigma \models (\eta \lor \xi)$ iff $\sigma \models \eta$ or $\sigma \models \xi$.
4. $\sigma \models (\Box \eta)$ iff $|\sigma| \geq 2$ and $\sigma^2 \models \eta$.
5. $\sigma \models (\eta : \xi)$ iff there exist $\sigma_1 \in \Sigma^+$ and $\sigma_2 \in \Sigma^\infty - \{\epsilon\}$ such that $\sigma = \sigma_1 \sigma_2, \sigma_1 \models \eta$ and $\sigma_2 \models \xi$ or $|\sigma| = \omega$ and $\sigma \models \eta$.
6. $\sigma \models (\exists \eta)$ iff there exist $\sigma_i \in \Sigma^+ (i = 1, \ldots, m - 1)$ and $\sigma_m \in \Sigma^\infty - \{\epsilon\}$ such that $\sigma = \sigma_1 \sigma_2 \ldots \sigma_m$ and $\sigma_i \models \eta$ for all $i$ or there exist an infinite number of finite sequences $\sigma_i \in \Sigma^+$ such that $\sigma = \sigma_1 \sigma_2 \ldots$ and $\sigma_i \models \eta (i = 1, 2, \ldots)$.

In the following, ‘$\land$’, $V_T$ and $V_F$ represent ‘conjunction’, ‘tautology’ and ‘invalid’ respectively. Unary operators have higher precedence than binary operators. If there is no ambiguity, ‘$($’ and ‘$)$’ are omitted.

Finite RTL is defined as a subclass of $\infty$RTL, whose semantics domain is restricted to $\Sigma^+$. Finite RTL is exactly the same as $\epsilon$-free RTL[2].

2.2 Regular Temporal Logic and Regular Sets

First, we introduce several notations. $\text{Len1}$ holds along a set of sequences whose length is 1. ‘$\Diamond$’(‘sometime’) and ‘$\Box$’(‘always’) correspond to the temporal operators used traditionally in other temporal logic. $\text{Inf}$ and $\text{Fin}$ represent infinite sequences and finite sequences respectively.
\[\begin{align*}
\bullet \text{Len1} & \overset{\text{def}}{=} \lnot \bigcirc V_T. \\
\bullet \Diamond \eta & \overset{\text{def}}{=} \eta \lor (V_T: \eta). \\
\bullet \Box \eta & \overset{\text{def}}{=} \lnot \Diamond \eta = \eta \land \lnot (V_T: \lnot \eta). \\
\bullet \text{Inf} & \overset{\text{def}}{=} (V_T: V_F). \\
\bullet \text{Fin} & \overset{\text{def}}{=} \lnot \text{Inf} = \lnot (V_T: V_F).
\end{align*}\]

In order to discuss the relation between \(\infty\)RTL and regular sets, we define \(L(\Sigma, I)(\eta) \overset{\text{def}}{=} \{\sigma | \sigma \in \Sigma^\infty - \{\epsilon\}, \sigma \models \eta\}\), \(L_f(\Sigma, I)(\eta) \overset{\text{def}}{=} \{\sigma | \sigma \in \Sigma^+, \sigma \models \eta\}\) and \(L_\omega(\Sigma, I)(\eta) \overset{\text{def}}{=} \{\sigma | \sigma \in \Sigma^\omega, \sigma \models \eta\}\).

If there is no confusion, \(L(\eta)\) etc. are used, omitting \(\langle \Sigma, I \rangle\).

**Theorem 1** For an arbitrary RTL formula \(\eta\) and an arbitrary model \((\Sigma, I)\), \(L(\Sigma, I)(\eta)\) is an \(\epsilon\)-free infinitary regular set. Conversely, for an arbitrary \(\epsilon\)-free infinitary regular set \(R\) over \(\Sigma\), we can construct an RTL formula \(\eta\) such that \(L(\Sigma, I)(\eta) = R\), by introducing, for each state \(s \in \Sigma\), an atomic proposition \(p_s\) such that \(I(s) = \{p_s\}\).

This theorem is proved in [1].

**Corollary 1** \(L_f(\eta)\) and \(L_\omega(\eta)\) are an \(\epsilon\)-free regular set and an omega regular set respectively.

From the definition of \(L(\eta)\), \(L_f(\eta)\) and \(L_\omega(\eta)\), we can see that an RTL formula \(\eta\) can be used to specify some property of sequences, and \(L(\eta)\) is a set of the sequences that have the property.

### 3 Formal Verification between Two Different Levels

#### 3.1 Formal Framework for Describing Relations between Two Different Levels

In this section, we provide a formal framework to explicitly describe relations between the two different levels. We assume two different levels, that is, a higher level for a specification and a lower level for an implementation.
As an implementation to be verified, we consider a Mealy type deterministic sequential machine $M$ with $n$ binary input signals $x_1, x_2, \ldots, x_n$ and $m$ binary output signals $z_1, z_2, \ldots, z_m$. Let $M = (X, Z, S, \delta, \lambda, s_0)$ be a Mealy type deterministic sequential machine with an initial state, where $X$, $Z$, and $S$ are finite, nonempty sets of binary input signals, binary output signals, and states, respectively. $s_0 \in S$ is the initial state.

$\delta : 2^X \times S \rightarrow S$ is the state transition function (We assume that at least one next state is defined for each state in $S$). $\lambda : 2^X \times S \rightarrow 2^Z$ (We assume that the $\lambda$ is defined so long as $\delta$ is defined).

A possible input-output sequence of the sequential machine $M$ is an infinite or finite sequence $\rho$ over $2^{X \cup Z}$ such that $x_i \in \rho(k)$ iff $x_1 = 1$ at the $k$th input and $z_j \in \rho(k)$ iff $z_j = 1$ at the $k$th output, where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$ and $k = 1, 2, \ldots, |\rho|$.

We can regard the behavior of the machine as the set of all of its possible input-output sequences. Furthermore, we can identify a possible input-output sequence with a sequence of states of $\infty$RTL, by introducing atomic propositions $p_{x_i}$ and $p_{z_j}$ associated with input signal $x_i$ and output signal $z_j$ respectively, such that $p_{x_i}$ is true iff $x_i = 1$ and $p_{z_j}$ is true iff $z_j = 1$. From Theorem 1 and Corollary 1, we can specify the behavior of the sequential machine in finite RTL or $\infty$RTL.

In [2], specifications are described for finite possible input-output sequences by using finite RTL. While finite RTL can express any behavior of sequential machines, fairness constraints[4], which are important in describing input constraints, cannot be described. In this paper, we

1. adopt $\infty$RTL to describe specifications and
2. focus our attention to only infinite possible input-output sequences.

When we describe a specification at the higher level, we assume that there are possible input-output sequences at the level, even if there does not exist a realized machine, and we specify the property of the sequences
by an RTL formula. In describing relations between two different levels formally, we should pay attention to higher-level and lower-level sequences of states of \(\infty\)RTL. We formalize the relations as mappings from lower-level sequences to higher-level ones.

The framework for describing the relation of two state sequences of \(\infty\)RTL is formalized by the following the transformation rule and abstraction mapping. Here subscripts \(H\) and \(L\) are used to distinguish two objects which belong to the higher level and the lower level respectively.

**Definition 3 Transformation Rule**

For two given sets of atomic propositions \(AP_H\) and \(AP_L\), \(\langle \eta_L, SI \rangle\) is called a transformation rule, where

- \(\eta_L\) is a finite RTL formula,
- \(SI = \bigcup_{p_H \in AP_H} \{ p_H \leftarrow f_L|f_L\text{ is a lower-level (finite) RTL formula.}\}\).

\(\eta_L\) is called a time marker and \(p_H \leftarrow f_L\) a state interpreter.

**Definition 4 Abstraction Mapping**

For a lower-level sequence \(\langle I_L, \sigma_L \rangle\) and a transformation rule \(A = \langle \eta_L, SI \rangle\), it is called transformation of \(\sigma_L\) by \(A\) to obtain a higher-level sequence \(\langle I_H, \sigma_H \rangle\) such that, if \(s_{L1} s_{L2} \cdots s_{Li} \models \eta_L\), then \(\sigma_H(i)\) is a higher-level state such that \(I_H(\sigma_H(i)) \ni p_{Hi}\) iff \(s_{L1} s_{L2} \cdots s_{Li} \models \xi_{Li}\) for all \(p_{Hi} \leftarrow \xi_{Li} \in SI\), otherwise \(\sigma_H(i) = \epsilon\).

Let us consider the example of Figure 1; a specification is assumed to be written at the higher level, and an implementation is given at the lower level. The higher-level adder calculates the addition (mod 16) of two integers \(P, Q\) given as inputs and then, after a higher-level unit delay, it outputs the result. The lower-level adder serially adds two integers as 4-bit binary numbers. And then, after a lower-level unit delay, starts to output the result.
In Figure 1, a higher-level state corresponds to the lower-level sequences which end with four consecutive bits of inputs, and the output 0010 at the lower level corresponds to 4 at the higher level.

A transformation rule $A = \langle \eta, SI \rangle$ of the example of Figure 1 is shown as follows, where $P, Q, R$ are represented in binary representation using atomic propositions, i.e., $(p_3, p_2, p_1, p_0), (q_3, q_2, q_1, q_0), (r_3, r_2, r_1, r_0)$ respectively. $p_3, q_3$ and $r_3$ are the most significant bits. Here the higher-level integers are regarded as binary numbers, for simplicity.

$$\eta \overset{\text{def}}{=} \Box Len4$$
$$SI \overset{\text{def}}{=} \{ p_0 \leftarrow last(4, a), p_1 \leftarrow last(4, 0a), \cdots$$
$$:$$
$$r_0 \leftarrow last(7, c), r_1 \leftarrow last(7, 0c),$$
$$r_2 \leftarrow last(7, 00c), r_3 \leftarrow last(7, 000c) \}$$

where $last(i, \eta) \overset{\text{def}}{=} (\eta \wedge Len i) \vee (V_T : (\eta : Len i))$. $Len i$ holds only along the sequences that consist of exactly $i$ states.

The example of the transformation from a lower-level sequence to a higher-level sequence of the adders of Figure 1 is shown in Figure 2.
Although the detail is omitted in this paper, we can prove that the abstraction mapping can be simulated by a generalized sequential machine (gsm)[5]. Because regular sets and infinitary regular sets are closed under gsm mapping[5], any higher-level sequence obtained through the abstraction mapping can be characterized by $\infty$RTL.

### 3.2 A Formal Verification Method Considering Two Different Levels

In this section, we show the outline of a formal verification method for an implementation and a specification given at two different levels.

We regard that a transformation rule is a part of an implementation.

Here a structure model is introduced to handle possible input-output sequences easily.

**Definition 5 Structure model**

$K = (\Sigma, I, R, \Sigma_0)$ is called a structure model, where $(\Sigma, I)$ is a linear model of $\infty$RTL. $R \subseteq \Sigma \times \Sigma$ is a total binary relation on $\Sigma$ and denotes the possible transitions between states. $\Sigma_0 \subseteq \Sigma$ is a set of initial states.

An RTL formula $\eta$ is said to be universally $K$-true, if $\eta$ holds along all finite and all infinite paths $\pi$ from $s_0$ for all $s_0 \in \Sigma_0$ in the structure model $K$. Otherwise universally $K$-false.
For a Mealy machine $M_l = (X, Z, S, \delta, \lambda, s_0)$, its corresponding structure $K_l = (\Sigma, I, R, \Sigma_0)$ is constructed as follows:

- $\Sigma = \{s_{i,j,k}' | s_i \in S, j \in 2^X, k \in 2^Z, \lambda(j, i) = k\}$
- $I(s_{i,j,k}') = \{p_x | x \in j\} \cup \{p_z | z \in k\}$
- $R = \{(s_{i,j,k}, s_{i',j',k'}) | s_{i,j,k}, s_{i',j',k'} \in \Sigma, \delta(j, s_i) = s_i, s_{i'} = s_{i'} \}$
- $\Sigma_0 = \{s_{0,j,k}' \in \Sigma\}$

Figure 3: Generation of a Structure Model from a Sequential Machine [2]

When we focus to only infinite paths on the structure model $K$, the term universally $K$-omega true (or false) is employed.

A structure model $K$ corresponding to a designed sequential machine $M$ is obtained so that the possible input-output sequences of $M$ have one-to-one correspondence with paths on $K$. The ways of generating a structure model from a given Mealy machine are shown in Figure 3.

Then formal verification is to make sure that a given specification formula holds along all the higher-level state sequences obtained by the transformation rule from all the lower-level state sequences.

To do this, firstly, we generate a higher-level structure model $K_H$ from the lower-level structure model $K_L$ corresponding to the machine. The transformation is performed by applying the abstraction mapping to all the paths of $K_L$. Its algorithm is omitted in this paper. Our remaining work is to check whether a specification formula is universally $K_H$-omega true. The outline of its algorithm is shown in [7].

4 Considerations

In this paper, we show a formal framework based on $\infty$RTL for describing relations between two different levels of abstraction and a verification method for them.

The size of the higher-level structure model obtained from a lower-
level one can be larger than that of the lower-level one. In order to avoid the increase of the size, some restriction will be necessary to the framework of abstraction mapping. However, describing the correspondence between a higher-level sequence and a lower-level one explicitly, seems a suitable approach for formal verification of hierarchical design.

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