数理解析研究所講究録
第 712 巻 1990 年 73-92

Invariant Hyperfunctions on a Rank One Semisimple Symmetric Space

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1. Introduction

Let G be a connected semisimple Lie group, σ an involutive automorphism of G and H an open subgroup of the group of fixed points of σ. Then G/H is called a semisimple symmetric space. Let $\mathfrak{g}$ be the Lie algebra of G and $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{q}$ the decomposition into $+1$ and -1 eigenspaces with respect to σ. Choose a maximal abelian subspace $\mathfrak{a} \subset \mathfrak{q}$ consisting of semisimple elements. Then all such $\mathfrak{a}$ have the same dimension, which we call the rank of G/H. In what follows, we assume that the rank of G/H is one. It is clear that $\mathfrak{q}$ can be identified with the tangent space at the origin of G/H. Moreover, H acts on $\mathfrak{q}$ by means of the restriction of the adjoint representation to $\mathfrak{q}$.

If E is a real vector space, we put $E_{\mathbf{C}}$ the complexification of E and $S(E_{\mathbf{C}})$ the symmetric algebra of $E_{\mathbf{C}}$. For any $p \in S(\mathfrak{q}_{\mathbf{C}})$, we define, as usual, the differential operator $\partial(p)$ on $\mathfrak{q}$. Let $B(\ \ ,\ \ )$ be the Killing form on $\mathfrak{g}$, and write $\omega(X) = B(X,X)$ for all $X \in \mathfrak{q}$, the *Casimir polynomial* of $\mathfrak{q}$, then the polynomial ω is an H-invariant nondegenerate quadratic form on $\mathfrak{q}$. Identifying $\mathfrak{q}$ with its dual by means of the Killing form, $\square = \partial(\omega)$ can be viewed as an H-invariant constant coefficients second order differential operator on $\mathfrak{q}$. We call $\square$ the *pseudo-Laplacian* of $\mathfrak{q}$. We set

$$(\widetilde{H})_1 = \{T \in GL(\mathfrak{q}); \omega(Tx) = \omega(x) \text{ for all } x \in \mathfrak{q}\}.$$

Let $(\widetilde{H})_0$ be the identity component of $(\widetilde{H})_1$, and $\widetilde{\mathfrak{h}}$ be the Lie algebra of $(\widetilde{H})_0$, then $\mathrm{ad}_{\mathfrak{q}}(\mathfrak{h}) \subset \widetilde{\mathfrak{h}}$.

The dimensions of H-invariant eigendistributions of $\square$ on $\mathfrak{q}$ is given by careful considerations to the radial part of pseudo-Laplacian $\square$ by van Dijk [2]. On the other hand by a result of Cerezo [1] we can also give the dimensions of $\widetilde{H}$-invariant eigenhyperfunction of $\square$ on $\mathfrak{q}$. By comparison of these results, Kowata introduce the conjecture that every H-invariant eigenhyperfunction of $\square$ on $\mathfrak{q}$ is $(\widetilde{H})_0$-invariant, and he has shown that this conjecture holds if the eigenvalue of $\square$ is not equal to zero.

In this paper we give the answer to this conjecture except for the case of one of the exceptional type $F_{4(4)}/Spin(4,5)$. In §2 we rewrite the problem in terms of the systems of differential equations ($\mathcal{D}$-modules) and in §3 we compare these two $\mathcal{D}$-modules related to H and $(\widetilde{H})_0$ in the case of the classical type. In §4 we make more precise investigation by using the H-orbits decomposition of $\mathfrak{q}$. We must remark that the conjecture of Kowata does not hold in one case (Proposition 18).

I express my sincere gratitude to Prof .T.Oshima, whose constant encouragements and guidances made me write this paper. I would also like to thank Prof .T.Kobayashi and Prof .N.Tose for many advices.

2. Preliminaries

Let $\widetilde{H}$ be the subgroup of $GL(\mathfrak{q})$ generated by $(\widetilde{H})_0$ and $\mathrm{Ad}_{\mathfrak{q}}(H)$. Then H is contained in $\widetilde{H}$ and $(\widetilde{H})_0$ is the identity component of $\widetilde{H}$. Let $\mathcal{B}$ be the sheaf of hyperfunctions on $\mathfrak{q}$.

We define $\mathcal{B}^H(\mathfrak{q})$, $\mathcal{B}^H_\lambda(\mathfrak{q})$, $\mathcal{B}^{\widetilde{H}}(\mathfrak{q})$ and $\mathcal{B}^{\widetilde{H}}_\lambda(\mathfrak{q})$ for $\lambda \in \mathbf{C}$ by

$$\mathcal{B}^H(\mathfrak{q}) = \{f \in \mathcal{B}(\mathfrak{q}); f \text{ is } H\text{-invariant}\}$$
$$\mathcal{B}^H_\lambda(\mathfrak{q}) = \{f \in \mathcal{B}^H(\mathfrak{q}); (\Box - \lambda)f = 0\}$$
$$\mathcal{B}^{\widetilde{H}}(\mathfrak{q}) = \{f \in \mathcal{B}(\mathfrak{q}); f \text{ is } \widetilde{H}\text{-invariant}\}$$
$$\mathcal{B}^{\widetilde{H}}_\lambda(\mathfrak{q}) = \mathcal{B}^{\widetilde{H}}(\mathfrak{q}) \cap \mathcal{B}^H_\lambda(\mathfrak{q}).$$

A hyperfunction $u \in \mathcal{B}^H(\mathfrak{q})$ is said to be *H-invariant* and a hyperfunction $u \in \mathcal{B}^H_\lambda(\mathfrak{q})$ is called *an invariant spherical hyperfunction*. In general, we have $\mathcal{B}^{\widetilde{H}}_\lambda(\mathfrak{q}) \subset \mathcal{B}^H_\lambda(\mathfrak{q})$ and $\mathcal{B}^{\widetilde{H}}(\mathfrak{q}) \subset \mathcal{B}^H(\mathfrak{q})$.

Let $\mathcal{D}$ be the sheaf of the ring of holomorphic coefficient differential operators on $\mathfrak{q}_{\mathbf{C}}$. For any $X \in \mathfrak{gl}(\mathfrak{q})$, define a vector field D_X on $\mathfrak{q}$ by

$$(D_X f)(y) = \frac{d}{dt} f(e^{-tX}y)\Big|_{t=0} \qquad (y \in \mathfrak{q}, f \in C^\infty(\mathfrak{q})).$$

Then D_X is a first order differential operator with polynomial coefficients on $\mathfrak{q}_{\mathbf{C}}$. We define the systems of differential equations *defining H-invariants* and *$\widetilde{H}$-invariants*

$$\mathcal{M} = \mathcal{D}/\mathcal{I} = \mathcal{D}/\sum_{X \in \mathfrak{h}} \mathcal{D}D_X$$

$$\mathcal{N} = \mathcal{D}/\mathcal{J} = \mathcal{D}/\sum_{X \in \widetilde{\mathfrak{h}}} \mathcal{D}D_X,$$

and also define

$$\mathcal{M}_\lambda = \mathcal{D}/\mathcal{I}_\lambda = \mathcal{D}/(\mathcal{I}_\lambda + \mathcal{D}(\Box - \lambda))$$
$$\mathcal{N}_\lambda = \mathcal{D}/\mathcal{J}_\lambda = \mathcal{D}/(\mathcal{J}_\lambda + \mathcal{D}(\Box - \lambda)).$$

In general, we have surjective homomorphisms $\mathcal{M} \longrightarrow \mathcal{N}$ and $\mathcal{M}_\lambda \longrightarrow \mathcal{N}_\lambda$.

In these notations, our problems are written as follows.

PROBLEM I. *When is* $\mathcal{B}^H_\lambda(\mathfrak{q}) = \mathcal{B}^{\widetilde{H}}_\lambda(\mathfrak{q})$ *?*

PROBLEM II. *When is* $\mathcal{B}^H(\mathfrak{q}) = \mathcal{B}^{\widetilde{H}}(\mathfrak{q})$ *?*

PROBLEM III. *When is* $\mathcal{M}_\lambda \xrightarrow{\sim} \mathcal{N}_\lambda$ *?*

PROBLEM IV. *When is* $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$ *?*

REMARK 1.

(1) *If (IV) holds, then (II) and (III) hold. If (II) or (III) hold, then (I) hold.*
(2) *Problem III and IV depend only on the complexification* $(\mathfrak{g}_{\mathbf{C}}, \mathfrak{h}_{\mathbf{C}})$ *of* $(\mathfrak{g}, \mathfrak{h})$. ∎

Our approach to these problems is case by case calculation. The following table shows the non-Riemannian semisimple symmetric spaces G/H of rank one ([**4**, Chapter X], [**2**, § 6.3]).

TABLE 2.

No.	G/H
1°	$SO_0(p,q+1)/SO_0(p,q)$
2°	$SO(p,q+1)/S(O(p,q)\times O(1))$
3°	$SU(p,q+1)/S(U(p,q)\times U(1))$
4°	$Sp(p,q+1)/Sp(p,q)\times Sp(1)$
5°	$F_{4(-20)}/Spin(1,8)$
6°	$SL(m+1,\mathbf{R})/S(GL_+(m,\mathbf{R})\times GL_+(1,\mathbf{R}))$
7°	$SL(m+1,\mathbf{R})/S(GL(m,\mathbf{R})\times GL(1,\mathbf{R}))$
8°	$Sp(m+1,\mathbf{R})/Sp(m,\mathbf{R})\times Sp(1,\mathbf{R})$
9°	$F_{4(4)}/Spin(4,5)$

Here $p\geq 1$, $q\geq 1$, $m\geq 2$.

We sometimes write $p+q=m$ *in cases* $1°,2°,3°,4°$.

In cases 1° and 2°, then we have $\mathrm{ad}_{\mathfrak{q}}(\mathfrak{h})=\widetilde{\mathfrak{h}}$, hence Problem IV and therefore all problems are true. On the other hands, we have $\mathrm{ad}_{\mathfrak{q}}(\mathfrak{h})\neq\widetilde{\mathfrak{h}}$ in the rest cases, hance the whole problems have meanings.

3. Problem III and IV

3.1. The Spaces 6° : $G/H=SL(m+1,\mathbf{R})/S(GL_+(m,\mathbf{R})\times GL_+(1,\mathbf{R}))$. ▮

Let $(x,y)=(x_1,\dots,x_m,y_1,\dots,y_m)$ be a coordinate system in $\mathfrak{q}\cong\mathbf{R}^{2m}$. We denote the action of $T\in GL(\mathfrak{q})$ on $\begin{pmatrix}x\\y\end{pmatrix}\in\mathfrak{q}$ by left multiplication. The Casimir polynomial is $\omega=x_1y_1+\cdots+x_my_m$. We realize $\mathfrak{h}$ and $\widetilde{\mathfrak{h}}$ as subalgebras of $\mathfrak{gl}(\mathfrak{q})$.

$$\begin{cases}\mathfrak{h}=\mathfrak{gl}(m,\mathbf{R})\cong\left\{\begin{pmatrix}A&0\\0&-{}^tA\end{pmatrix}:A\in M(m,\mathbf{R})\right\}.\\ \widetilde{\mathfrak{h}}=\mathfrak{so}(m,m)\\ \quad=\left\{\begin{pmatrix}A&B\\C&-{}^tA\end{pmatrix}:A,B,C\in M(m,\mathbf{R}),{}^tB=-B,{}^tC=-C\right\}.\end{cases}$$

First we deal with Problem IV.

Proposition 3. *If $m \geq 3$, then $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$.*

Proof: We write $\mathcal{I}$ and $\mathcal{J}$ explicitly on $\mathfrak{q}_{\mathbf{C}}$ with the above coordinates

$$\mathcal{I} = \sum_{i,j=1}^{m} \mathcal{D}(x_i\partial_{x_j} - y_j\partial_{y_i}).$$

$$\mathcal{J} = \mathcal{I} + \sum_{i,j=1}^{m} \mathcal{D}(x_i\partial_{y_j} - x_j\partial_{y_i}) + \sum_{i,j=1}^{m} \mathcal{D}(y_i\partial_{x_j} - y_j\partial_{x_i}).$$

We will show $x_i\partial_{y_j} - x_j\partial_{y_i} \in \mathcal{I}$ for $1 \leq i < j \leq m$. Since $m \geq 3$, there is an integer k ($1 \leq k \leq m$) such that i, j, k are different to one another. Since the right hand side of the formula

$$\begin{aligned} x_i\partial_{y_j} - x_j\partial_{y_i} &= (x_j\partial_{y_k} - x_k\partial_{y_j})(x_i\partial_{x_k} - y_k\partial_{y_i}) \\ &+ (x_k\partial_{y_i} - x_i\partial_{y_k})(x_j\partial_{x_k} - y_k\partial_{y_j}) \\ &+ (x_i\partial_{y_j} - x_j\partial_{y_i})(x_k\partial_{x_k} - y_k\partial_{y_k}) \end{aligned}$$

is contained in $\mathcal{I}$, then we have $x_i\partial_{y_j} - x_j\partial_{y_i} \in \mathcal{I}$. Hence Proposition 3 is proved. ∎

Next we deal with the case $m = 2$. We define two left $\mathcal{D}$–Modules

$$\begin{aligned} \mathcal{B}_{\{x=0\}|\mathfrak{q}_{\mathbf{C}}} &= \mathcal{D}/\mathcal{D}(x_1, x_2, \partial_{y_1}, \partial_{y_2}) \\ \mathcal{B}_{\{y=0\}|\mathfrak{q}_{\mathbf{C}}} &= \mathcal{D}/\mathcal{D}(y_1, y_2, \partial_{x_1}, \partial_{x_2}), \end{aligned}$$

and two morphisms as left $\mathcal{D}$-Modules

$$(3.1) \qquad \mathcal{B}_{\{x=0\}|\mathfrak{q}_{\mathbf{C}}} \longrightarrow \mathcal{M}, \qquad P \longmapsto P(y_1\partial_{x_2} - y_2\partial_{x_1})$$

$$(3.2) \qquad \mathcal{B}_{\{y=0\}|\mathfrak{q}_{\mathbf{C}}} \longrightarrow \mathcal{M}, \qquad P \longmapsto P(x_1\partial_{y_2} - x_2\partial_{y_1}).$$

By (3.1) and (3.2), we get the complex of left $\mathcal{D}$-modules.

$$(3.3) \qquad 0 \longrightarrow \mathcal{B}_{\{x=0\}|\mathfrak{q}_{\mathbf{C}}} \oplus \mathcal{B}_{\{y=0\}|\mathfrak{q}_{\mathbf{C}}} \longrightarrow \mathcal{M} \longrightarrow \mathcal{N} \longrightarrow 0.$$

PROPOSITION 4. *The complex (3.3) is exact.*

PROOF: It is sufficient to prove the injectivity of the first morphism, which follows from Proposition 6 below. ∎

Next we consider Problem III.

PROPOSITION 5. *If $m = 2$ and $\lambda \neq 0$, then $\mathcal{M}_\lambda \xrightarrow{\sim} \mathcal{N}_\lambda$.*

PROOF: Since the pseudo-Laplacian $\Box$ is written as $\Box = \sum_{i=1}^{m} \partial_{x_i}\partial_{y_i}$ by the explicit coordinates, then $\Box - \lambda = \partial_{x_1}\partial_{y_1} + \partial_{x_2}\partial_{y_2} - \lambda$. We find that $(y_1\partial_{x_2} - y_2\partial_{x_1}) \in \mathcal{I}_\lambda$ by the formula

$$\begin{aligned}\lambda(y_1\partial_{x_2} - y_2\partial_{x_1}) = &-(y_1\partial_{x_2} - y_2\partial_{x_1})(\Box - \lambda) \\ &+ \partial_{x_1}^2(x_1\partial_{x_2} - y_2\partial_{y_1}) - \partial_{x_2}^2(x_2\partial_{x_1} - y_1\partial_{y_2}) \\ &- \partial_{x_1}\partial_{x_2}(x_1\partial_{x_1} - y_1\partial_{y_1}) + \partial_{x_1}\partial_{x_2}(x_2\partial_{x_2} - y_2\partial_{y_2}).\end{aligned}$$

Hence Proposition 5 is proved. ∎

The rest is in case $\lambda = 0$.

PROPOSITION 6. *The following complex (3.4) induced by the complex (3.3) is exact.*

$$(3.4) \qquad 0 \longrightarrow \mathcal{B}_{\{x=0\}|\mathfrak{q}_{\mathbf{C}}} \oplus \mathcal{B}_{\{y=0\}|\mathfrak{q}_{\mathbf{C}}} \longrightarrow \mathcal{M}_0 \longrightarrow \mathcal{N}_0 \longrightarrow 0.$$

PROOF: It is sufficient to prove the injectivity of the first morphism. First we consider the complex (3.4) outside the origin. We may assume

$y_2 \neq 0$ without loss of generality. Then remark that $\mathcal{B}_{\{y=0\}|\mathfrak{q}\mathbf{c}} = 0$ on the open set $\{y_2 \neq 0\}$. Moreover the following coordinate transformation makes sense on the open set $\{y_2 \neq 0\}$:

$$(3.5) \qquad \left\{ \begin{aligned} \widetilde{x}_1 &= x_1 \\ \widetilde{x}_2 &= x_1 y_1 + x_2 y_2 \\ \widetilde{y}_1 &= y_1 \\ \widetilde{y}_2 &= y_2 \end{aligned} \right\} \qquad \left\{ \begin{aligned} x_1 &= \widetilde{x}_1 \\ x_2 &= \frac{\widetilde{x}_2 - \widetilde{x}_1 \widetilde{y}_1}{\widetilde{y}_2} \\ y_1 &= \widetilde{y}_1 \\ y_2 &= \widetilde{y}_2 \end{aligned} \right\}.$$

We rewrite everything in the new coordinates. Then

$$\begin{aligned} \mathcal{B}_{\{x=0\}|\mathfrak{q}\mathbf{c}} &= \mathcal{D}/\mathcal{D}(\widetilde{x}_1, \widetilde{x}_2, \partial_{\widetilde{y}_1}, \widetilde{y}_2 \partial_{\widetilde{y}_2} - 1), \\ \mathcal{M}_0 &= \mathcal{D}/\mathcal{D}(\partial_{\widetilde{y}_1}, \partial_{\widetilde{y}_2}, \widetilde{x}_1 \partial_{\widetilde{x}_1}, \widetilde{x}_2 \partial_{\widetilde{x}_1}, \partial^2_{\widetilde{x}_2} \widetilde{x}_2), \end{aligned}$$

and the morphism (3.1) is the right multiplication by $\widetilde{y}_2 \partial_{\widetilde{x}_1}$, which is the composition of the following two injective morphisms

$$\mathcal{B}_{\{x=0\}|\mathfrak{q}\mathbf{c}} \xrightarrow[\sim]{\cdot \widetilde{y}_2} \mathcal{D}/\mathcal{D}(\widetilde{x}_1, \widetilde{x}_2, \partial_{\widetilde{y}_1}, \partial_{\widetilde{y}_2}) \overset{\cdot \partial_{\widetilde{x}_1}}{\hookrightarrow} \mathcal{M}_0.$$

This implies the complex (3.4) is exact outside the origin, i.e. the kernel of $\mathcal{B}_{\{x=0\}|\mathfrak{q}\mathbf{c}} \oplus \mathcal{B}_{\{y=0\}|\mathfrak{q}\mathbf{c}} \longrightarrow \mathcal{M}_0$ is supported in the origin. Moreover it follows from the involutivity of the characteristic varieties that the kernel must be zero. Thus we finish the proof of Proposition 6 and Proposition 4. ∎

3.2. The Spaces 3° : $SU(p,q+1)/S(U(p,q)\times U(1))$

By Remark 1, we can get the following Proposition 7 for spaces 3° as a corollary of the previous results for spaces 6°. Let $m=p+q$, $\mathfrak{q}=\mathbf{C}^m$. We take coordinates $z=(z_1,\dots,z_m)$ of $\mathfrak{q}$ and those of $\mathfrak{q}_{\mathbf{C}}$ as $(z,w)\in\mathbf{C}^{2m}$ with $z,w\in\mathbf{C}^m$, $\mathfrak{q}=\{(z,w)\in\mathfrak{q}_{\mathbf{C}};z=\bar{w}\}$.

PROPOSITION 7.

(1) *If* $m\geq 3$, *then* $\mathcal{M}\xrightarrow{\sim}\mathcal{N}$.
If $m=2$, *then we have the following exact sequence,*

$$0\longrightarrow\mathcal{B}_{\{z=0\}|\mathfrak{q}_{\mathbf{C}}}\oplus\mathcal{B}_{\{w=0\}|\mathfrak{q}_{\mathbf{C}}}\longrightarrow\mathcal{M}\longrightarrow\mathcal{N}\longrightarrow 0.$$

(2) *If* $m\geq 3$ *or* $\lambda\neq 0$, *then* $\mathcal{M}_\lambda\xrightarrow{\sim}\mathcal{N}_\lambda$.
If $m=2$ *and* $\lambda=0$, *then we have the exact sequence,*

$$0\longrightarrow\mathcal{B}_{\{z=0\}|\mathfrak{q}_{\mathbf{C}}}\oplus\mathcal{B}_{\{w=0\}|\mathfrak{q}_{\mathbf{C}}}\longrightarrow\mathcal{M}_0\longrightarrow\mathcal{N}_0\longrightarrow 0.$$

(3) *We have an isomorpism* $\underline{Hom}_{\mathcal{D}}(\mathcal{N},\mathcal{B})\xrightarrow{\sim}\underline{Hom}_{\mathcal{D}}(\mathcal{M},\mathcal{B})$.
In particular, $\mathcal{B}^{\widetilde{H}}(\mathfrak{q})=\mathcal{B}^H(\mathfrak{q})$ *and* $\mathcal{B}^{\widetilde{H}}_\lambda(\mathfrak{q})=\mathcal{B}^H_\lambda(\mathfrak{q})$.

PROOF: (1) and (2) follow from Proposition 3, 4, 5 and 6.
(3) is led by $\underline{Hom}_{\mathcal{D}}(\mathcal{B}_{\{z=0\}|\mathfrak{q}_{\mathbf{C}}},\mathcal{B})=\underline{Hom}_{\mathcal{D}}(\mathcal{B}_{\{w=0\}|\mathfrak{q}_{\mathbf{C}}},\mathcal{B})=0$. ∎

3.3. The Spaces 8° : $Sp(m+1,\mathbf{R})/Sp(m,\mathbf{R}) \times Sp(1,\mathbf{R})$

We set $Sp(m,\mathbf{R}) = \left\{ g \in SL(2m,\mathbf{R}); g \begin{pmatrix} 0 & 1_m \\ -1_m & 0 \end{pmatrix} {}^t g = \begin{pmatrix} 0 & 1_m \\ -1_m & 0 \end{pmatrix} \right\}$. We take coordinates

$$\begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} \in M(2m,2;\mathbf{R}) \cong \mathfrak{q}, \quad x_1 = \begin{pmatrix} x_{11} \\ \vdots \\ x_{m1} \end{pmatrix}, x_2, y_1, y_2 \in \mathbf{R}^m.$$

In this coodinates, the Casimir polynomial ω is written as

$$\omega = {}^t\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \begin{pmatrix} 0 & 1_m \\ -1_m & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = {}^t x_1 \cdot y_2 - {}^t y_2 \cdot x_2 = \sum_{k=1}^{m} (x_{k1} y_{k2} - y_{k1} x_{k2}).$$

The group $H = Sp(m,\mathbf{R}) \times Sp(1,\mathbf{R})$ acts on $\mathfrak{q}$ by

(3.6) $\quad (A,B) \cdot C = ACB^{-1} \qquad (A \in Sp(m,\mathbf{R}), B \in Sp(1,\mathbf{R}), C \in \mathfrak{q}).$

THEOREM 8. *For $m \geq 1$, we have $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$.*

PROOF: First we consider the problem outside the origin. This step contains rather tedious calculation. We may assume $x_{11} \neq 0$ and the following coordinate transformation makes sense on the locus $\{x_{11} \neq 0\}$:

$$\begin{cases} \widetilde{x}_{j1} = x_{j1}, & \widetilde{y}_{j1} = y_{j1} & (1 \leq j \leq m) \\ \widetilde{x}_{12} = x_{12}, & \widetilde{y}_{12} = \omega(x,y) & \\ \widetilde{x}_{k2} = x_{k2} - \dfrac{x_{12} x_{k1}}{x_{11}}, & \widetilde{y}_{k2} = y_{k2} - \dfrac{x_{12} y_{k1}}{x_{11}} & (2 \leq k \leq m). \end{cases}$$

We recall the generators of $\mathcal{I}$ by old coordinates (cf. (3.6)).

$$\begin{cases} (a): & x_{j1}\partial_{x_{i1}} - y_{i1}\partial_{y_{j1}} + x_{j2}\partial_{x_{i2}} - y_{i2}\partial_{y_{j2}} \qquad (1 \le i,j \le m) \\ (b): & y_{j1}\partial_{x_{i1}} + y_{i1}\partial_{x_{j1}} + y_{j2}\partial_{x_{i2}} + y_{i2}\partial_{x_{j2}} \qquad (1 \le i,j \le m) \\ (c): & x_{j1}\partial_{y_{i1}} + x_{i1}\partial_{y_{j1}} + x_{j2}\partial_{y_{i2}} + x_{i2}\partial_{y_{j2}} \qquad (1 \le i,j \le m) \\ (d): & \sum_{i=1}^{m}(x_{i1}\partial_{x_{i1}} + y_{i1}\partial_{y_{i1}} - x_{i2}\partial_{x_{i2}} - y_{i2}\partial_{y_{i2}}) \\ (e): & \sum_{i=1}^{m}(x_{i1}\partial_{x_{i2}} + y_{i1}\partial_{y_{i2}}) \\ (f): & \sum_{i=1}^{m}(x_{i2}\partial_{x_{i1}} + y_{i2}\partial_{y_{i1}}). \end{cases}$$

In new coordinates, we exchange generators of $\mathcal{I}$:

$$(3.7)\quad \begin{cases} (e) = x_{11}\partial_{\tilde{x}_{12}}, & \partial_{\tilde{x}_{12}} \in \mathcal{I}. \\ ((c), i=j=1) = 2x_{11}\partial_{\tilde{y}_{11}}, & \partial_{\tilde{y}_{11}} \in \mathcal{I}. \\ ((a), i=j=1) = x_{11}\partial_{\tilde{x}_{11}} + x_{12}\partial_{\tilde{x}_{12}} - y_{11}\partial_{\tilde{y}_{11}}, & \partial_{\tilde{x}_{11}} \in \mathcal{I}. \\ ((c), i \ge 2, j=1) = x_{11}\partial_{\tilde{y}_{i1}} + x_{i1}\partial_{\tilde{y}_{11}}, & \partial_{\tilde{y}_{i1}} \in \mathcal{I}. \\ ((a), i \ge 2, j=1) = x_{11}\partial_{\tilde{x}_{i1}} - y_{i1}\partial_{\tilde{y}_{11}}, & \partial_{\tilde{x}_{i1}} \in \mathcal{I}. \end{cases}$$

$$(3.8)\quad (d) = -\sum_{k=2}^{m}(\widetilde{x}_{k2}\partial_{\tilde{x}_{k2}} + \widetilde{y}_{k2}\partial_{\tilde{y}_{k2}}) + \sum_{k=2}^{m}(x_{k1}\partial_{\tilde{x}_{k1}} + y_{k1}\partial_{\tilde{y}_{k1}}) + x_{11}\partial_{\tilde{x}_{11}} + y_{11}\partial_{\tilde{y}_{11}},$$

$$\sum_{k=2}^{m}(\widetilde{x}_{k2}\partial_{\tilde{x}_{k2}} + \widetilde{y}_{k2}\partial_{\tilde{y}_{k2}}) \in \mathcal{I}.$$

$$
(3.9)\quad \begin{cases} ((a), i \geq 2, j \geq 2) = \widetilde{x}_{j2}\partial_{\tilde{x}_{i2}} - \widetilde{y}_{i2}\partial_{\tilde{y}_{j2}} + x_{j1}\partial_{\tilde{x}_{i1}} - y_{i1}\partial_{\tilde{y}_{j1}}, \\ \qquad\qquad \widetilde{x}_{j2}\partial_{\tilde{x}_{i2}} - \widetilde{y}_{i2}\partial_{\tilde{y}_{j2}} \in \mathcal{I}. \\ ((b), i \geq 2, j \geq 2) = \widetilde{y}_{j2}\partial_{\tilde{x}_{i2}} + \widetilde{y}_{i2}\partial_{\tilde{x}_{j2}} + y_{j1}\partial_{\tilde{x}_{i1}} + y_{i1}\partial_{\tilde{x}_{j1}}, \\ \qquad\qquad \widetilde{y}_{j2}\partial_{\tilde{x}_{i2}} + \widetilde{y}_{i2}\partial_{\tilde{x}_{j2}} \in \mathcal{I}. \\ ((c), i \geq 2, j \geq 2) = \widetilde{x}_{j2}\partial_{\tilde{y}_{i2}} + \widetilde{x}_{i2}\partial_{\tilde{y}_{j2}} + x_{j1}\partial_{\tilde{y}_{i1}} + x_{i1}\partial_{\tilde{y}_{j1}}, \\ \qquad\qquad \widetilde{x}_{j2}\partial_{\tilde{y}_{i2}} + \widetilde{y}_{i2}\partial_{\tilde{y}_{j2}} \in \mathcal{I}. \end{cases}
$$

Since ω is $\widetilde{H}$-invariant, then

$$
\mathcal{J} \subset \mathcal{D}(\partial_{\tilde{x}_{ij}}, \partial_{\tilde{y}_{ij}}; 1 \leq i \leq m, 1 \leq j \leq 2, \text{ except for } \partial_{\tilde{y}_{12}}).
$$

If we can prove

$$
(3.10)\quad \mathcal{I} \supset \mathcal{D}(\partial_{\tilde{x}_{ij}}, \partial_{\tilde{y}_{ij}}; 1 \leq i \leq m, 1 \leq j \leq 2, \text{ except for } \partial_{\tilde{y}_{12}}),
$$

then $\mathcal{I} = \mathcal{J}$. We prepare two Lemmas.

LEMMA 9. *Two groups $Sp(n, \mathbf{R}) \subset SL(2n, \mathbf{R})$ act on $\mathbf{R}^{2n}$ by left multiplication ($n \geq 1$). Then the system $\mathcal{M}_1$ of differential equations defining $Sp(n, \mathbf{R})$-invariants equals the system $\mathcal{M}_2$ of differential equations defining $SL(2n, \mathbf{R})$-invariants.*

PROOF: We set $\mathcal{M}_1 = \mathcal{D}/\mathcal{I}_1$, $\mathcal{M}_2 = \mathcal{D}/\mathcal{I}_2$. We can express $\mathcal{I}_1$ and $\mathcal{I}_2$

in coordinates.

$$\mathcal{I}_1 = \mathcal{D}\begin{pmatrix} x_i\partial_{x_j} - y_j\partial_{y_i} & ; & 1 \le i,j \le m \\ x_i\partial_{y_j} + x_j\partial_{y_i} & ; & 1 \le i,j \le m \\ y_i\partial_{x_j} + y_j\partial_{x_i} & ; & 1 \le i,j \le m \end{pmatrix}$$

$$\mathcal{I}_2 = \mathcal{D}\begin{pmatrix} x_i\partial_{x_i} - y_j\partial_{y_j} & ; & 1 \le i,j \le m \\ x_i\partial_{y_j}, \quad y_j\partial_{x_i} & ; & 1 \le i,j \le m \\ x_i\partial_{x_j}, \quad y_i\partial_{y_j} & ; & i \ne j \end{pmatrix}.$$

One after another,

$$\begin{aligned} x_i^2\partial_{x_i} &= x_i(x_i\partial_{x_i} - y_i\partial_{y_i}) + y_i x_i\partial_{y_i} \quad \in \mathcal{I}_1. \\ x_i^2\partial_{y_j} &= x_i(x_i\partial_{y_j} + x_j\partial_{y_i}) - x_j x_i\partial_{y_i} \ \in \mathcal{I}_1. \\ x_i\partial_{y_j} &= \frac{1}{2}(\partial_{x_i}x_i^2\partial_{y_j} - \partial_{y_j}x_i^2\partial_{x_i}) \in \mathcal{I}_1, \quad y_i\partial_{x_j} \in \mathcal{I}_1. \end{aligned}$$

Lie brackets of these elements in $\mathcal{I}_1$ span all generators of $\mathcal{I}_2$. ∎

LEMMA 10. *Let us consider that the group $GL_+(n, \mathbf{R})$ acts on $\mathbf{R}^n$ by left multiplication. Then the system of differential equations $\mathcal{M}_1$ defining $GL_+(n, \mathbf{R})$-invariants is de Rham system.*

PROOF: We set $\mathcal{M}_1 = \mathcal{D}/\mathcal{I}_1$, $\mathcal{I}_1 = \sum_{i,j=1}^n \mathcal{D}(x_i\partial_j)$.

$$\partial_j = \partial_i x_i \partial_j - \partial_j x_i \partial_i \in \mathcal{I}_1 \qquad (\text{for } i \ne j).$$

Then $\mathcal{I}_1 = \sum_{j=1}^m \mathcal{D}\partial_j$. ∎

Now we return to the proof of (3.10). Applying Lemma 9 to (3.9) and then applying Lemma 10 to (3.8), we conclude

$$\partial_{\bar{x}_{i2}}, \quad \partial_{\bar{y}_{i2}} \in \mathcal{I} \qquad (2 \le i \le m). \tag{3.11}$$

(3.7) and (3.11) imply (3.10). Thus $\mathcal{M}|_{\mathfrak{q}-\{0\}} \xrightarrow{\sim} \mathcal{N}|_{\mathfrak{q}-\{0\}}$. This isomorphism and following Proposition 11 imply $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$. Hence Theorem 8 is proved. ∎

PROPOSITION 11. *Let V be an n-dimensional vector space over $\mathbf{R}$ and $H_1 \subset H_2 \subset SL(V)$ be two closed subgroups. Suppose H_1 and H_2 are reductive in $SL(V)$, by definition ${}^tH_1 = H_1$, ${}^tH_2 = H_2$, here ${}^tH_i = \{{}^th^{-1} \in SL(V); h \in H_i\}$. Let $\mathcal{M}_i$ be the system of differential equations defining H_i-invariants ($i = 1, 2$). We assume*

$$\mathcal{M}_1|_{V_{\mathbf{C}}-\{0\}} \xrightarrow{\sim} \mathcal{M}_2|_{V_{\mathbf{C}}-\{0\}}, \tag{3.12}$$

then $\mathcal{M}_1 \xrightarrow{\sim} \mathcal{M}_2$.

PROOF: $\mathcal{M}_i = \mathcal{D}/\mathcal{I}_i$, $\mathcal{I}_i = \sum_{X \in \mathfrak{h}_i} \mathcal{D}D_X$, here $\mathfrak{h}_i$ is the Lie algebra of H_i ($i = 1, 2$).

Let A be a Weyl algebra over $V_{\mathbf{C}}$ and $F : \mathsf{A} \to \mathsf{A}$ be the algebra automorphism defined by $x_i \mapsto -\partial_i$, $\partial_i \mapsto x_i$ (cf. [5]). By the assumption (3.12) and Hilbert zero point theorem, there exists a positive integer N such that $(x_1, \ldots, x_n)^N D_X \subset \mathcal{I}_1$ for all $X \in \mathfrak{h}_2$, i.e. for any multi-index α satisfying ($|\alpha| \geq N$), there are $P_j(x, \partial) \in \mathsf{A}$,

$$x^\alpha D_X = \sum_{X_j \in \mathfrak{h}_1} P_j(x, \partial) D_{X_j}.$$

Let $P_j^\circ(x, \partial) \in \mathsf{A}$ be the homogeneous part of $P_j(x, \partial)$ of degree $-|\alpha|$, here we assign the degree 1 and -1 to the element ∂_i and x_i respectively. Then

$$x^\alpha D_X = \sum_{X_j \in \mathfrak{h}_1} P_j^\circ(x, \partial) D_{X_j}. \tag{3.13}$$

The transform of (3.13) by F is

$$(-\partial)^\alpha D_{-{}^t X} = \sum_{X_j \in \mathfrak{h}_1} P_j^\circ(-\partial, x) D_{-{}^t X_j}.$$

Then the left ideal $\{P \in \mathcal{A}; PD_X \in \mathcal{I}_1 \text{ for any } X \in \mathfrak{h}_2\}$ of $\mathcal{A}$ contains both $(x_1, \dots, x_n)^N$ and $(\partial_1, \dots, \partial_n)^N$, but such an ideal must be $\mathcal{A}$ itself. This means that $D_X \in \mathcal{I}_1$ for any $X \in \mathfrak{h}_2$, hence Proposition 11 is proved. ∎

3.4. The Spaces 4° : $Sp(p, q+1)/Sp(p, q) \times Sp(1)$

As a corollary of Theorem 8 and Remark 1,

THEOREM 12. *For $p, q \geq 1$, we have $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$.* ∎

4. Problem I and II

In this section, we treat Problem I and II with the H-orbit decomposition of $\mathfrak{q}$. Define $\mathfrak{q}_{reg} = \{x \in \mathfrak{q}\,; \operatorname{codim}_{\mathfrak{q}} H \cdot x = 1\}$ and $S = \mathfrak{q} - \mathfrak{q}_{reg}$, then $\mathfrak{q}_{reg}$ is H-invariant open subset of $\mathfrak{q}$. Define $\omega' = \omega\big|_{\mathfrak{q}_{reg}}$ and $\omega_1 = \omega\big|_{\mathfrak{q}-\{0\}}$.

4.1.

We quote the following Proposition from [2, § 6.3].

PROPOSITION 13. *In cases 3° ∼ 9° except for the case 6° and* $m = 2$.

(1) $\omega' : \mathfrak{q}_{reg} \longrightarrow \mathbf{R}$ *gives H-orbit parametrization, by definition,* *grad* $\omega' \neq 0$ *everywhere and* $\omega^{-1}(t)$ *is H-orbit for any* $t \in \mathbf{R}$.

(2) $S \subset \omega^{-1}(0)$ *and S decomposes finite H-orbits.*

(3) $\omega_1 : \mathfrak{q} - \{0\} \longrightarrow \mathbf{R}$ *gives* $\widetilde{H}$*-orbit parametrization.*

(4) *As a consequence,*

$$(4.1)\qquad \begin{cases} \mathcal{B}^H(\mathfrak{q}_{reg}) = \omega'^*(\mathcal{B}_{\mathbf{R}}(\mathbf{R})) \\ \mathcal{B}^{\widetilde{H}}(\mathfrak{q} - \{0\}) = \omega_1^*(\mathcal{B}_{\mathbf{R}}(\mathbf{R})). \end{cases} \qquad \blacksquare$$

We quote the following theorem due to Cerezo [1].

THEOREM 14. *A connected Lie group* $SO_0(p,q)$ *acts on* $\mathbf{R}^{p+q}$ *naturally. If* $(p,q) \neq (1,1)$, *then both*
$\mathcal{B}^{SO_0(p,q)}(\mathbf{R}^{p+q}) \longrightarrow \mathcal{B}^{SO_0(p,q)}(\mathbf{R}^{p+q} - \{0\})$ *and*
$\mathcal{B}^{SO(p,q)}(\mathbf{R}^{p+q}) \longrightarrow \mathcal{B}^{SO(p,q)}(\mathbf{R}^{p+q} - \{0\})$ *are surjective.* ∎

Now we prepare Lemma 15 for the proof of Proposition 16.

LEMMA 15. *Set* $\mathcal{F}[0] = \{u \in \mathcal{F}(\mathfrak{q}); supp\ u \subset \{0\}\}$. *Then in case* 1° ∼ 9°,

$$\mathcal{B}^{\widetilde{H}}[0] = \mathcal{B}^H[0].$$

PROOF: After Fourier transformation, Lemma 15 is easily shown by the fact that the ring of H-invariant polynomial on $\mathfrak{q}$ is generated by Casimir polynomial ω. ∎

Proposition 16. *In the same case in Proposition 13, if the assumption (A) :* $\mathcal{B}^H(\mathfrak{q}-\{0\})[S-\{0\}]=0$ *holds, then we have* $\mathcal{B}^{\widetilde{H}}(\mathfrak{q})=\mathcal{B}^H(\mathfrak{q})$.

Proof: Take $f \in \mathcal{B}^H(\mathfrak{q})$. For the restriction $f\big|_{\mathfrak{q}_{reg}} \in \mathcal{B}^H(\mathfrak{q}_{reg})$, there is a $g \in \mathcal{B}(\mathbf{R})$ such that $f\big|_{\mathfrak{q}_{reg}} = g\circ\omega'$ by (4.1). For $g\circ\omega_1 \in \mathcal{B}^{\widetilde{H}}(\mathfrak{q}-\{0\})$, there is an $\widetilde{f} \in \mathcal{B}^{\widetilde{H}}(\mathfrak{q})$ such that $\widetilde{f}\big|_{\mathfrak{q}-\{0\}} = g\circ\omega'$ by Theorem 14. Then $f-\widetilde{f} \in \mathcal{B}^H(\mathfrak{q})[S]$. For $(f-\widetilde{f})\big|_{\mathfrak{q}-\{0\}} \in \mathcal{B}^H(\mathfrak{q}-\{0\})[S-\{0\}]$, it must be zero by the assumption (A). Then $f-\widetilde{f} \in \mathcal{B}^H[0]=\mathcal{B}^{\widetilde{H}}[0]$, therefore $f \in \mathcal{B}^{\widetilde{H}}(\mathfrak{q})$. ∎

We can check the assumption (A) in some cases.

Proposition 17.

(1) *The singular set* $S=\{0\}$ *in cases* 1°, 2°, 3°, 4° *and* 5°. *In particular, the assumption (A) is satisfied.*
(2) *The assumption (A) is satisfied in cases* 6°, 7°.

Proof:

(1) This is the definition of isotropic symmetric pairs. [2]
(2) If $m \geq 3$, then Theorem 3 implies this. Thus we may assume $m=2$. Since the problem is outside the origin, we adopt the coordinates (3.5). In this coordinates, $S=\{\widetilde{x}_1=\widetilde{x}_2=0, \widetilde{y}_2 \neq 0\}$ and $\mathcal{M}=\mathcal{D}/\mathcal{I}=\mathcal{D}/\mathcal{D}(\partial_{\widetilde{y}_1}, \partial_{\widetilde{y}_2}, \widetilde{x}_1\partial_{\widetilde{x}_1}, \widetilde{x}_2\partial_{\widetilde{x}_1})$, then $\underline{Hom}_{\mathcal{D}}(\mathcal{M}, \Gamma_S\mathcal{B}) \subset \underline{Hom}_{\mathcal{D}}(\mathcal{D}/\mathcal{D}\widetilde{x}_1\partial_{\widetilde{x}_1}, \Gamma_{\{\widetilde{x}_1=0\}}\mathcal{B})=0$. ∎

Summing up the results, we have $\mathcal{B}^{\widetilde{H}}(\mathfrak{q})=\mathcal{B}^H(\mathfrak{q})$ in cases 3°$\tilde{8}$° except for the case 6° and $m=2$.

4.2. The Space : $SL(3,\mathbf{R})/S(GL_+(2,\mathbf{R}) \times GL_+(1,\mathbf{R}))$

We consider case 6°, $m = 2$. We adopt the notations in § 3.1. We introduce an element $f \in \Gamma(\mathbf{R}^4 - \{0\}, \mathcal{B})$

$$f = \begin{cases} \delta(x_1y_1 + x_2y_2)\{Y(x_1)Y(y_2)+Y(x_2)Y(y_1)\} \\ \qquad\qquad\qquad \in \Gamma(\{x_1 \neq 0 \text{ or } y_2 \neq 0\}, \mathcal{B}) \\ \delta(x_1y_1 + x_2y_2)\{Y(x_2)Y(-y_1) + Y(-x_2)Y(y_1)\} \\ \qquad\qquad\qquad \in \Gamma(\{x_2 \neq 0 \text{ or } y_1 \neq 0\}, \mathcal{B}) \end{cases}$$

Then f is H-invariant, $\Box f = 0$, and

$$(x_1\partial_{y_2} - x_2\partial_{y_1})f = \delta(y) = \delta(y_1, y_2)$$
$$(y_1\partial_{x_2} - y_2\partial_{x_1})f = \delta(x),$$

thus f is not $\widetilde{H}$-invariant. Let $\theta = x_1\partial_{x_1} + x_2\partial_{x_2} + y_1\partial_{y_1} + y_2\partial_{y_2}$, then $(\theta + 2)f = 0$. We take $\widetilde{f} \in \mathcal{B}(\mathbf{R}^4)$ such that $\widetilde{f}|_{\mathbf{R}^4-\{0\}} = f$. Since the mapping $(\theta + 2) : \mathcal{B}[0] \longrightarrow \mathcal{B}[0]$ is bijective, there is a unique $g \in \mathcal{B}[0]$ such that $(\theta + 2)g = (\theta + 2)\widetilde{f}$. We define $f_0 = \widetilde{f} - g \in \mathcal{B}(\mathfrak{q})$, then $(\theta + 2)f_0 = 0$. For any $X \in \mathfrak{h}$, $[D_X, \theta] = 0$ implies that $(\theta + 2)D_Xf_0 = D_X(\theta+2)f_0 = 0$, then $D_Xf_0 = 0$, i.e. $f_0 \in \mathcal{B}^H(\mathfrak{q})$. Moreover $\Box f_0 \in \mathcal{B}[0]$ is homogeneous of degree -4, then $\Box f_0 = c\delta(x, y)$ by some constant $c \in \mathbf{C}$. The relation $(\Box f_0)(-x_1, x_2, y_1, -y_2) = -\Box f_0(x, y)$ and $c\delta(-x_1, x_2, y_1, -y_2) = c\delta(x, y)$ imply $\Box f_0 = 0$. Therefore $f_0 \in \mathcal{B}_0^H(\mathfrak{q})$, $f_0 \notin \mathcal{B}^{\widetilde{H}}(\mathfrak{q})$ and $supp(f_0) = \overline{N_+} = \{(x, y) \in \mathbf{R}^4; \omega(x, y) = 0,\ x_1y_2 - x_2y_1 \geq 0\}$. Conversely an element $\mathcal{B}^H(\mathfrak{q} - \{0\})$ whose support is contained in $\overline{N_+} - \{0\}$ is a constant multiple of $f_0|_{\mathfrak{q}-\{0\}}$.

Unfortunately $\omega' : \mathfrak{q}_{reg} \longrightarrow \mathbf{R}$ does not give H-orbit parametrization because $\omega'^{-1}(0)$ has two connected components $N_\pm = \{(x, y) \in$

$\mathbf{R}^4; \omega(x,y) = 0,\ \pm(x_1y_2 - x_2y_1) > 0\}$, Therefore Proposition 13 must be modified in this case. Let $\mathbf{R}'$ be the non-Hausdorff manifold obtained by taking copies of $\mathbf{R}$, say $\mathbf{R}_+$, $\mathbf{R}_-$, and sticking together the positive parts of $\mathbf{R}_+$ and $\mathbf{R}_-$ and the negative parts of those [1]. Let $\mathbf{R}' \xrightarrow{\rho} \mathbf{R}$ be the canonical map and define $\mathcal{B}_{\mathbf{R}'} = \rho^{-1}\mathcal{B}_{\mathbf{R}}$, the sheaf of hyperfunctions on $\mathbf{R}'$. We define a real analytic mapping $\omega'' : \mathfrak{q}_{reg} \longrightarrow \mathbf{R}'$ by $\omega''^{-1}(t) = \omega'^{-1}(t)$ for any $t \in \mathbf{R}^\times$ and $\omega''^{-1}(0\pm) = (N_\pm)$.

$$
\begin{array}{ccccc}
\mathfrak{q}_{reg} & \subset & \mathfrak{q} - \{0\} & \subset & \mathfrak{q} \\
\omega'' \downarrow & \omega' \searrow & \downarrow \omega_1 & \swarrow \omega & \\
\mathbf{R}' & \underset{\rho}{\longrightarrow} & \mathbf{R} & &
\end{array}
$$

Then $\omega'' : \mathfrak{q}_{reg} \longrightarrow \mathbf{R}'$ gives H-orbit parametrization, and therefore

$$
\mathcal{B}^H(\mathfrak{q}_{reg}) = \omega''^*(\mathcal{B}_{\mathbf{R}'}(\mathbf{R}')). \tag{4.2}
$$

PROPOSITION 18. *Retain the above notation.*

(1) $\mathcal{B}^H(\mathfrak{q}) = \mathcal{B}^{\widetilde{H}}(\mathfrak{q}) \oplus \mathbf{C}f_0$.

(2) $\mathcal{B}_0^H(\mathfrak{q}) = \mathcal{B}_0^{\widetilde{H}}(\mathfrak{q}) \oplus \mathbf{C}f_0$.

PROOF: (1) Take $f \in \mathcal{B}^H(\mathfrak{q})$. For $f|_{\mathfrak{q}_{reg}} \in \mathcal{B}^H(\mathfrak{q}_{reg})$, there is a $g \in \mathcal{B}(\mathbf{R}')$ such that $f|_{\mathfrak{q}_{reg}} = g \circ \omega''$ by (4.2). For $g|_{\mathbf{R}_-} \in \mathcal{B}_{\mathbf{R}'}(\mathbf{R}_-) = \mathcal{B}_{\mathbf{R}}(\mathbf{R})$, $(g|_{\mathbf{R}_-}) \circ \omega_1 \in \mathcal{B}^{\widetilde{H}}(\mathfrak{q} - \{0\})$, then there is an $\widetilde{f} \in \mathcal{B}^{\widetilde{H}}(\mathfrak{q})$ such that $\widetilde{f}|_{\mathfrak{q}-\{0\}} = (g|_{\mathbf{R}_-}) \circ \omega_1$ by Theorem 14. Then $f - \widetilde{f} \in \mathcal{B}^H(\mathfrak{q})[\overline{N_+}]$. For $(f - \widetilde{f})|_{\mathfrak{q}-\{0\}} \in \mathcal{B}^H(\mathfrak{q} - \{0\})[\overline{N_+} - \{0\}]$, there is a constant $c \in \mathbf{C}$ such that $(f - \widetilde{f})|_{\mathfrak{q}-\{0\}} = cf_0$. Then $(f - \widetilde{f} - cf_0) \in \mathcal{B}^H[0] = \mathcal{B}^{\widetilde{H}}[0]$. Therefore $f \in \mathcal{B}^{\widetilde{H}}(\mathfrak{q}) \oplus \mathbf{C}f_0$.
(2) It is obvious from (1) because $f_0 \in \mathcal{B}_0^H$. ∎

Because f_0 is a distribution and tempered, the table in [2, § 6.3] must be corrected : case 7, $m = 3$ and $\lambda = 0$, in his notation, then $\dim D'_{\lambda,H}(\mathfrak{q}) = 3$, and $\dim S'_{\lambda,H}(\mathfrak{q}) = 3$.

以前 [8] で記したことのうち、ここの §4.2 に相当する部分は誤っています。正しくは、ここのようになります。おわびして訂正します。

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