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HIGHER ORDER ACCURATE COMPUTATIONS OF THE SIVASHINSKY EQUATION

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ABSTRACT

The Sivashinsky equation which describes the turbulent-like flow of a disturbed flame front is calculated by both the classical centered difference method and the higher order accurate modified differential quadrature (MDQ) method. Since the Sivashinsky equation is a nonstationary nonlinear integro-differential equation and represents a chaotic fluid motion, a higher order accurate computational method is needed. Although compuatations by the former method present erroneous results in the coarse meshes, the solutions independent of mesh points are obtained by the latter method. The results of the present computations show that the higher order accurate MDQ method is one of the most appropriate computational methods for the Sivashinsky equation.

1. Introduction

Sivashinsky found that the nonstationary nonlinear integro-differential equation describes the evolution of a disturbed plane flame in the premixed combustion of the gas. The Sivashinsky equation includes effects of instability, dissipation and dispersion which are important in turbulent motions. This equation is shown to describe the chaotic motion. The same type equations are deribed for the reaction-diffusion motion of the n-component system and for the long waves on a viscous flow down an inclined plane.

The Sivashinsky equation as well as other equations has to be solved ultimately numerically due its nonlinearity. In general, small initial disturbances are amplified and noticeable changes are observed through the time in the chaotic turbulent motions. In this respect the high accurate computational methods are needed for solving this equation, while computations are carried out so far by making use of the classical centered difference method.

In the present paper we calculate the Sivashinsky equation using both the centered difference scheme and the higher order accurate MDQ method. The accuracy and efficiency of the MDQ method are verified for direct numerical simulations on instability of laminar flows[1-3] and the shear flow turbulence in a plane channel.[4] We carry out computations of the Sivashinsky equation for different mesh sizes, and show that the MDQ method is appropriate for solving this equation while the centered difference method gives erroneous results.[5]

2. Computational Results

Computational methods used in present computations and detailed results are to be depicted in the full paper.[5] Here we only compare results using the MDQ method with ones using the centered difference method. The figure 1 shows the computational results at four time stages at τ = 6, 20, 60 and 80. At early time stages there is any appreciable difference between four results. At later time stages, however, two results using centered difference method differs significantly each other. Both results using the MDQ method show an excellent agreement, while these results differs considerably with ones using the centered difference method.

References

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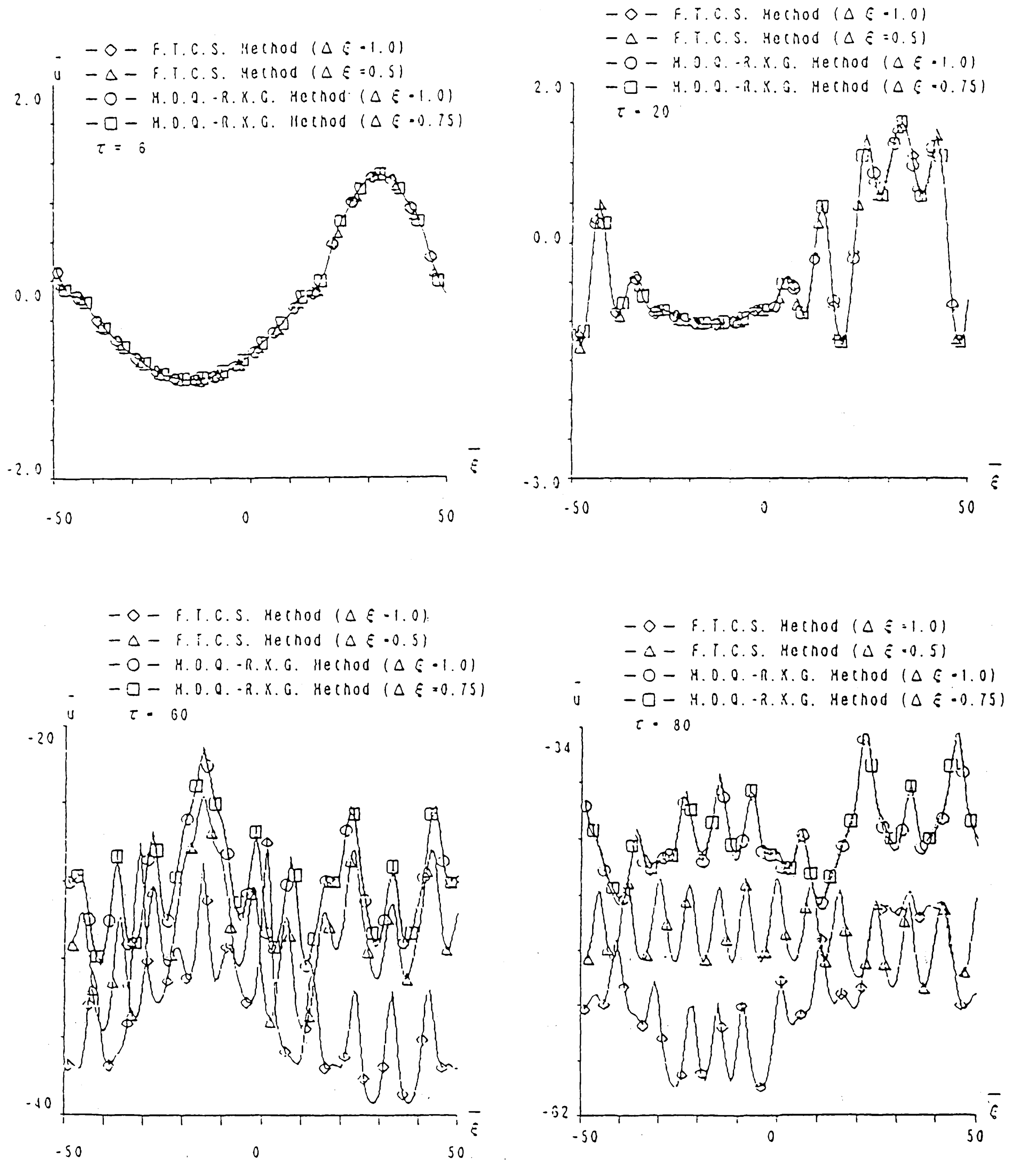

F.T.C.S. Method (Δ ξ = 1.0)
F.T.C.S. Method (Δ ξ = 0.5)
H.D.Q.-R.K.G. Method (Δ ξ = 1.0)
H.D.Q.-R.K.G. Method (Δ ξ = 0.75)
τ = 6
u
2.0
0.0
-2.0
-50
0
50
ξ
τ = 20
-3.0
τ = 60
-20
-40
τ = 80
-34
-52