Some Problems in Characteristic P>0 (Recent Topics in Algebraic Geometry)

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Some problems in char. $p > 0$

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Let $X$ be a smooth proper scheme over $k$, $k$ being algebraically closed of char. $p > 0$.

I. Concerning De-Rham-Witt complex:
   a) If $X$ is an abelian scheme, try to compute $H^i(X, W_n^j)$ in terms of $H^1_{\text{crys}}(X, W)$.

   If it is not possible, what are the new invariants one has to introduce?

   b) Define a Poincaré duality in terms of D.R.W. (?)

   Probably one will have to lift in char. 0 the Residue calculus.

   c) If $Y$ is another smooth scheme, what are the relations between $DRW(X \times Y)$ and $DRW(X), DRW(Y)$?

   d) Look for some geometric interpretation of the Cartier modules $H^i(X, W_n^j)/V$-Torsion, generalizing the Cartier modules of formal Brauer groups $H^i(X, W_n^0)/V$-Torsion.

II. Torsion phenomena in problems of lifting from char. $p$ to char. 0:

   Let $R$ be a complete discrete valuation ring of unequal characteristics, and of ramification index $e$. Let $X \to R$ be a smooth proper scheme with closed fibre $\bar{X} \to k$.

   1) If $e < p - 1$ (or $2e < p - 1$?), can we have non-closed 1-forms on $\bar{X}$?

   2) Let $L$ be an ample invertible sheaf on $x$. 

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If $e \leq p - 1$, I have proved in my paper at Colloque de Rennes (cf. Asterisque. 64 (1979)) that $H^1(\mathcal{X}, L^{-1}) = 0$.

If $\mathcal{X} \to \mathbb{R}$ is of relative dimension $\geq 3$, what can be said about $H^2(\mathcal{X}, L^{-1})$?

If the dimension of the formal Brauer group does not jump from generic fibre to closed fibre, my proof works also for the $H^2$. So, can the dimension of the Brauer groups jump?

III. Problems on surfaces in char. $p > 0$:

1) (Analog in char. $p$ of the Castelnuovo theorem) Suppose $c_2(X) < 0$ ($c_2$ = top. Euler characteristic). Does $X$ admit a fibration $f: X \to C$ such that genus $C \geq 2$ and the generic fibre of $f$ is of geometric genus $0$? (A surface with such a fibration is called a false ruled surface, in case it is not a (true) ruled surface.)

2) Let $f: X \to C$ be a false ruled surface with genus $C > 2$ and with generic fibre of arithmetic genus $\geq 2$.

Is $\chi(\mathcal{O}_X) \geq 0$? (Notice that $12\chi(\mathcal{O}_X) = c_1^2 + c_2$. The interesting case is where $c_1^2 > 0$ and $c_2 < 0$.)

3) Let $X$ be a K3 or an abelian surface, $H$ a general hyperplane section.

Is the difference between jacobian of $H$ and jacobian of $X$ (Picard variety) ordinary?