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Stability in 0L System

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0. *Introduction*

The theory of L systems originated from the work of Lindenmayer(3,4). The original aim of this theory was to provide mathematical models for the development of simple filamentous organisms. At the beginning L systems were defined as linear arrays of finite automata, later however they were reformulated into the more suitable framework of grammar-like costructs. From then on, the theory of L systems has been developed essentially as a branch of formal language theory(1,5,6,7). In the theory of L systems the development or the change of the organisms is expressed by the mapping on the strings of symbols.

A string is changed into some strings by the L system, and these are in turn changed into some other strings, and so on. For a string x, there are many descendants of x produced by the mapping of the given L system. Among these descendent strings some might go back to the original string x after several operations of the mapping. If every descendent string has a path which goes back to the original string x, we think the string x has a kind of stability. We call such a string 'recurrent' with respect to the L system. Walker and Herman defined an adult string(2,8), which is entirely mapped onto itself. In other words, an adult string is not changed under the L system. Obviously, an adult string is a special case of our recurrent string, and our definition is a natural extension of that of Walker's.

From the biological point of view, the recurrentness corresponds to some sort of maturity. Matured organisms seem to make no essential chenges. According to our definition a recurrent organism can always come back to itself even if it changes into some other one. The study of recurrentness in L system will make some characteristics of such matured organisms clear.

In this paper we give a brief introduction of L system theory and the formal definition of recurrent string. Then after some definitional preparation, we prove a factorization theorem for recurrent strings. Finally we establish our main theorem, which determines that for a given 0L system whether or not the system derives any recurrent strings.

1. *Preliminaries*

Most of definitions and the notations of this section are taken from Herman and Rozenberg(1). Although L system has various forms, we only discuss in this paper the basic form: 0L system. The symbol 0 means the system under consideration is interactionless, that is, the next state of a cell is determined without being affected by its neighbors.

The spatial sequence of cells of a filamentous organism is represented by a string of symbols. We assume the number of symbols used in the representation is finite. The set of all the symbols, called alphabet, is denoted as Σ. Σ^* denotes all the finite strings over the symbols in Σ, including the null string λ which is the string of length 0. For $x \in \Sigma^*$ alph(x) denotes the set of symbols appearing in x. For a set Σ, cardΣ denotes the cardinality of Σ.

Definition 1.1. A 0L scheme is a pair $S=\langle\Sigma,P\rangle$, where Σ (the alphabet of S) is a finite, nonempty set, and P (the set of productions of S) is a finite, nonempty subset of $\Sigma\times\Sigma^*$, such that for any $a\in\Sigma$ there is $x\in\Sigma^*$ and $(a,x)\in P$. □

We write $a\to x\in P$ or $x\in P(a)$ instead of $(a,x)\in P$. A 0L scheme S defines a relation $\underset{S}{\Longrightarrow}$ over Σ^* as follows.

Definition 1.2. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. For $x,y\in\Sigma^*$, we write $x\underset{S}{\Longrightarrow}y$ if and only if $x=x_1x_2\ldots x_n$, $x_i\in\Sigma$, $y=y_1y_2\ldots y_n$, $y_i\in\Sigma^*$ and $x_i\to y_i\in P$ for $i=1,2,\ldots,n$. □

Definition 1.3. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. For $x,y\in\Sigma^*$, we write $x\overset{n}{\underset{S}{\Longrightarrow}}y$ if and only if there exist n+1 strings $x_0,x_1,\ldots,$ x_n such that $x=x_0\underset{S}{\Longrightarrow}x_1\underset{S}{\Longrightarrow}x_2\underset{S}{\Longrightarrow}\ldots\underset{S}{\Longrightarrow}x_n=y$. □

We shall use $\Longrightarrow$ and $\overset{n}{\Longrightarrow}$ instead of $\underset{S}{\Longrightarrow}$ and $\overset{n}{\underset{S}{\Longrightarrow}}$, respectively, whenever S is understood. By definition, $x\overset{0}{\Longrightarrow}x$ for every $x\in\Sigma^*$

and $\lambda \overset{n}{\Longrightarrow} \lambda$ for any nonnegative integer n. We use the notation $\overset{*}{\Longrightarrow}$ which means $\overset{n}{\Longrightarrow}$ for some $n \geq 0$ and $\overset{+}{\Longrightarrow}$ which means $\overset{n}{\Longrightarrow}$ for some $n \geq 1$. In L system theory $\Longrightarrow$ is usually called a derivation.

Although the following lemmas are easy to prove, they are very useful, and we often use them without explicit citation.

Lemma 1.1. For any 0L scheme $S=<\Sigma,P>$, for any nonnegative integer n, and for any strings x_1,x_2,y_1,y_2, and z in Σ^*, if $x_1 \overset{n}{\Longrightarrow} y_1$ and $x_2 \overset{n}{\Longrightarrow} y_2$, then $x_1x_2 \overset{n}{\Longrightarrow} y_1y_2$. Conversely, if $x_1x_2 \overset{n}{\Longrightarrow} z$, then there exist strings z_1,z_2 in Σ^*, such that $z=z_1z_2$, $x_1 \overset{n}{\Longrightarrow} z_1$, and $x_2 \overset{n}{\Longrightarrow} z_2$. □

Lemma 1.2. For any 0L scheme $S=<\Sigma,P>$, for any nonnegative integers n and m, and for any strings x,y, and z in Σ^*, if $x \overset{n}{\Longrightarrow} y$ and $y \overset{m}{\Longrightarrow} z$, then $x \overset{l}{\Longrightarrow} z$, where $l=n+m$. □

Definition 1.4. A 0L system is a triple $G=<\Sigma,P,\omega>$, where $S=<\Sigma,P>$ is a 0L scheme, and ω is in Σ^* and is called the axiom. □

Definition 1.5. Let $G=<\Sigma,P,\omega>$ be a 0L system. A derivation in G is defined by the derivation in S where $S=<\Sigma,P>$. □

Definition 1.6. Let $G=<\Sigma,P,\omega>$ be a 0L system. The language generated by G or simply the language of G, denoted by L(G), is defined as $L(G)=\{x \mid \omega \overset{*}{\Longrightarrow} x\}$. □

Definition 1.7. A language L is said to be a 0L language if and only if $L=L(G)$ for some 0L system G. □

Now we give an illustrative example of a 0L scheme which will be used in the sequel.

Example 1.1. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme, where $\Sigma=\{a,b,c,d,e\}$ and $P=\{a\to ae, a\to b, b\to b, c\to ab, c\to ec, d\to e, d\to c, e\to\lambda, e\to e\}$. If we consider a 0L system $G=\langle\Sigma,P,a\rangle$, then some of the derivations are $a\Longrightarrow ae\Longrightarrow aee\Longrightarrow b$, $a\Longrightarrow b\Longrightarrow a\Longrightarrow ae$ and $a\Longrightarrow ae\Longrightarrow be$. It is easily seen that $L(G)=(a\cup b)e^*$. □

2. *Definitions and Lemmas*

In this section we give the definitions of a recurrent string and a closed strongly connected set. We establish some basic results.

Definition 2.1. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. $x\in\Sigma^*$ is said to be recurrent with respect to S if for any $z\in\Sigma^*$ such that $x\overset{*}{\Longrightarrow}z$, we have $z\overset{*}{\Longrightarrow}x$. □

Definition 2.2. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme and A be a subset of Σ^*.

i) A is said to be closed with respect to S if for any $x\in A$ and $y\in\Sigma^*$ such that $x\overset{*}{\Longrightarrow}y$ we have $y\in A$.

ii) A is said to be strongly connected with respect to S if for any $x,y\in A$ we have $x\overset{*}{\Longrightarrow}y$. □

Proposition 2.1. With respect to a 0L scheme $S=\langle\Sigma,P\rangle$, $x\in\Sigma^*$ is recurrent if and only if $x\in A$ where A is a closed strongly connected subset of Σ^*.

Proof. If part: For any $z\in\Sigma^*$ such that $x\overset{*}{\Longrightarrow}z$, we have $z\in A$ because A is closed. As A is also strongly connected, we have $z\overset{*}{\Longrightarrow}x$, which means that x is recurrent.

Only if part: Let $A=L(G)$ where $G=\langle\Sigma,P,x\rangle$. Then A is closed by the definition of $L(G)$. For any $y,z\in A$, there exist derivations $x\overset{*}{\Longrightarrow}y$, $x\overset{*}{\Longrightarrow}z$ and $y\overset{*}{\Longrightarrow}x$ the last one due to the recurrentness of x. So we have $y\overset{*}{\Longrightarrow}z$, and A is strongly connected. □

Example 2.1. Consider the 0L scheme $S=\langle\Sigma,P\rangle$ in Example 1.1. Then a is recurrent with respect to S. $(a\cup b)e^*$ is closed strongly connected with respect to S. □

If a string $x\neq\lambda$ has a derivation $x\overset{+}{\Longrightarrow}\lambda$, then x cannot be recurrent. So we must pick up the 'mortal' symbols as follows.

Definition 2.3. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. The set of vital symbols $\Sigma_v\subset\Sigma$ is given by

$$\Sigma_v=\{a\mid a\in\Sigma \text{ and } a\overset{*}{\Longrightarrow}x \text{ implies } x\neq\lambda\}.$$

The set of mortal symbols $\Sigma_m\subset\Sigma$ is given by

$$\Sigma_m=\Sigma-\Sigma_v$$

or

$$\Sigma_m=\{b\mid b\in\Sigma \text{ and there is a derivation } b\overset{*}{\Longrightarrow}\lambda\}. \quad □$$

Definition 2.4. Let $x\in\Sigma^*$. The vitality of x (denoted as $v(x)$)

equals the number of vital symbols in x. □

If a symbol b is mortal, then there is a derivation $b \overset{k}{\Longrightarrow} \lambda$ such that $k \le \mathrm{card}\Sigma$. Therefore Σ_m and Σ_v are effectively constructed and the vitality of a string is effectively computable.

Lemma 2.2. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. For any $x,y \in \Sigma^*$, we have the followings.

1) If $x \overset{*}{\Longrightarrow} y$, then $v(x) \le v(y)$.

2) If x is recurrent and $x \overset{*}{\Longrightarrow} y$, then $v(x)=v(y)$.

Proof. Obvious. □

The above Lemma tells us that every rewriting rule for a symbol in a recurrent string must be vitality preserving. This motivates us to define a further classification of Σ_m and a subscheme of a given 0L scheme as follows.

Definition 2.5. The set of ever mortal symbols $\Sigma_{mm} \subset \Sigma_m$ is given by

$$\Sigma_{mm}=\{a \mid a \in \Sigma_m \text{ and } a \overset{*}{\Longrightarrow} x \text{ implies } v(x)=0\}. \ \square$$

We denote the remainder part of Σ_m, i.e., $\Sigma_m - \Sigma_{mm}$ as Σ_{mv}. If a is in Σ_{mv} then there is a derivation $a \overset{k}{\Longrightarrow} x$ and $v(x) \ge 1$. In this case we can assume $k \le \mathrm{card}\Sigma$. Hence it is decidable whether or not a given symbol a is in Σ_{mm}.

Definition 2.6. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. The vitality preserving scheme of S is a 0L scheme $S'=\langle\Sigma',P'\rangle$ where

$$\Sigma'=\{a \mid a\in\Sigma_v \text{ and } P(a)\subset\Sigma^*_{mm}\Sigma_v\Sigma^*_{mm}\}\cup\Sigma_{mm}$$

and

P' is the restriction of P to $\Sigma'\times\Sigma'^*$. □

Note that $\Sigma'_m=\Sigma'_{mm}=\Sigma_{mm}$ and $\Sigma'_v\subset\Sigma_v$. Note that in the vitality preserving scheme $\langle\Sigma',P'\rangle$ $x\overset{*}{\Longrightarrow}y$ implies $v(x)=v(y)$ for any $x,y\in\Sigma'^*$.

Proposition 2.3. Let $S=\langle\Sigma,P\rangle$ and $S'=\langle\Sigma',P'\rangle$ be a 0L scheme and its vitality preserving scheme, respectively. A string is recurrent with respect to S if and only if it is recurrent with respect to S .

Proof. Let $x\in\Sigma^*$ be recurrent with respect to S. By virtue of Lemma 2.2 x must be in Σ'^*. Because S' is a subscheme of S, x is also recurrent in S'. If $x\in\Sigma'^*$ is recurrent with respect to S', then it is easy to see that x is recurrent in S. □

Let $S'=\langle\Sigma',P'\rangle$ be the vitality preserving scheme of a 0L scheme $S=\langle\Sigma,P\rangle$. We define vital recurrent symbols in Σ'_v as follows.

Definition 2.7. The set of vital recurrent symbols Σ'_{vr} satisfies the following condition:

$a\in\Sigma'_{vr} \Longleftrightarrow a\in\Sigma'_v$ and for any z such that $a\overset{*}{\Longrightarrow}z$ there exists a derivation $z\overset{*}{\Longrightarrow}x$ where x contains a. □

Note that it is decidable whether or not a given symbol is in

Σ'_{vr}.

Example 2.2. Consider the 0L scheme $S=\langle\Sigma,P\rangle$ in Example 1.1. Then $\Sigma_v=\{a,b,c\}$, $\Sigma_m=\{d,e\}$, and $\Sigma_{mm}=\{e\}$. The vitality preserving scheme is $\langle\{a,b,e\},\{a\to ae, a\to b, b\to b, e\to\lambda, e\to e\}\rangle$. $\Sigma'_{vr}=\{a,b\}$. □

3. *Factorization Theorem and Decision Problem for the Recurrent String*

In this section we prove a factorization theorem for the recurrent string, which divides a given recurrent string of vitality k into k segments each of which is recurrent and contains one vital recurrent symbol. We need a lemma to prove the theorem.

Lemma 3.1. Let $S=\langle\Sigma,P\rangle$ and $S'=\langle\Sigma',P'\rangle$ be a 0L scheme and its vitality preserving scheme, respectively. Then the following conditions are equivalent:

1) x is recurrent with respect to S and $v(x)=1$.

2) $x=lar$ for some $lr\in\Sigma'^{*}_{m}$ and $a\in\Sigma'_{vr}$ such that $a\overset{*}{\Longrightarrow}x\overset{+}{\Longrightarrow}x$.

Proof. 1)→2): By Proposition 2.3, x is recurrent with respect to S' and we can write $x=lar$ for some $lr\in\Sigma'^{*}_{m}$ and $a\in\Sigma'_{v}$. If $lr=\lambda$, then a is in Σ'_{vr} and there is a derivation $a\overset{+}{\Longrightarrow}a$ because $a=x$ is recurrent. Assume $lr\neq\lambda$, then there is a derivation $lr\overset{+}{\Longrightarrow}\lambda$. Thus we can have a derivation $x\overset{+}{\Longrightarrow}y$ such that $a\overset{+}{\Longrightarrow}y$. By the recurrent property of x, we also have a derivation $y\overset{*}{\Longrightarrow}x$. Therefore, $a\overset{+}{\Longrightarrow}y\overset{*}{\Longrightarrow}x\overset{+}{\Longrightarrow}y\overset{*}{\Longrightarrow}x$. This proof guarantees that a is in Σ'_{vr}.

2)→1): Let $x \overset{*}{\Longrightarrow} y$. Because $a \in \Sigma'_{vr}$, there is a derivation $y \overset{*}{\Longrightarrow} l_1 a r_1$ where $l_1 r_1 \in \Sigma'^{*}_m$. By the assumption that $a \overset{*}{\Longrightarrow} x \overset{+}{\Longrightarrow} x$, we have $l_1 a r_1 \overset{*}{\Longrightarrow} x$. Thus $x \overset{*}{\Longrightarrow} y \overset{*}{\Longrightarrow} l_1 a r_1 \overset{*}{\Longrightarrow} x$ for any possible y, and we see that x is recurrent with respect to S´ and hence with respect to S. □

Theorem 3.2. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme and $x \in \Sigma^*$ where $v(x)=k$ for a nonnegative integer k. Then x is recurrent with respect to S if and only if $x=x_1 x_2 \ldots x_k$ such that $v(x_i)=1$ and x_i is recurrent with respect to S for $i=1,2,\ldots,k$.

Proof. If part: It is sufficient to show that if x and y are recurrent so is xy. Let $xy \overset{n}{\Longrightarrow} z_1 z_2$ such that $x \overset{n}{\Longrightarrow} z_1$ and $y \overset{n}{\Longrightarrow} z_2$. As there are derivations $z_1 \overset{m_1}{\Longrightarrow} x$ and $z_2 \overset{m_2}{\Longrightarrow} y$ for some positive integers m_1 and m_2, we have derivations $z_1 \overset{p}{\Longrightarrow} x$ and $z_2 \overset{p}{\Longrightarrow} y$ where $p=(n+m_2-1)(n+m_1)+m_1=(n+m_1-1)(n+m_2)+m_2$.

Only if part: Let $S'=\langle\Sigma',P'\rangle$ be the vitality preserving scheme of S. If x is recurrent with respect to S´ such that $v(x)=k$, we can write $x=b_1 a_1 b_2 a_2 \ldots b_k a_k b_{k+1}$ where $a_i \in \Sigma'_{vr}$ $(i=1,2,\ldots,k)$ and $b_1 b_2 \ldots b_{k+1} \in \Sigma'^{*}_m$. Then there exists a nonnegative integer n such that $b_1 b_2 \ldots b_{k+1} \overset{n}{\Longrightarrow} \lambda$ and $a_i \overset{n}{\Longrightarrow} l'_i a'_i r'_i$, $l'_i r'_i \in \Sigma'^{*}_m$ for $i=1,2,\ldots,k$. Let $y=l'_1 a'_1 r'_1 l'_2 a'_2 r'_2 \ldots l'_k a'_k r'_k$. Because $x \overset{*}{\Longrightarrow} y$ and x is recurrent there is a derivation $y \overset{*}{\Longrightarrow} x$. Then x can be written as $x=l_1 a_1 r_1 l_2 a_2 r_2 \ldots l_k a_k r_k$ where $l_i r_i \in \Sigma'^{*}_m$ such that $l'_i a'_i r'_i \overset{*}{\Longrightarrow} l_i a_i r_i$ in that derivation for $i=1,2,\ldots,k$. Because $l_1 r_1 l_2 r_2 \ldots l_k r_k = b_1 b_2 \ldots b_{k+1} \overset{n}{\Longrightarrow} \lambda$, we have $l_i a_i r_i \overset{n}{\Longrightarrow} l'_i a'_i r'_i$ and $a_i \overset{*}{\Longrightarrow} l_i a_i r_i \overset{+}{\Longrightarrow} l_i a_i r_i$. Hence by Lemma 3.1, $l_i a_i r_i = x_i$ is recurrent for $i=1,2,\ldots,k$. □

Next let us consider the problem to decide for a given 0L system G whether or not there exist recurrent strings in L(G). For example, L(G) in Example 1.1 consists of recurrent strings only. On the other hand, if we consider $G=<\Sigma,P,c>$ where Σ and P are those of Example 1.1, some of the strings in L(G) are not recurrent. Now we must consider the symbols which can derive vital recurrent symbols.

Definition 3.1. Let $S=<\Sigma,P>$ be a 0L scheme. We define two sets of symbols which can derive the vital recurrent symbols as follows

$$\Sigma_{vd}=\{a\,|\,a\in\Sigma_v \text{ and there is a derivation } a\overset{*}{\Longrightarrow}x \text{ such that } x\in(\Sigma_m\cup\Sigma'_{vr})^+\}$$

$$\Sigma_{md}=\{b\,|\,b\in\Sigma_m \text{ and there is a derivation } b\overset{*}{\Longrightarrow}x \text{ such that } x\in(\Sigma_m\cup\Sigma'_{vr})^+ \text{ and } v(x)\geq 1\}.$$ □

Obviously $\Sigma'_{vr}\subset\Sigma_{vd}$. If $a\in\Sigma_{vd}$ $(\in\Sigma_{md})$, then there exists a nonnegative integer $k\leq\mathrm{card}\Sigma$ such that $a\overset{k}{\Longrightarrow}x$ and $x\in(\Sigma_m\cup\Sigma'_{vr})^+$. Therefore it is decidable whether or not a given symbol a is in Σ_{vd} (Σ_{md}).

Theorem 3.3. Let $G=<\Sigma,P,\omega>$ be a 0L system. L(G) contains a recurrent string if and only if the following condition holds.
In case $v(\omega)\geq 1$; $\mathrm{alph}\omega\subset\Sigma_{vd}\cup\Sigma_m$.
In case $v(\omega)=0$; $\mathrm{alph}\omega\cap\Sigma_{md}\neq\emptyset$ or $\omega=\lambda$.
Proof. If part: obvious.

Only if part: First assume $v(\omega)\geq 1$. If $alph\omega\subset\Sigma_{vd}\cup\Sigma_m$ fails to hold, in other words if there exists $a\in alph\omega\cap(\Sigma_v-\Sigma_{vd})$, then there exists a symbol $b\in\Sigma_v-\Sigma_{vr}$ in any descendant of a. By the factorization theorem, the vital symbols contained in a recurrent string must be vital recurrent. This is a contradiction. Next assume $v(\omega)=0$ and $\omega\neq\lambda$. Let x be a recurrent string in L(G). Then some symbol b in ω must derive a substring x' of x which contains some elements of Σ'_{vr}. From the definition of Σ_{md}, b is in Σ_{md}. □

From Theorem 3.3 we have the following

Theorem 3.4. Let $G=\langle\Sigma,P,\omega\rangle$. It is decidable whether or not there are recurrent strings in L(G). □

4. *Discussions*

In this article we have investigated the recurrentness only from the mathematical point of view. We have shown that a recurrent string has a factorization (Theorem 3.2) and that it is decidable for a 0L system G whether or not L(G) contains a recurrent string (Theorem 3.4). Now we consider some biological meaning of our results. Recurrent string may be interpreted as a matured or stable organism. The concept of mortal or vital recurrent symbol introduced here will be also useful in biological interpretation.

Lemma 3.1 and Theorem 3.2 tell us that the symbols in a recurrent string are to be rewritten in a special manner, i.e., either $P(a) \subset \Sigma_m'^* \Sigma_{vr}' \Sigma_m'^*$ or $P(a) \subset \Sigma_m'^*$ for every symbol a in a recurrent string. Thus we can interpret that only two types of cells are contained in matured or stable organisms. One is the cell in which some mortal mechanism is built-in. The other, which corresponds to the vital recurrent symbols Σ_{vr}', includes e.g., stem cell. The cell which divides into the same two cells as its only possible devision rule, in L system terminology $a \to aa$ is the only possible rewriting rule for that symbol, cannot be involved in the recurrent organism. Otherwise the number of cells in the organism will propagate forever like cancer.

If all the developmental rules are known, then from Theorem 3.4 it is decidable for a seed or an egg whether or not it will develop into a matured organism. Futher, we think that Theorem 3.3 may be interpreted as follows: If a seed or an egg develop into a matured organism, then the cells which will appear in the development must be able to derive vital recurrent cells.

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