数理解析研究所講究録
427 巻 1981 年 51-55

REFERENCES ON DISSECTION

W. T. Tutte

Department of Combinatorics
and Optimization
Faculty of Mathematics
University of Waterloo

[1] R. L. Brooks, C. A. B. Smith, A. H. Stone and W. T. Tutte: "The dissection of rectangles into squares," Duke Math, J., 7, pp.312-340, 1940.

[2] R. L. Brooks, C. A. B. Smith, A. H. Stone and W. T. Tutte: "Leaky electricty and triangulated triangles," Philips Research Reports, 30, pp.205-219, 1975.

[3] W. T. Tutte: "The dissection of equilateral triangles into equilateral triangles," Proc. Cambridge Phil. Soc., 44, pp.463-482, 1948.

[4] W. T. Tutte: "The rotor effect with generalized electrical flows," ABS Combinatoria, 1, pp.3-31, 1976.

For other references see:

P. J. Federico: "Squaring Rectangles and Squares-A Historical Review with Annotated Bibliography," Graph Theory and Related Topics, Edit. J. A. Bondy and U. S. R. Murty, Academic Press, N. Y., 1979.

A REMARK ON PLANARITY

W. T. Tutte

Department of Combinatorics
and Optimization
Faculty of Mathematics
University of Waterloo

A classical theorem of Hassler Whitney states that a 3-connected planar graph G can be drawn in the plane in essentially one way, meaning that the circuits that bound faces are uniquely determined by the structure of G. [3]. The simplest way to prove this is to characterize those circuits combinatorially. In [2] they are identified with the <u>peripheral</u> circuits of G, that is circuits having only one bridge. In [1] such circuits are called <u>non-separating</u>: if a peripheral circuit is contracted into a single vertex the graph G remains non-separable.

A <u>convex representation</u> of G is a drawing of G in the plane in which each edge is represented by a straight segment and each face is the interior or exterior of a convex polygon. Moreover the vertices of the bounding circuit of the face are to be drawn as the vertices of the convex polygon. Of course only one face, the outer or infinite face, is the exterior of a convex polygon.

In the theory of convex representations of G we first suppose that a circuit J of G, necessarily peripheral, is chosen to the boundary of the outer face. It is then supposed that J is drawn in the plane as the boundary of an arbitrary convex polygon Q, with appropriate number of vertices. The drawing of J is defined by a 1 - 1 mapping of the vertices of J onto Q, preserving cyclic order.

The mapping of f is now supposed to be extended as a mapping of the vertex-set V(G) of G onto some set of points of the plane. Initially the extended mapping f need not be one to one. We call J the frame of the extended mapping. We now ask such questions as "Can the extended mapping f be chosen so as to determine a convex representation of G?" and "What restrictions on f will ensure that it defines a convex representation?" The answer to the first of these questions is "Yes". See for example the existence proof in [1]. A partial answer to the second question is found in [2]: if each vertex not in J is drawn at the centroid of the representative points of its neighbours in G, then a convex representation results. Moreover such a "barycentric" representation can indeed be found as the solution of a set of linear equations.

It seems convenient to point out here one obvious property of convex representations. Let us say that the mapping f is enclosing if it satisfies the following condition: for each vertex v of G not in J the point f(v) lies in the interior of some convex polygon whose vertices are the representative points of some of the neighbours of v. Clearly if f defines a convex representation of G, then f is an enclosing mapping.

Let us make another definition. We call f a balanced mapping if for each vertex v of G not in J the following assertion is true: if a line through f(v) has the representative point of a neighbour of v on one side, then it has the representative point of another neighbour of v on the other side. Clearly every enclosing mapping is balanced and every barycentric mapping is balanced.

Consider the proof in [2] that a barycentric mapping determines a convex representation. The only property of a barycentric mapping used in this proof is that of being balanced. Accordingly, after a few verbal changes, the proof shows that if f is balanced it determines a convex representation of G. Incidentally we see that for the 3-connected graph G a balanced mapping is necessarily an enclosing one.

The purpose of this Note is to draw attention to the following theorem.

THEOREM. <u>The mapping of f, with frame J, of G into the plane defines a convex representation of G if and only if it is enclosing(or, equivalently, balanced)</u>.

It seems to the author that this theorem is worthy of explicit statement, but he is not aware of any such explicit statement in the literature.

References.

[1]. C. Thomassen, Planarity and duality of finite and infinite graphs. J. Combinatorial Theory B, 29, pp.244-271, 1980.

[2] W. T. Tutte, How to draw a graph, Proc. London Math. Soc. (Ser. 3), 13, pp.743-767, 1963.

[3]. H. Whitney, 2-isomorphic graphs, Amer. J. Math., 55, pp.245-254, 1933.