Dependency Integration of Locally Independent Relational Databases into a Distributed Database

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INTRODUCTION

In the research on distributed database systems, much effort has been devoted to the study of systems architecture, query processing, concurrency control, multiple copy update problems and file allocation etc. These results have been also used for the actual implementations of several distributed database management systems (for short, D-DBMS's). Basically, D-DBMS's can be classified into

(1) homogeneous type (for example, SDD-1)
(2) heterogeneous type (for example, POLYPHEM, Multibase)

As for the logical distributed database design, the following approaches are known:
(i) top-down design approach
(ii) bottom-up design approach

The top-down design approach often suits well for a homogeneous type D-DBMS. The bottom-up design approach is often adopted in heterogeneous type D-DBMS's since most of heterogeneous type D-DBMS's are constructed in order to integrate several locally independent DBMS's and databases which already exist.

In this paper, we mainly discuss a logical integration problem of local relational database constraints. We assume (1) a homogeneous type D-DBMS (a relational DBMS) and (ii) the bottom-up design approach in this paper. Our choice of the combination
(1)-(ii) is motivated from the following reasons:
(a) When we choose a heterogeneous type D-DBMS and the bottom-up design approach (that is, (2)-(ii)), it is also necessary to consider how to integrate homogeneous local databases by the bottom-up design approach since heterogeneous local databases are often logically translated into homogeneous local databases\textsuperscript{2,3,16}. Thus, our results in this paper are also useful for the construction of heterogeneous distributed database systems.
(b) As described later, the bottom-up design approach often offers more flexibility than the top-down approach in maintaining constraints of local databases, and in this sense, this approach is more practical.
(c) It is possible to transform CODASYL-type databases or hierarchical model databases into relational databases\textsuperscript{3}. Furthermore, several commercial relational DBMS's have become available on many kinds of computers.

Each local relational database has its own semantics and constraints, and therefore, the logical integration of these local databases must be performed very carefully. As the result of the logical integration, one or more global views are provided to users. In this case, it is important to consider what kinds of semantic constraints are provided to users by the global views of the distributed database. Some semantic constraints imposed on a local database may be violated when creating a global view. Furthermore, some semantic constraints, which are not imposed on any local database, may appear in a global view. When some semantic constraint on a local database is violated and does not hold on a global view, the distributed database designer must decide whether or not such a violation is allowable (or meaningful), and he must find a method for solving the problem if it is not allowed. Furthermore, semantic constraints imposed on global views are useful information for performing distributed query processing efficiently.

In this paper, we discuss a method for calculating semantic constraints (especially, dependency constraints) imposed on a global view. As shown later, some join dependency holding on a local
relational database may be violated when creating a global view by integrating local databases by join, projection and renaming operations. We show the relationship between a class of semantic constraints of a global view and a class of semantic constraints of local relational databases. A method is given for testing whether or not an embedded join dependency holds on a global view. This method is a simple modification of the 'chase' procedure by Maier et al. Immediately from our method, we also show a necessary and sufficient condition for a join dependency holding on a local database to hold on a global view.

**BASIC CONCEPTS**

A *relation* is defined as a finite set of mappings from the set of attributes to the corresponding domains. *Attributes* are symbols taken from a given finite set of symbols. A *domain* is simply a set of all the possible values, which can appear as the corresponding attribute values. We use $A,B,C,...$ (possibly with subscripts) to denote single attributes, and ...,X,Y,Z (possibly with subscripts) to denote sets of attributes. For a set of attributes $R=[A_1,...,A_n]$ and an associated domain $D_i$ for each attribute $A_i$, a relation $r$ on $R$ is a finite set of mappings $t$ such that $t: \{A_1,...,A_n\} \rightarrow D$, where $D$ is the union of the $D_i$'s. The mapping must map each attribute in $R$ to a member of its corresponding domain. Hereafter, we often represent a set of attributes by omitting commas and set braces, so that, for example, $\{A_1,...,A_n\}$ is written as $A_1...A_n$. The union of sets of attributes is also represented by concatenation, e.g., $X U Y U Z$ is written as XYZ.

Let $R$ be a given finite set of attributes. A *local (database) scheme* (for short, LS) at site $i$ ($i=1...n$) is a subset of $R$ of attributes. A *global database scheme* for LS's $R_1,...,R_n$ is defined as a relational algebra expression using restriction, join, projection etc. for the local schemes. A
**global join scheme** (for short, GJS) for LS's is defined by only join operators as follows:

\[ R_1 \ast \ldots \ast R_n \]  
(or denoted by \( \prod_{i=1}^{n} R_i \)).

In this paper, we consider the semantic constraints of only GJS's.

For a mapping \( t \) on a set \( R \) of attributes and a set \( X \subseteq R \), we denote the restriction of \( t \) to \( X \) by \( t[X] \). Let \( r \) be a relation on \( R \). The **projection** of \( r \) on \( X \), denoted by \( r[X] \), is the set \( r[X] = \{ t' \mid t' \in r, \ t' = t[X] \} \). The projection \( r[X] \) is a relation on \( X \) whose elements are the restrictions of the mappings in \( r \) to \( X \). For \( 1 \leq i \leq n \), let \( r_i \) be a relation on the set \( R_i \) of attributes. The (natural) join of \( r_1, \ldots, r_n \), denoted by \( r_1 \ast \ldots \ast r_n \) or \( \prod_{i=1}^{n} r_i \), is the relation on \( \prod_{i=1}^{n} R_i \) defined by \( \prod_{i=1}^{n} r_i \) if \( t \in \prod_{i=1}^{n} r_i \) is a mapping on \( \prod_{i=1}^{n} R_i \) such that \( t[R_i] \) is in \( r_i \) for all \( i, 1 \leq i \leq n \).

An **semantic constraint** \( c \) for a set \( R_i \) of attributes is a predicate, that assigns to each relation on \( R_i \) either 'true' or 'false'. When a relation \( r_i \) is assigned to 'true', \( r_i \) is said to satisfy \( c \), and \( c \) is said to hold in \( r_i \). For a given set \( C_i \) of semantic constraints, we denote the set of relations on \( R_i \) that satisfy every constraint in \( C_i \) by \( \text{SAT}(R_i, C_i) \). We say a set \( C_i \) of semantic constraints holds in \( R_i \) if and only if every meaningful relation on \( R_i \) must satisfy \( C_i \).

A **functional dependency** (FD)\(^6\) is a semantic constraint denoted by \( X \rightarrow Y \). It can be defined for every relation on \( R \) such that \( XY \subseteq R \). The FD \( X \rightarrow Y \) holds in relation \( r \) if and only if, for all mappings \( t_1, t_2 \) in \( r \), \( t_1[X] = t_2[X] \) implies \( t_1[Y] = t_2[Y] \). An **embedded join dependency** (EJD)\(^5,7\) is a semantic constraint denoted by \( *[S_1, \ldots, S_m] \), which can be defined for every relation on \( R \) such that \( R \supseteq i=1 S_i \). The EJD \( *[S_1, \ldots, S_m] \) holds in relation \( r \) if and only if

\[ \prod_{i=1}^{m} r[S_i] = r[\prod_{i=1}^{m} S_i] \]

If the union of \( S_i \)'s is equal to \( R \), then \( *[S_1, \ldots, S_m] \) is said to be a **join dependency** (JD) on \( R \). Let \( d \) be a single dependency that can
be defined on \( R \). The dependency \( d \) is said to be **logically implied** by a set \( D \) of dependencies on \( R \) if and only if \( d \) holds in any relation on \( R \) that satisfies all the dependencies in \( D \). If \( D \) logically implies \( d \), then we denote it by \( D \models d \).

Let \( R_1 \ldots R_n \) be a GJS such that \( \bigcup_{i=1}^{n} R_i = R \) and \( D_i \) denotes a set of FDs and EJDs that hold in \( R_i \) for each \( i \), \( 1 \leq i \leq n \).

1. Let \( X \rightarrow Y \) be an arbitrary FD such that \( XY \subseteq R \). We say \( R_1 \ldots R_n \) preserves \( X \rightarrow Y \) if and only if \( \bigcup_{i=1}^{n} R_i \) satisfies \( X \rightarrow Y \) for any distributed database \( \{r_1, \ldots, r_n\} \) such that \( r_i \) belongs to \( \text{SAT}(R_i, D_i) \) for each \( i \).

2. Let \( *[S_1, \ldots, S_m] \) be an arbitrary EJD such that \( R \supseteq \bigcup_{j=1}^{m} S_j \). We say \( R_1 \ldots R_n \) preserves \( *[S_1, \ldots, S_m] \) if and only if \( \bigcup_{i=1}^{n} R_i \) satisfies \( *[S_1, \ldots, S_m] \) for any distributed database \( \{r_1, \ldots, r_n\} \) such that \( r_i \) belongs to \( \text{SAT}(R_i, D_i) \) for each \( i \).

Here, \( \text{SAT}(R_i, D_i) \) denotes a set of all the relations on \( R_i \) that satisfy every dependency in \( D_i \).

**EXAMPLE**

Let us consider a distributed database consisting of two sites. Site 1 has an LS \( R_1 = \{\text{DEPT}, \text{RES}, \text{COM}, \text{DATE}\} \) and site 2 has an LS \( R_2 = \{\text{RES}, \text{COM}, \text{SOFTWARE}, \text{LAN} \} \). \text{DEPT} denotes a department name, \text{RES} a researcher name, \text{COM} a computer, \text{DATE} an installed date, \text{SOFTWARE} a software name, \text{LAN} a programming language to write the corresponding software. Fig.1 shows example relations \( r_1 \) on \( R_1 \) and \( r_2 \) on \( R_2 \). Every tuple \((d, r, c, i)\) in \( r_1 \) means that a researcher \( r \) belongs to department \( d \) and that computer \( c \) was installed at department \( d \) on date \( i \). A researcher may belong to more than one department. A department may have more than one researcher. More than one department may have the same type of computer. Each installed date of a computer is uniquely determined by the department name and the computer name. Then, we have the following dependency constraints for \( R_1 \):

\[
\text{FD: } \{\text{DEPT,COM}\} \rightarrow \text{DATE},
\]
JD: \[*\{\text{DEPT,RES}\}, \{\text{DEPT,COM,DATE}\}\].

Each tuple \((r,c,s,p)\) in \(r_2\) means that researcher \(r\) developed software \(s\) on computer \(c\) by programming language \(p\). We assume that the following FD holds in \(r_2\):

FD: \{\text{RES,COM,SOFT}\} \rightarrow \text{LAN}.

The relation \(r_1\) satisfies \{\text{DEPT,COM} \rightarrow \text{DATE}\} and \[*\{\text{DEPT,RES}\}, \{\text{DEPT,COM,DATE}\}\]. The relation \(r_2\) satisfies \{\text{RES,COM,SOFT} \rightarrow \text{LAN}\}.

If a distributed database designer defines \(R_1^{*}R_2\) as a GJS, then the relation \(r_1^{*}r_2\) in Fig.2 becomes a current instance for the global view. It is easily proved that the relation \(r_1^{*}r_2\) does not satisfy the EJD \[*\{\text{DEPT,RES}\}, \{\text{DEPT,COM,DATE}\}\], while \(r_1^{*}r_2\) satisfies the FDs \{\text{DEPT,COM} \rightarrow \text{DATE}\} and \{\text{RES,COM,SOFT} \rightarrow \text{LAN}\}. In this sense, the EJD \[*\{\text{DEPT,RES}\}, \{\text{DEPT,COM,DATE}\}\] cannot be preserved by the GJS \(R_1^{*}R_2\). Since it is easily proved that any FD holding in an LS is preserved by any GJS containing the LS, the two FDs above are preserved by \(R_1^{*}R_2\).

If the distributed database designer wishes to have a global view, in which any tuple \((d,r,c,i,s,p)\) means that researcher \(r\) developed a software \(s\) by language \(p\) for computer \(c\) which was installed on date \(i\) at his department \(d\), this GJS agrees with his intention.

If he wishes to preserve the EJD \[*\{\text{DEPT,RES}\}, \{\text{DEPT,COM,DATE}\}\] on a global view, one of the following renamings of attributes can be used:

1. Rename attribute RES of \(R_2\) into DEV(eloper) and create the GJS \(R_1^{*}R_2\).
2. Rename attribute COM of \(R_2\) into MAC(hine for the software) and create the GJS \(R_1^{*}R_2\).
3. Perform the renamings in (1) and (2), and create the GJS \(R_1^{*}R_2\).

Fig.3 shows a relation \(r_1^{*}r_2\) in which \(r_2\) is a relation on \(R_2\) = \{\text{DEV,COM,SOFT,LAN}\}. In this relation, we can find that the EJD \[*\{\text{DEPT,RES}\}, \{\text{DEPT,COM,DATE}\}\] is satisfied. Let \(r = r_1^{*}r_2\). Then, the
join \ r([\text{DEPT}, \text{RES}]) \bowtie r([\text{DEPT}, \text{COM}, \text{DATE}]) \text{ is equal to} \\
r([\text{DEPT}, \text{RES}, \text{COM}, \text{DATE}]). \text{ Therefore, } r \text{ satisfies the EJD} \\
\bowtie ([\{\text{DEPT}, \text{RES}\}, \{\text{DEPT}, \text{COM}, \text{DATE}\}]. \text{ r also satisfies the FDs shown} \\
\text{above.}

The semantic constraint \bowtie ([\{\text{DEPT}, \text{RES}\}, \{\text{DEPT}, \text{COM}, \text{DATE}\}] \text{ is} \\
\text{important at Site 1 since it means that} \\
(a) \text{ there is a many-to-many correspondence between DEPT and RES,} \\
(b) \text{ there is a many-to-many correspondence between DEPT and} \\
\{\text{COM}, \text{DATE}\}, \\
(c) \text{ RES and } \{\text{COM}, \text{DATE}\} \text{ are mutually independent.} \\
\text{Therefore, this semantic constraint is maintained at Site 1 for} \\
\text{several update commands. As shown previously, however, if a} \\
distributed database designer regards that the constraint is \\
unnecessary in his global scheme, the semantic constraint is \\
vviolated as shown in Fig.2. In this sense, each local database has} \\
\text{its own semantics and is semantically closed, and the semantics can} \\
\text{be maintained independently from what global schemes are defined} \\
\text{from the local databases. This offers more flexibility for the} \\
maintenance of local databases.}

\text{It is also important in distributed query processing to} \\
calculate semantic constraints on global views. For example, the JD} \\
\bowtie ([\text{XY}, \text{XZ}], \text{ which holds in a global view or in a user's query} \\
\text{expression, implies that the resultant data can be sent to the user} \\
in a more reduced form. That is, the data } (x_1, y_1, z_1), (x_1, y_1, z_2), \\
(x_1, y_2, z_1), (x_1, y_2, z_2) \text{ can be transferred as } x_1/y_1, y_2/z_1, z_2/ \text{ if we} \\
\text{know that } \bowtie ([\text{XY}, \text{XZ}] \text{ holds in a result. Furthermore, when a user} \\
\text{issues a query, if a D-DBMS can calculate semantic constraints} \\
\text{preserved on the user's query expression, the user can understand} \\
the semantic of his query well.}

\text{As shown in this example, a GJS must be created very carefully} \\
since the the semantics of a GJS depends very much on what} 
\text{dependencies can be preserved by the GJS. Therefore, it is} 

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important to test whether or not a dependency is preserved by a defined GJS, and to solve the problem of the violation on a dependency if it is not allowed.

TESTING CONSTRAINTS ON A GJS

The example of the previous section shows that the following issues are important in the logical integration of local relational databases:

1. To specify correctly a set of dependency constraints which must be preserved by a GJS.
2. To test whether or not a specified dependency can be preserved by the GJS.
3. To rename some attributes in some LS's so that every specified dependency may be preserved by the GJS.

As for (1), it is important to consider what class of dependency constraints can be preserved by a GJS for given LS's. First, we will show some conditions, which clarify the relationship between the class of preserved dependencies and the local dependency constraints. As for (2), a testing algorithm is required, which actually tests whether or not a specified dependency is preserved. We will show a method for testing whether or not an EJD is preserved by modifying the 'chase' procedure. As for (3), it is necessary to find a condition for a specified, but not preserved dependency to be preserved after renaming some attributes. Such a condition can be obtained immediately from our testing method.

Let \{R_1, \ldots, R_n\} be an arbitrary set of LS's, where \( D_i \) denotes a set of FDs and EJDs that hold on \( R_i \) \((i=1, \ldots, n)\). The following theorem provides a necessary condition for an FD or an EJD to be preserved by the GJS \( R_1 \ast \ldots \ast R_n \).

**Theorem 1**

If \( \bigcup_{i=1}^{n} R_i \) preserves a dependency \( d \) (FD or EJD), then \( d \) must be
logically implied by the union of \{*[R_1, \ldots, R_n]\} and D_i's.

Theorem 1 states that the set of all the dependencies preserved by the GJS must be contained in the closure of \[ D_1 \cup \{*[R_1, \ldots, R_n]\} \]. Here, the closure of a given set of dependencies means a set of all the FDs and EJDs that are logically implied by the given set. It should be noted that the converse of Theorem 1 does not always hold since some JD in some D_i may be violated, as shown in the previous section.

The following theorem provides a sufficient condition for an FD or an EJD to be preserved by the GJS \( R_1 \ast \ldots \ast R_n \).

**Theorem 2**
Let \( F_i \) denote a set of all the FDs that are logically implied by \( D_i \). For a given dependency \( d \) (FD or EJD), if the union of \( F_i \)'s (\( i=1, \ldots, n \)) and \{*[R_1, \ldots, R_n]\} logically implies \( d \), then \( \bigcup_{i=1}^{n} R_i \) preserves \( d \).

For a given GJS \( \bigcup_{i=1}^{n} R_i \), let \( \text{SPECIFIED}(\bigcup_{i=1}^{n} R_i) \) denote a set of FDs and EJDs that a database designer wishes to preserve by the GJS. Let \( \text{PRESERVED}(\bigcup_{i=1}^{n} R_i) \) denote a set of all the preserved dependencies (FDs and EJDs) by the GJS. Generally, the relationships among \( \text{SPECIFIED}(\bigcup_{i=1}^{n} R_i) \), \( \text{PRESERVED}(\bigcup_{i=1}^{n} R_i) \), \( D_i \)'s and \( F_i \)'s can be illustrated as shown in Fig. 4.

As a special case, if each \( D_i \) of dependency constraints is equivalent to a certain set of FDs, we can prove that the condition of Theorem 1 becomes a necessary and sufficient condition as follows:

**Corollary 1**
Assume that each \( D_i \) is equivalent to a certain set \( F_i \) of FDs. That is, \( D_i \) is logically implied by \( F_i \) and \( F_i \) is also logically implied
by $D_i$. In this case, a necessary and sufficient condition for a
dependency $d$ (FD or EJD) to be preserved by GJS $\bigwedge_{i=1}^{n} R_i$ is that the
union of $\{*[R_1, \ldots, R_n]\}$ and $D_i$’s logically implies $d$.

The result in Corollary 1 is strongly related to the result of
Beeri and Rissanen. Beeri and Rissanen has already shown the
similar result under the (pure) universal relation assumption. That is,
each (distributed) database $\{r_1, \ldots, r_n\}$ must be a set of
projections of a universal relation. The major difference of our
work from theirs is that we do not have such an assumption. An
example to illustrate the difference will be shown later in
Example 1.

Before describing our testing method, we briefly summarize some
preliminary concepts such as ‘tableau’ and ‘chase’ since our method
is based on a modified chase method. The definitions of tableau and
chase follow the papers by Maier et al. and by Sciore.

A tableau for a set $R$ of attributes is a set of rows, each with
one variable for each attribute in $R$. The variables are either
distinguished or nondistinguished. In any tableau, each variable can
be associated with only one attribute, and there can only be one
distinguished variable associated with each column. We use $a_i$ for
distinguished variables, and $b_i$ for nondistinguished variables. Let
$T$ be a tableau, $K$ the set of variables, and $D$ the union of the
domains of $R$. A valuation $\rho$ of $T$ is a mapping from $K$ to $D$ such that
$\rho(k) \in D_i$ whenever $k$ is associated with attribute $A_i$ and the domain
of $A_i$ is $D_i$. Valuations are extended to rows in the following way:
$\rho(k_1, \ldots, k_p) = \rho(k_1) \cdots \rho(k_p)$. The chase process is performed for a
tableau $T$ by the following FD-rule and JD-rule:

**FD-rule** For a given FD $X \rightarrow A$, whenever $T$ has rows $w_1$ and $w_2$ that
agree in all $X$-columns but does not agree in $A$-column, replace one
of the $A$-column variable in $T$ by the other $A$-column variable, and
remove duplicate rows. If one of the $A$-column variables is
distinguished, the other one must be renamed to that distinguished
variable. If both are nondistinguished, rename the variable with
larger subscript to be the variable with smaller subscript.
**JD-rule** For a given \( JD *[Y_1, \ldots, Y_m] \) for \( T \), add a row \( w \) to \( T \) if \( T \) contains rows \( w_1, \ldots, w_m \) (not necessarily distinct) such that for all \( 1 \leq j \leq m \), \( w_j \) agrees with \( w \) on \( Y_j \).

[Testing Algorithm for Preserving EJDS]^{9,15}

Inputs: (1) Local schemes (LS's) \( R_1, \ldots, R_n \), (2) sets of dependencies (FDs and JDs) \( D_1, \ldots, D_n \) such that \( D_i \) holds in \( R_i \) for each \( i \), \( 1 \leq i \leq n \), and (3) an EJD \( *[S_1, \ldots, S_m] \) such that \( S_j \bigcup_{i=1}^{n} R_i \) and \( S \subseteq R \).

Output: If \( \bigcup_{i=1}^{n} R_i \) preserves \( *[S_1, \ldots, S_m] \), then 'YES' otherwise 'NO'.

Step 1: (Construction of an initial tableau \( T \)) Construct an initial tableau \( T \) for \( *[S_1, \ldots, S_m] \) such that (a) \( T \) consists of \( m \) rows \( w_1, \ldots, w_m \), (b) row \( w_j \) has \( A_k \) in the \( k \)-th column if \( S_j \) contains attribute \( A_k \), and (c) all other entries in \( T \) are distinct nondistinguished variables.

Step 2: (Tableau projection) For each \( i \), \( 1 \leq i \leq n \), take a projection of \( T \) onto \( R_i \), denoted by \( T[R_i] \), considering \( T \) as a relation on \( R \). Let \( T_i = T[R_i] \).

Step 3: (Chase and interactions) For each \( i \), \( 1 \leq i \leq n \), apply the FD-rule or JD-rule to \( T_i \) by \( D_i \). Whenever a nondistinguished variable in some \( T_i \) is changed by applying FD-rule, change the nondistinquished variable appearing in other tableaux to the same variable. If any FD-rule or JD-rule or the above interaction cannot be applied, then let the final tableau be \( \bigcup_{i=1}^{n} D_i(T_i) \) for each \( i \), \( 1 \leq i \leq n \).

Step 4: (Join of tableau) Take a join of \( \bigcup_{i=1}^{n} D_i(T_i) \) for all \( i \), denoted by \( T' = \bigcup_{i=1}^{n} \text{chase}_{D_i}(T_i) \). If \( T' \) contains a row \( w \) such that \( w \) has distinguished variables in every column corresponding to an attribute in \( S \), then output 'YES' else output 'NO'. Terminate.

**Theorem 3**

The above testing algorithm works correctly. That is, if the algorithm outputs 'YES', then the GJS \( \bigcup_{i=1}^{n} R_i \) preserves the EJD \( *[S_1, \ldots, S_m] \), and if it outputs 'NO', then the GJS does not preserve \( *[S_1, \ldots, S_m] \).
Example 1
Let \( R_1 = \{A \to B, [BC, AB]\} \) and \( R_2 = \{[AC, AD]\} \) hold in \( R_1 \) and \( R_2 \), respectively. First, let us consider the preservability of the JD \( *[ABC, AD] \) on \( R_1 \cup R_2 \) by \( R_1 \ast R_2 \). Fig. 5(a) shows the initial tableau \( T \) for \( *[ABC, AD] \), the projected tableaux \( T_1 \) and \( T_2 \), chase_1(T_1) \) and chase_2(T_2), and the join chase_1(T_1) \ast chase_2(T_2), which are constructed by the above testing algorithm. Since we can obtain the row \( (a_1, a_2, a_3, a_4) \) in the join chase_1(T_1) \ast chase_2(T_2), we conclude that the GJS \( R_1 \ast R_2 \) preserves the JD \( *[ABC, AD] \). That is, for any relations \( r_1 \) on \( R_1 \) and \( r_2 \) on \( R_2 \), such that \( r_1 \in \text{SAT}(R_1, D_1) \) and \( r_2 \in \text{SAT}(R_2, D_2) \), \( r_1 \ast r_2 \) always satisfies the JD \( *[ABC, AD] \).

Next, let us consider the preservability of the JD \( *[ABD, BC] \). Fig. 5(b) shows the initial tableau \( T \) for \( *[ABD, BC] \) and several succeeding tableaux generated by the testing algorithm. We cannot obtain the row \( (a_1, a_2, a_3, a_4) \) in the final tableau chase_1(T_1) \ast chase_2(T_2) \) and thus, \( R_1 \ast R_2 \) cannot preserve the JD \( *[ABD, BC] \). As described in the proof of Theorem 3, the tableaux chase_1(T_1) \) and chase_2(T_2) \) in Fig. 5(b) can be considered to be relations on \( R_1 \) and \( R_2 \), respectively such that chase_1(T_1) \in \text{SAT}(R_1, D_1) \), chase_2(T_2) \in \text{SAT}(R_2, D_2) \) and the join of them does not satisfy the JD \( *[ABD, BC] \).

It is difficult to obtain the result only by the result by Beeri and Rissanen. They assumed so called (pure) universal relation assumption (Fagin et al.), which enforces that all relations must be projections of a large common relation at every time. Under this assumption, Beeri and Rissanen showed that for relation schemes \( R_1 \)
(i=1, ..., n) and a JD j defined on \( \bigcup_{i=1}^{n} R_i \), j is preserved by \( R_1 \cdots R_n \) if and only if the union of \( D_i \)'s and the JD \( \star [R_1, \ldots, R_n] \) logically implies j, where \( D_i \) denotes a set of FDs and JDs on \( R_i \). By the original chase procedure shown in Fig.5(c), we can prove that
\[
D_1 \cup D_2 \cup \{ \star [ABC, ACD] \} \models \star [ABD, BC].
\]
However, \( \star [ABD, BC] \) cannot be preserved by \( R_1 \star R_2 \) as shown above.

From the testing algorithm and Theorem 3, we can obtain the following theorem immediately:

**Theorem 4**

Let \( \{ R_1, \ldots, R_n \} \) be an arbitrary set of local schemes such that a JD \( \star [S_1, \ldots, S_m] \) holds in some \( R_i \). Then, the GJS \( \bigcup_{i=1}^{n} R_i \) preserves the EJD \( \star [S_1, \ldots, S_m] \) if and only if for each j (1 ≤ j ≤ n, j ≠ i), chase\( R_j \) contains a row that has distinguished variables in all columns corresponding to attributes in \( R_i \cap R_j \) (proof omitted).

**Example 2**

Let \( R_1 = BCDEFG \) and \( R_2 = AECG \) be local schemes such that the JD \( \star [BCDE, BCFG] \) holds in \( R_1 \) and on \( R_2 \) are arbitrary. Fig.6 shows the initial tableau T for the EJD \( \star [BCDE, BCFG] \), the projected tableaux \( T_1 \) and \( T_2 \), and a tableau \( T_1' \) obtained by applying JD-rule to \( T_1 \). Since \( R_1 \cap R_2 = CEG \), \( R_1 \star R_2 \) can preserve \( \star [BCDE, BCFG] \) if and only if \( T_2 \) can be changed into a tableau which contains a row \( (b, a_3, a_5, a_7) \). Here, b denotes an arbitrary nondistinguished variable in \( T_2 \). The following conditions provide several sufficient conditions for \( \star [BCDE, BCFG] \) to be preserved:

(a) \( BC \rightarrow E \) holds in \( R_1 \) or \( BC \rightarrow G \) holds in \( R_1 \).
(b) \( C \rightarrow E \) holds in \( R_2 \) or \( C \rightarrow G \) holds in \( R_2 \).
(c) \( \star [CE, CG] \) holds in \( R_2 \).
We can easily prove that each of the conditions (b) and (c) enforces that $T_2$ has a row $(b, a_3, a_5, a_7)$ in the chase process. The condition (a) seems to be a little strange since neither $BC \rightarrow E$ nor $BC \rightarrow G$ cannot be defined on $R_2$. If (a) holds (for example, $BC \rightarrow E$ holds), then we can obtain $T_1'$ from $T_1$ by applying FD-rule $(BC \rightarrow E)$. Then, at Step 3, the interaction occurs and the nondistinguished variable $b_6$ in $T_2$ is also changed into the distinguished variable $a_5$. Therefore, a row $(b, a_3, a_5, a_7)$ is generated by condition (a).

From Theorem 4 and Example 2, we can obtain the following theorems immediately. This theorem provides a useful sufficient condition for a JD on $R_1$ to be preserved by $\{R_1, R_2\}$.

**Theorem 5**

Let $\{R_1, R_2\}$ be an arbitrary set of local schemes, where the JD $[S_1, ..., S_m]$ holds in $R_1$. Then, $R_1*R_2$ preserves the EJD $[S_1, ..., S_m]$ if the EJD $[S_1 \cap R_2, ..., S_m \cap R_2]$ holds in $R_2$ (proof omitted).

**Theorem 6**

Assume that a JD $*[XY, XZ]$ holds in $R_1=XYZ$. $R_1*R_2$ preserves the EJD $*[XY, XZ]$ if at least one of the following conditions holds: (a) $*[XY \cap R_2, XZ \cap R_2]$ holds in $R_2$, (b) $X \rightarrow Y \cap R_2$ (or $X \rightarrow Z \cap R_2$) holds in $R_1$, (c) $X \cap R_2 \rightarrow Y \cap R_2$ (or $X \cap R_2 \rightarrow Z \cap R_2$) holds in $R_2$ (proof omitted).

Theorem 5 and Theorem 6 provide useful sufficient conditions when we need to rename some attributes so that some EJD may be preserved by a GJS. In the example shown previously, the EJD $*\{(DEPT, RES), (DEPT, COM, DATE)\}$ cannot be preserved by the GJS $R_1*R_2$. If we rename attribute RES into DEV in $R_2$, then we can verify that the condition (a) of Theorem 6 holds. That is, $*[XY \cap R_2, XZ \cap R_2]$ is equal to $*[\emptyset, (COM)]$, which always holds. Here, $X=\{DEPT\}$, $Y=\{RES\}$, $Z=\{COM, DATE\}$ and $R_2=\{DEV, COM, SOFT, LAN\}$.

**RENAMEING PLAN**
Let $j$ be a join dependency which holds in some $R_k \ (1 \leq k \leq n)$. Assume that $R_1 \times \ldots \times R_n$ does not preserve $j$. The following renaming plan selects a set of attributes whose renaming makes $j$ be preserved:

[Renaming Plan]

(1) Apply our testing algorithm to examine the preservability of $j$ by the GJS $R_1 \times \ldots \times R_n$.

(2) Let $T_i \ (i \neq k)$ be a tableau for $R_i$ such that at least one row is changed by the tableau interaction until $T_i$ is transformed into $\text{chase}_{D_i}(T_i)$. For each of these $T_i$'s, let $\text{CANDIDATE}(T_i)$ be a set of subsets of $R_i$ such that each subset $X$ of $R_i$ satisfies the following conditions: (a) $T_i$ has a row whose $R_k \cap (R_i - X)$ value consists of only distinguished variables, and (b) $X$ is a minimal subset of $R_i$ satisfying the condition (a).

(3) Let $T_i \ (i \neq k)$ be a tableau for $R_i$ such that any tableau interaction is not performed during its transformation into $\text{chase}_{D_i}(T_i)$. For each of these $T_i$'s, let $\text{CANDIDATE}(T_i)$ be a set of subsets of $R_i$ such that each subset $X$ satisfies the following conditions: (a) $\text{chase}_{D_i}(T_i)$ has a row whose $R_k \cap (R_i - X)$ value consists of only distinguished variables, and (b) $X$ is a minimal subset of $R_i$ satisfying the condition (a).

(4) Find a subset $Y$ of $R$ (the union of $R_i$'s) that satisfies the following conditions: (a) For every $\text{CANDIDATE}(T_i) \ (i=1, \ldots, n, i \neq k)$, $Y$ contains at least one element of $\text{CANDIDATE}(T_i)$, and (b) The number of attributes in $Y$ is minimum.

(5) Rename the names of attributes in $R_i \cap Y$ for each relation scheme $R_i \ (i=1, \ldots, n, i \neq k)$.

Example 3
Let \( R_1 \) and \( R_2 \) be LS's defined as follows:
\[
R_1 = \text{ABCD}, \quad D_1 = \{*[AB,ACD], *[ABC,BD]\} \\
R_2 = \text{BCDE}, \quad D_2 = \{*[BC,BDE]\}.
\]
As shown in Fig.7(a), \( *[AB,ACD] \) (a JD of \( R_1 \)) cannot be preserved by \( R_1 \times R_2 \) since the chase \( D_2(T_2) \) does not contain a row \( (a_2,a_3,a_4,b) \). In this case, no interaction occurs at Step 3 in our testing algorithm, we obtain

Therefore, if we rename attribute \( B \) into \( B' \) of \( R_2 \), \( *[AB,ACD] \) is preserved by \( R_1 \times R_2 \). This is shown in Fig.7(b). Without renaming any attributes, the JD \( *[BC,BDE] \) of \( R_2 \) is preserved by \( R_1 \times R_2 \) (see Fig.7(c)). However, if we rename attribute \( B \) of \( R_2 \) into \( B' \) in order to preserve \( *[AB,ACD] \), we cannot preserve \( *[B'C,B'DE] \) of \( R_2 \) by \( R_1 \times R_2 \) (see Fig.7(d)). This is because any JD-rule cannot be applied to \( T_1 \). This example shows that some renaming plan to preserve a JD \( j_1 \) may cause other JD \( j_2 \) not to be preserved, where without any renaming, \( j_2 \) was preserved and \( j_1 \) was not preserved. Therefore, in order to preserve a set of JDs, we must select renaming plans so that any renaming plan does not violate the preservability of other JDs.

Fig.7(e) shows another example, which implies the necessity of taking a minimum cover of \( \text{CANDIDATE}(T'_i) \)'s at (5) in our renaming plan if we wish to reduce the number of renamed attributes.
CONCLUDING REMARKS

In this paper, we discussed the logical integration problem of dependency constraints of local relational databases into a global view of a distributed database. When creating a global view by the bottom-up approach, some semantic constraints on a local database may be violated. It is important to calculate semantic constraints (especially, dependency constraints in the case of relational databases) of a global view that can be preserved by the global view. We have shown the relationship between a class of dependencies (FDs and EJDs) preserved by a global view and a class of dependencies of local relational databases. Here, any global view is assumed to be constructed by join, projection and attribute-renaming operations. A method is given for testing whether or not an embedded join dependency is preserved by a global view, where the global view is created by only join operations, and only FDs and JDs are given to each local database. By utilizing this testing method, we can examine whether or not a given FD or a JD is preserved by a global view, which is constructed by join plus projection operations. We also discussed a method to remedy the violation of embedded join dependencies by renaming attributes. Our renaming plan makes an EJD (which was not preserved) to be preserved by renaming as few attributes as possible.

Recently, Klug\textsuperscript{12} showed a method for calculating constraints (functional dependency and some non-dependency constraints) on relational expressions. The difference of our results from Klug's result is that we showed a technique for testing the preservability of not only functional but also embedded join dependencies. Furthermore, we also showed that the preservability problem of dependency constraints is important to create a logical scheme of a distributed database. As described in INTRODUCTION, our results will be useful to construct a heterogeneous distributed database since our approach is the bottom-up design approach. Recently, Motro et al.\textsuperscript{17} and Masunaga\textsuperscript{18} discussed a tool for integrating
(heterogeneous) independent databases into a global view. They suggested that Smith's generalization and aggregation concept is useful to construct a global view. The relationship between their higher level abstraction facility and our dependency integration technique should be investigated as a future problem since their combination will provide a useful 'database integration tool'.

The results in this paper are useful not only to handle the logical integration problem, but also to discuss the dependency equivalence of two centralized relational database schemes. Indeed, our study is motivated from the problem to test whether or not two relational database schemes can enforce the same dependency constraints, under the non-existence of the pure universal relation assumption. This problem was partially treated in our previous work. In this paper, we provide further results to cope with this problem.

Our testing method in this paper can be simplified (Step 4 can be eliminated). If we obtain a row with all distinguished variables in necessary columns for each tableau during the chase and interaction process, we can conclude that a given EJD is preserved and can terminate. It will be necessary to find more efficient algorithms to test the preservability of dependencies.

ACKNOWLEDGEMENTS

The authors are grateful to Professor Yajima and the colleagues in Yajima Laboratory at Kyoto University for their helpful discussions. This work is partly supported by the Science Foundation Grant of the Ministry of Education, Science and Culture of Japan. The work of K.Tanaka is also partly supported by Sakko-kai Foundation.

REFERENCES


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Fig.1 Example local relational databases $r_1$ and $r_2$. 
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Fig. 2 A current instance $r_1 \times r_2$ for a global join scheme $R_1 \times R_2$.

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Fig. 3 A current instance $r_1 \times r_2$ for a global join scheme $R_1 \times R_2$, where $R_1 = \{\text{DEPT, RES, COM, DATE}\}$ and $R_2 = \{\text{DEV, COM, SOFT, LAN}\}$. The EJD $*\{(\text{DEPT,RES}),\{\text{DEPT,COM,DATE}\}\}$ holds in $r_1 \times r_2$. 
Fig. 4 Relationships among a class of specified dependencies, a class of actually preserved dependencies and the constraints of local relational databases.
(Here, + denotes a closure of a set of FDs and EJDS.)
(a) Testing the preservability of $\{\text{ABC, AD}\}$ by $R_1 \bowtie R_2$.

Step 1: Initial tableau $T$

Step 2: Project $T$ onto $R_1$ and $R_2$.

Step 3: Chase and interactions.

Apply FD-rule by $A \rightarrow B$ to $T_1$.

Apply JD-rule by $\{\text{AC, AD}\}$ to $T_2$.

Step 4: Join of tableaux.

The tuple $(a_1, a_2, a_3, a_4)$ is contained.

(b) Testing the preservability of $\{\text{ABD, BC}\}$ by $R_1 \bowtie R_2$.

Initial tableau $T$

Step 3: Chase and interactions.

Apply JD-rule by $\{\text{BC, AB}\}$ to $T_1$.

No FD-rule, JD-rule or interaction can be applied to $T_2$.

No interaction.

The tuple $(a_1, a_2, a_3, a_4)$ is not contained.

(c) Ordinary chase process using the result by Beeri and Rissanen.

T

Apply EJD-rule by $\{\text{BC, AB}\}$.

Apply EJD-rule by $\{\text{AC, AD}\}$.

Apply JD-rule by $\{\text{ABC, ACD}\}$.

Fig.5 Testing the preservability of EJDs for the GJS $R_1 \bowtie R_2$, where $R_1 \bowtie \text{ABC}$ and $R_2 \bowtie \text{ACD}$.
**T**: Initial Tableau

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**T_1**: Projection

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JD-rule by *[BCDE,BCFG]*

**T_1’**:

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When BC + E holds in \( R_1 \), then we obtain \( T_1'' \) as follows:

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(In this case, \( b_6 \) in \( T_2 \) is also replaced by \( a_5 \), as shown in (a).)

**T_2**: Projection

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(a) BC + E holds in \( R_1 \).

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(b) C + G holds in \( R_2 \).

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(c) *[CE,CG]* holds in \( R_2 \).

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<td>b_1  a_3  a_5  b_3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b_4  a_3  b_6  a_7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b_7  a_3  a_5  a_7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6 Conditions for the EJD *[BCDE,BCFG]* to be preserved by \( R_1 \ast R_2 \).
**Fig. 7 (a)** Preservability of *[AB, ACD]*.

**Fig. 7 (b)** Preservability test of *[AB, ACD]* after renaming attribute B of R₂ into B'.

**Fig. 7 (c)** Preservability test of *[BC, BDE]*.

**Fig. 7 (d)** Preservability test of *[B'C, B'DE]*.

**Fig. 7 (e)** An example to show the necessity of taking a minimum cover of (5) in our renaming plan.

---

### Diagrams

#### (a) Preservability of *[AB, ACD]*
- \( T_1: \) \( \begin{array}{cc}
A & B \\
\hline
a_1 & b_1 \\
\end{array} \)
- \( T_2: \) \( \begin{array}{cc}
A & B \\
\hline
a_1 & b_2 \\
\end{array} \)

---

#### (b) Preservability test of *[AB, ACD]* after renaming attribute B of R₂ into B'
- \( T_1: \) \( \begin{array}{cc}
A & B' \\
\hline
a_1 & b_1 \\
\end{array} \)
- \( T_2: \) \( \begin{array}{cc}
A & B \\
\hline
a_1 & b_2 \\
\end{array} \)

---

#### (c) Preservability test of *[BC, BDE]*
- \( T_1: \) \( \begin{array}{cc}
A & B \\
\hline
a_1 & b_1 \\
\end{array} \)
- \( T_2: \) \( \begin{array}{cc}
A & B \\
\hline
a_1 & b_2 \\
\end{array} \)

---

#### (d) Preservability test of *[B'C, B'DE]*
- \( T_1: \) \( \begin{array}{cc}
A & B \\
\hline
a_1 & b_1 \\
\end{array} \)
- \( T_2: \) \( \begin{array}{cc}
A & B \\
\hline
a_1 & b_2 \\
\end{array} \)

---

#### (e) Example of minimum cover necessity
- \( T_1: \) \( \begin{array}{cc}
A & C \\
\hline
a_1 & b_1 \\
\end{array} \)
- \( T_2: \) \( \begin{array}{cc}
A & C \\
\hline
a_1 & b_2 \\
\end{array} \)
- \( T_3: \) \( \begin{array}{cc}
A & D \\
\hline
a_1 & b_1 \\
\end{array} \)
- \( T_4: \) \( \begin{array}{cc}
A & F \\
\hline
a_1 & b_1 \\
\end{array} \)

---

**An initial tableau to test the preservability of *[ABG, CEG]*.**

---

**Fig. 7**