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Recent development of differentiable dynamical systems in Japan

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§1. Introduction

We shall give a brief survey on recent development of differentiable dynamical systems in Japan. The works surveyed in this article were published mostly between 1970 and 1980 by Japanese mathematicians and scientists.

The objects of the works surveyed in this article are mostly limited to the qualitative (topological) theory of differentiable dynamical systems and its closely related topics. We do not intend to be complete, but we would be very grateful if the reader would kindly point out the works left unmentioned in this article.

The general references for this field of research are the followings: D.V. Anosov and Ja.G. Sinai [2], R. Bowen [17], Ja.G. Sinai [122], and S. Smale [124].

§2. Stability and generic properties

The notion of structural stability was first introduced by A. Andronov and L. Pontrjagin in 1937. Study of structural stability was continued by Lefschetz's school. In 1960, M. Peixoto obtained the necessary and sufficient conditions for the smooth flows on 2-dimensional compact differentiable manifolds to be structurally stable generalizing the result of Andronov-Pontrjagin. He also obtained that the set of all structurally stable flows

of class C^1 on a 2-dimensional compact differentiable manifold is dense in the set of all C^1-flows on it in C^1-topology. Based on this fact, S. Smale introduced the notion of Morse-Smale system and started to begin the study of differentiable dynamical systems. Morse-Smale systems exist on any compact differentiable manifolds.

In 1962, D.V. Anosov introduced the notion of Anosov system and proved its structural stability. Using this result, he succeeded in proving the structural stability of geodesic flows on negatively curved manifolds. Anosov systems exist only on the very restricted manifolds. In 1970, J. Palis and S. Smale proved that Morse-Smale systems were structurally stable.

These results are unified and generalized as follows.

<u>Theorem</u> (J. Robbin, 1971; R.C. Robinson, 1974) Axiom A systems with strong transversality conditions are structurally stable.

Since Anosov systems and Morse-Smale systems satisfy Axiom A and strong transversality condition, the above theorem is a generalization of Anosov and Palis-Smale. Also, the following is proved.

<u>Theorem</u> (S. Smale, 1970; C. Pugh and M. Shub, 1970) Axiom A systems with no cycle condition are Ω-stable.

Converse of the above two theorems is conjectured and investigated by S. Smale, J. Palis, S. Newhouse, R. Mañé, V.A. Pliss, S.T. Liao, and others. Recently, A. Sannami succeeded in proving the converse problems for C^2-diffeomorphisms on 2-dimensional closed manifolds.

In 1966, Smale showed that the set of all structurally stable systems was not dense in the set of all dynamical systems in higher dimensions. Many tried to find a suitable class of dynamical systems which is dense in the set of all dynamical systems and is stable in some sense. Toward this problem, G. Ikegami introduced the notion of weak stability and studied it in [33], [34], and [36]. The set of all weakly stable systems properly contains the set of all structurally stable systems in general, but it is not dense in the set of all dynamical systems. Under Axiom A, weak stability implies structural stability.

Generic properties for dynamical systems are very important in the study of dynamical systems. A generic property holds for almost all dynamical systems. The first important and basic generic properties were obtained by I. Kupka and S. Smale in 1963. C. Pugh's closing lemma obtained in 1967 implies that Axiom A(b) is a generic property, where Axiom A(b) states that the set of all periodic points is dense in the nonwandering set.

T. Koike [60] says that the following is generic: If M is a 2-dimensional closed manifold and $f : M \to M$ is a diffeomorphism, then the interior of the nonwandering set of f is empty or f is an Anosov diffeomorphism. As a corollary to this, the following is generic: If M is a 2-dimensional manifold and is not a torus, then a diffeomorphism $f : M \to M$ has empty interior for the nonwandering set.

Y. Togawa [128], [129] show that the following properties are generic: (i) An Axiom A diffeomorphism has only trivial centralizers. (ii) A diffeomorphism has no k-root for any integer

k, $k \neq \pm 1$. As a corollary to (ii), we cannot generically imbed a diffeomorphism into a smooth flow.

§3. Anosov systems

D.V. Anosov [1] gives basic results on Anosov systems. Many important results concerning the ergodic properties of Anosov systems were obtained including Ja.G. Sinai's results. Besides these results, J. Moser, J. Franks, S. Newhouse, W.M. Hirsch, A. Manning, and others obtained various important results. The following results are contributions of our colleagues.

K. Shiraiwa [116] gives a necessary condition for the existence of an Anosov diffeomorphism and gives examples of manifolds which do not admit an Anosov diffeomorphism. K. Yano [150] shows that there are no transitive Anosov diffeomorphisms on negatively curved manifolds.

K. Takaki [126] shows that Anosov diffeomorphisms are structurally stable in the space of all lipemorphisms (Lipschitz homeomorphisms). K. Kato and A. Morimoto [53] shows the topological stability of Anosov flows and the triviality of centralizers of Anosov flows. The last property of Kato-Morimoto's result is generalized by M. Oka [93] for expansive flows.

There are other results by N. Otsuki [103], [104] and A. Morimoto [85].

§4. Hyperbolic sets, Axiom A, and related topics

The notion of Axiom A was introduced by Smale to include both Morse-Smale and Anosov systems and to give new examples such as horseshoe sytems. Theory of Axiom A systems plays the most important role in Smale's theory of differentiable dynamical

systems. Many important results are obtained by Smale's school and we need a book to describe them. We give here some results obtained by our colleagues.

M. Kurata [69] states that Axiom A(a) does not imply Axiom A(b) for a diffeomorphism $f : M \to M$ if $\dim M \geqq 4$. This gave a counter example of Smale's conjecture. Similar result was given by A. Dankner [21] independently.

Ja.G. Sinai [121] constructed Markov partitions for Anosov diffeomorphisms, and R. Bowen [15] generalized this for Axiom A diffeomorphisms restricted on their basic sets. A Markov partition of a system gives a semi-conjugacy from a suitable symbolic dynamical system to the system. It plays an important role in both ergodic theory and qualitative theory. M. Kurata [66], [67], [68] investigated hyperbolic sets and obtained Markov partitions of hyperbolic sets.

K. Kato and A. Morimoto [54] generalizes their work [53] and shows that an Axiom A flow with no C^0-Ω-explosion is topologically Ω-stable.

Pseudo-orbit tracing property (sometimes called stochastic stability) is important for the study of dynamical systems (For example, see R. Bowen [16], [17]). K. Sawada [109], A. Morimoto [86] and K. Kato [50] show that an Axiom A diffeomorphism with strong transversality condition has the pseudo-orbit tracing property. Using this result they affirmatively solved Takens' conjecture. Related topics to the above theorem is treated by K. Kato [51], A. Morimoto [87], T. Sasaki [107], K. Yano [147], and N. Aoki [7].

It seems that specification property introduced by R. Bowen and used by R. Bowen, K. Sigmund [120], D.A. Lind [71], and others is very important for both ergodic theory and qualitative theory of dynamical systems. N. Aoki [4], [5], [6], [7], N. Aoki, M. Dateyama, and M. Komuro [11], M. Dateyama [20] investigate the dynamics of the automorphisms of compact metric groups including specification property and obtained many interesting results.

Expansiveness is also important for the study of dynamical systems. Anosov diffeomorphisms and the restriction of Axiom A diffeomorphims on their basic sets are expansive. The followings are works related to expansive homeomorphisms: N. Aoki and M. Dateyama [10], N. Aoki and C. Saikawa [12], A. Koriyama [62], A. Koriyama and Y. Matsuoka [63], A. Koriyama and T. Nagase [64], M. Kouno [65], and M. Oka [93], [94].

§5. Topological entropy

The notion of topological entropy was introduced by R.L. Adler, A.G. Konheim, and M.H. McAndrew in 1965. It is closely related to the measure theoretic entropy. Many important contributions were done by many mathematicians. The followings are by our colleagues.

S. Ito [45] estimated the topological entropy of a C^1-diffeomorphism of a compact Riemannian manifold from above. This is generalized by R. Bowen for a C^1-map on a Riemannian manifold.

K. Sasano [108] investigated the topological entropy of the continuous maps of the circle in detail.

K. Yano [148] shows that the topological entropy of a

homeomorphism of a compact topological manifold of dimension greater than one is generically equal to the infinity. This holds for continuous maps, too.

Other results are as follows: N. Aoki [3], T. Hamachi and H. Totoki [22], M. Hata [23], T. Koike [61], T. Ohno [96], M. Osikawa and T. Hamachi [100], and K. Yano [149].

§6. Chaos and related topics

E.N. Lorenz [72] derived the following system of ordinary differential equation from the convection equation:

$$\begin{cases} x' = -\sigma x + \sigma y \\ y' = rx - y - xz \\ z' = -bz + xy. \end{cases}$$

He studied this equation numerically in the case of $\sigma = 10$, $b = \frac{8}{3}$, $r = 28$ and found that the solutions exhibited chaotic behavior. Later, Y. Ueda and H. Kawakami found the similar result from Duffing's equations. Rössler also found many examples of chaos from systems of ordinary differential equations in dimension three.

In 1974, R.M. May [83] found the similar phenomena for the difference equation of the first order arising from biological populations with nonoverlapping generations. M. Hénon [28] gave a two dimensional mapping exhibiting chaos which is related to both Lorenz model and May model.

Since then many investigations were done by many scientists. We list some of our colleagues' results:

(1) Concerning Lorenz model: J. Nagashima and I. Shimada [88], [89], I. Shimada [112]; I. Shimada and T. Nagashima [113], [114];

K. Tomita and I. Tsuda [133].

(2) Concerning May model: S. Matsumoto [75], [76]. S. Ushiki, M. Yamaguti, and H. Matano [140].

(3) Others: H. Daido [19], K. Ito [44], [44a], T. Kai and K. Tomita [47], K. Tomita and H. Daido [130], K. Tomita and T. Kai [131], K. Tomita and I. Tsuda [132], S. Ushiki [138].

Mathematical treatment for chaos began with the work of T. Li and J. Yorke [70] on the one dimensional mapping. Their result is generalized by F. Marotto [73] to a higher dimensional case. K. Shiraiwa and M. Kurata [119] generalizes both Marotto's result and S. Smale's work [123]. M. Yamaguti and H. Matano [141], M. Yamaguti and S. Ushiki [143] found chaos discretizing some ordinary differential equations.

Related works are as follows: M. Hata [24], S. Ito, S. Tanaka, and H. Nakada [46], Y. Oono [97], Y. Oono and M. Osikawa [98], Y. Oono and Y. Takahashi [99], M. Osikawa and Y. Oono [101], Y. Oshime [102], M. Yamaguti and S. Ushiki [142], [144].

§7. Dynamical systems from electrical networks

There are many interesting dynamical systems arising from electrical networks. R. Brayton and J. Moser gave a basic treatment on this problem, and S. Smale [125] extended and gave modern formulation to this problem.

The followings are a part of the results obtained by the collaboration of our colleagues: L.O. Chua, T. Matsumoto, and S. Ichiraku [18], S. Ichiraku [30], [31], [32]; T. Matsumoto [77], [78], [79]; T. Matsumoto, L.O. Chua, H. Kawakami, and S. Ichiraku [80]; T. Matsumoto, L.O. Chua, and A. Makino [81];

T. Matsumoto and G. Ikegami [82].

Duffing's equations arising from nonlinear oscilations and electrical networks are very interesting dissipative systems to study.

Followings are some of our results C. Hayashi and Y. Ueda [25]; C. Hayashi, Y. Ueda, and H. Kawakami [26]; N. Kakiuchi [48]; H. Kawakami [55], [56], [57]; F. Nakajima [92]; K. Shiraiwa [117], [118]; Y. Ueda [134], [135], [136], [137].

§8. Miscellany

(1) G. Ikegami [33], [34], [38] investigated the relation between diffeomorphisms and their suspension flows.

I. Ishii [42], [43] investigated minimal flows. Other results related to the differentiable dynamical systems are as follws: K. Hayashi [27], F. Ichikawa [29], G. Ikegami [35], S. Matsumoto [74], K. Sawada [110], S. Ushiki [138].

(2) Concerning the limit cycles of planary flows, K. Yamato's work [145] is interesting. G. Ikegami's work [40] is related to this problem.

M. Oka [95] treated difficult problem of classifications of a certain type of homogeneous differential equations on the plane.

Other results related to the ordinary differential equations are J. Kato and F. Nakajima [49], F. Nakajima [90], [91], and G.R. Sell and F. Nakajima [111].

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