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Numerical Calculation of Standing Gravity Waves in Deep Water

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Numerical solutions of inviscid equations that describe standing waves of finite amplitude on deep water are reported. The calculations are performed using a numerical scheme based on the expansions of the wave profile and the velocity potential in the Fourier series. It is suggested that the fast development of higher harmonics leads the breakdown while it does not occur as the wave amplitude is small. The same results are given by the other numerical method (MAC method).

1. Introduction

In this paper we consider standing gravity wave of finite amplitude on deep water. The fluid is assumed to be inviscid and incompressible. An analytical study of this problem was made by Penney and Price (1952). They calculated approximate solution for standing wave using a series expansion in wave amplitude to fifth order. One of the authors, H. Aoki (1980), calculated approximate solution to eighth order by the same method using the formula manipulation language REDUCE-2 (A.C. Hern) on the computer. In these papers they obtained stable highest wave profiles by assuming that the downward acceleration at the crest of the wave of greatest hight is g, where g is gravitational acceleration.

The validity of the arguments of Penney and Price was questioned by Taylor (1953). However, the results of his experiments to determine the highest standing wave profile were found to be consistent with the results of Penney and Price.

Saffman and Yuen (1979) presented numerical calculations of standing waves using a method based on that developed by Longuet-Higgins and Cokelet (1976) which solves the exact free surface unsteady flow problem of an inviscid, irrotational, incompressible fluid with periodic boundary conditions for prescribed initial conditions and external pressure. They show the existence of standing waves with H/L greater than 0.218, Penney and Price's results, on the condition that the downward crest acceleration of the highest stable standing wave is equal to g, where L is the wavelength and H is the peak-to-trough wave-hight.

We transformed the basic equations of the wave with the boundary conditions, all of which are partial differential equations, into the simultaneous ordinary differential equations of the coefficients by which the Fourier expansions of the wave profile and the velocity potential are generated. In analytical investigation these coefficients are also expanded in a power series of wave amplitude.

The basic equations are approximated by the above simultaneous ordinary differential equations to eighth order of the wave amplitude and the time development of the wave profiles is calculated numerically by Runge-Kutta scheme. Our numerical calculations show that if the wave amplitude is sufficiently large, the computation fails because the approximate equations are singular. It corresponds with the physical breakdown of the standing gravity wave. It also suggests that the fast development of higher harmonics occurs when the breakdown of the standing gravity wave occurs.

We also verify the above phenomenon by the other numerical method. This method was first developed by Harlow and Welch (1965) and improved by Hirt and Shannon (1968), Chan and Street (1970), etc.. It is called MAC (Marker-And-Cell) method and is the standard method for solving the free surface unsteady flow problem of viscous, incompressible fluid. We modified MAC method and adapted it to above problem by keeping the viscosity of the fluid small.

2. Numerical results

The basic equations are the eighth order approximate equations which are the simultaneous ordinary equations for Fourier coefficients of the wave profile and the velocity potential. These equations were reported in the other paper of the author (Aoki 1980). The numerical calculations for these equations are carried out by using the fourth order Runge-Kutta scheme. The initial wave profile is determined by the analytical eight mode solution (given by the author) at the time when the wave is at rest. The accuracy of the solutions is verified by computing the period which should coincide with the analytical result when the wave amplitude is small. We show these results in the following table 1.

A	Numerical	Analytical
0.1	6.29108	6.29108
0.2	6.31534	6.31536
0.3	6.35760	6.35787

Table 1.

The periods are calculated numerically and analytically when A=0.1, A=0.2 and A=0.3, where A, the wave amplitude, is very small.

We also show these calculations in Fig.1, Fig.2 and Fig.3 which correspond with the case of A=0.1, A=0.2 and A=0.3 respectively. When the wave amplitude is small, numerical and analytical calculations give the same results approximately. In our calculations the mass flux across the surface is exactly zero since we use the equations of the Fourier coefficients. The gradual increasing of the periods with A is due to the effects of finite amplitude.

Figures 1, 2 and 3 show the time plots of wave profile, which are the stable standing gravity wave.

Fig.4 shows the numerical results for A=0.6. The smooth profiles of standing wave with the large amplitude at the initial time become the oscillating wave profiles with the higher harmonics. Then the equations become singular and the numerical calculations fails. It gives the good agreement with Penney and Price's results and the results of author's analytical calculations. It also coincides with the results of Saffman and Yuen.

We also calculate the above problem by the numerical method

(MAC method) to verify the phenomenon that fast development of higher harmonics causes the breakdown of the gravity standing wave. This method was developed to simulate the unsteady flow problem of viscous, incompressible fluid including free surface. However if we adapt this method in its original form to our problem, the well-shaped initial wave becomes irregular after a few time steps of computation.

Considering that small variations of wave profiles within grid-size are meaningless in finite difference approximation, we used an adequate smoothing method (e.g. the method of least squares) to get reasonable results.

Our numerical calculations are shown in Fig.5, Fig.6 and Fig.7 by the MAC method and smoothing. These results also give the same rusults that the standing gravity waves are stable when the amplitude of the gravity waves is small. Fig.5 and Fig.6 show this results. They correspond the case of A=0.43 and A=0.53 respectively. Fig.6 shows that the wave profile with the higher harmonics appears when the amplitude of the gravity waves is large. However we can continue to calculate this wave corresponding to A=0.58 without the breakdown if the strong smoothing method is applied to this problem. It is shown in Fig.8. Therefore we can not give the greatest stable gravity standing wave accurately by the present methods (MAC method and smoothing method).

3. Conclusion

Numerical solutions of the inviscid equations that describe standing gravity waves of finite amplitude on deep water are given. It is shown that the fast development of higher harmonics in the Fourier series of the wave profile and the velocity potential cause the breakdown.

Numerical calculations were carried on Hitac 8800/8700 system and Hitac M-200H system in Computer Center of University of Tokyo.

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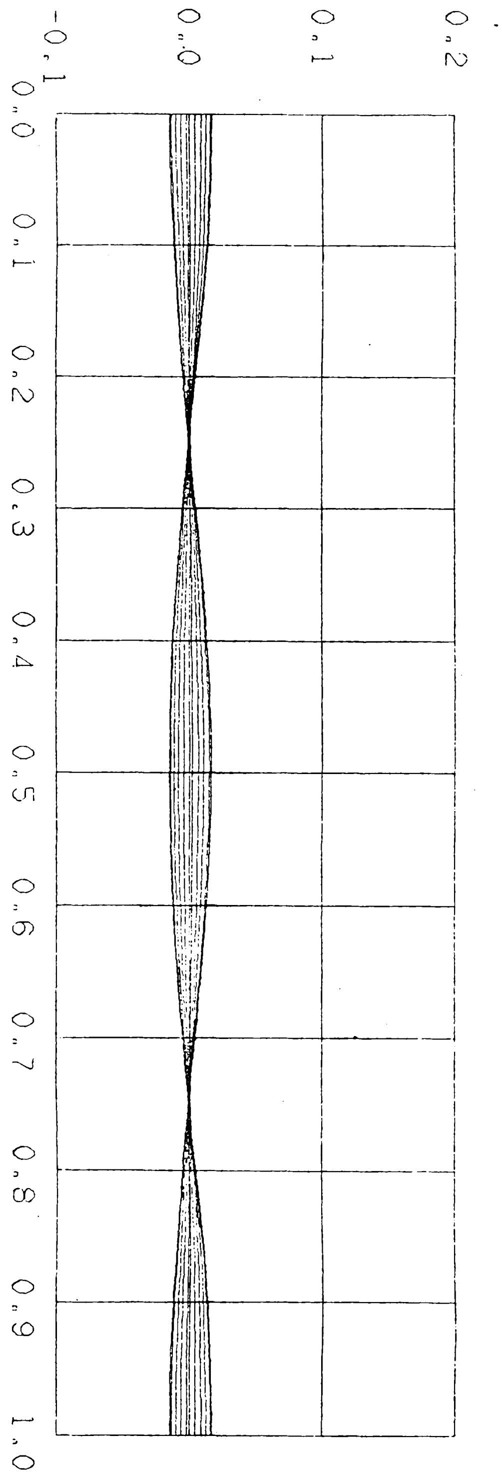


Fig. 1

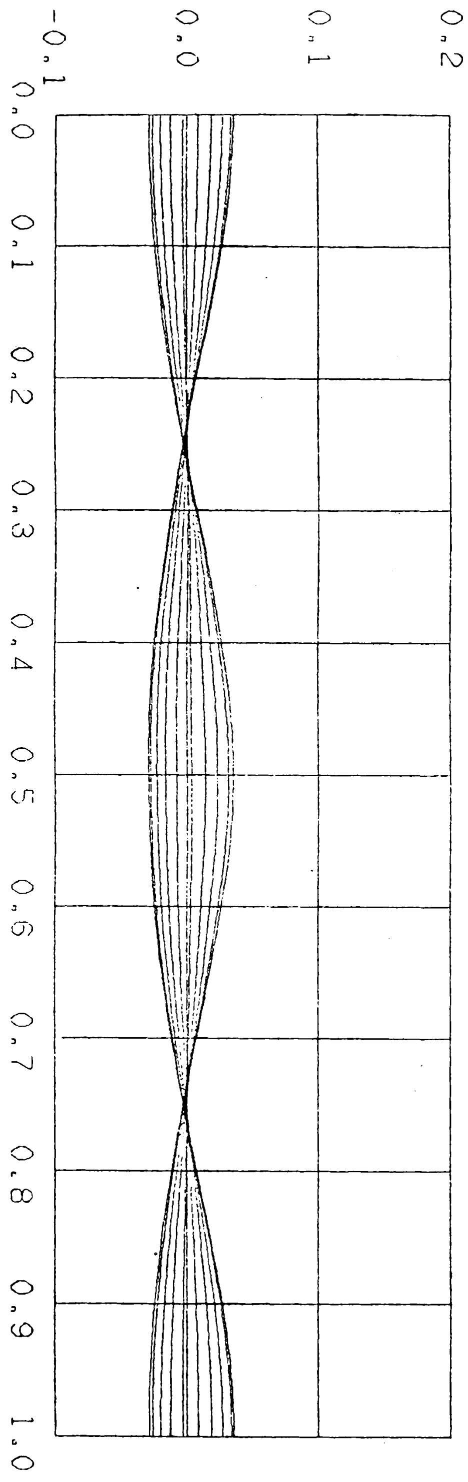


Fig. 2

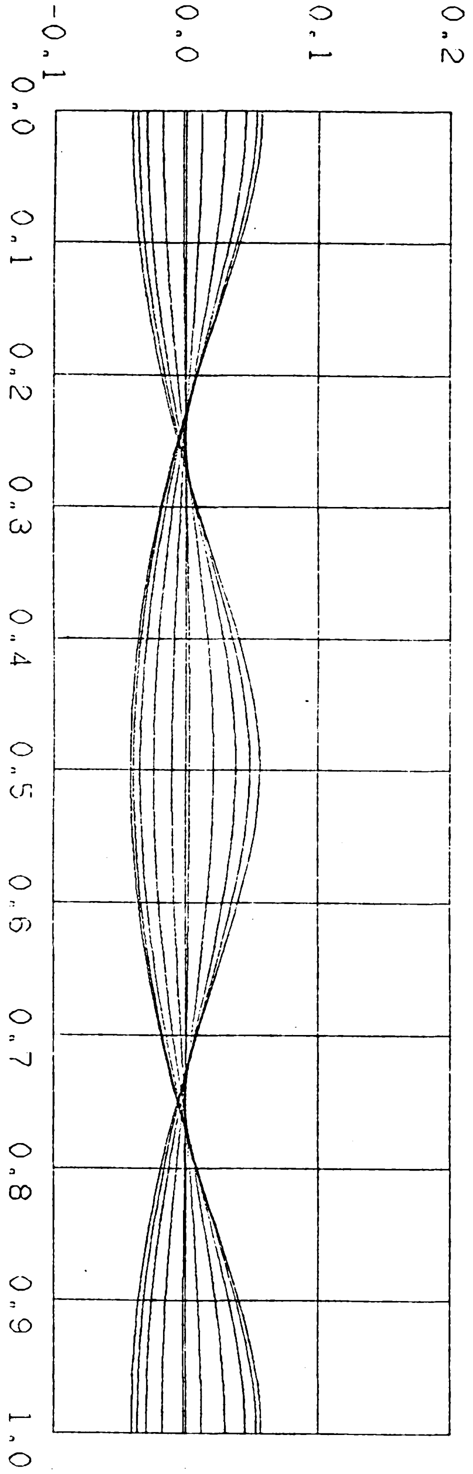


Fig. 3

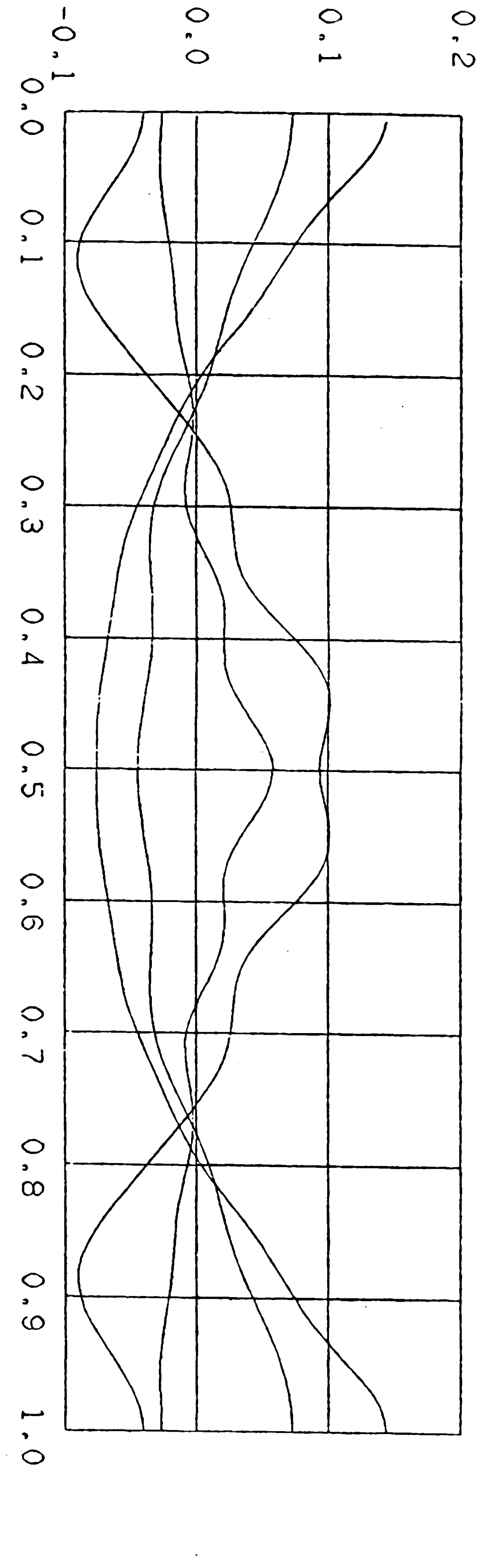
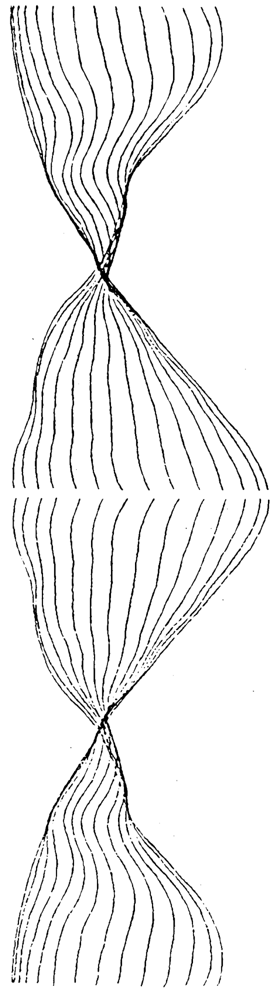
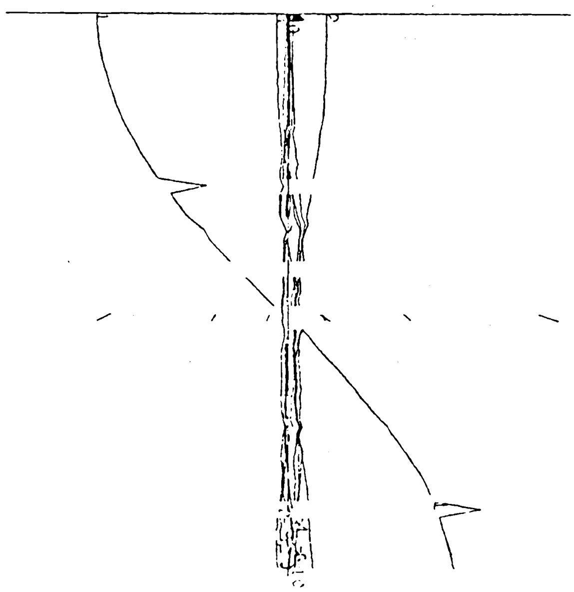


Fig. 4



A=0.43

Fig. 5



A=0.53

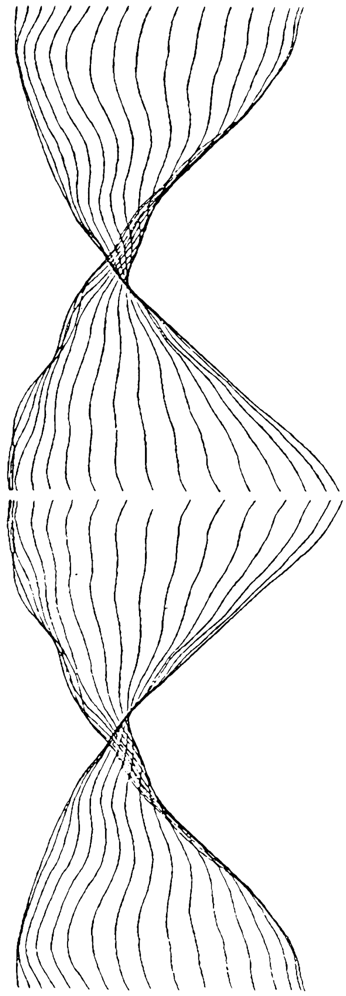
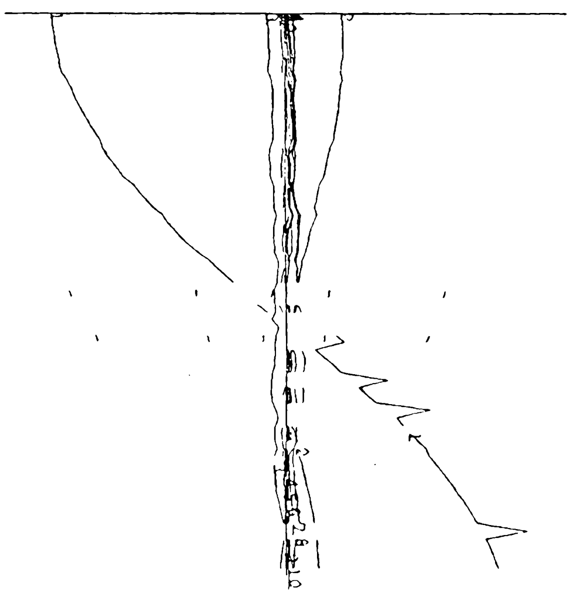
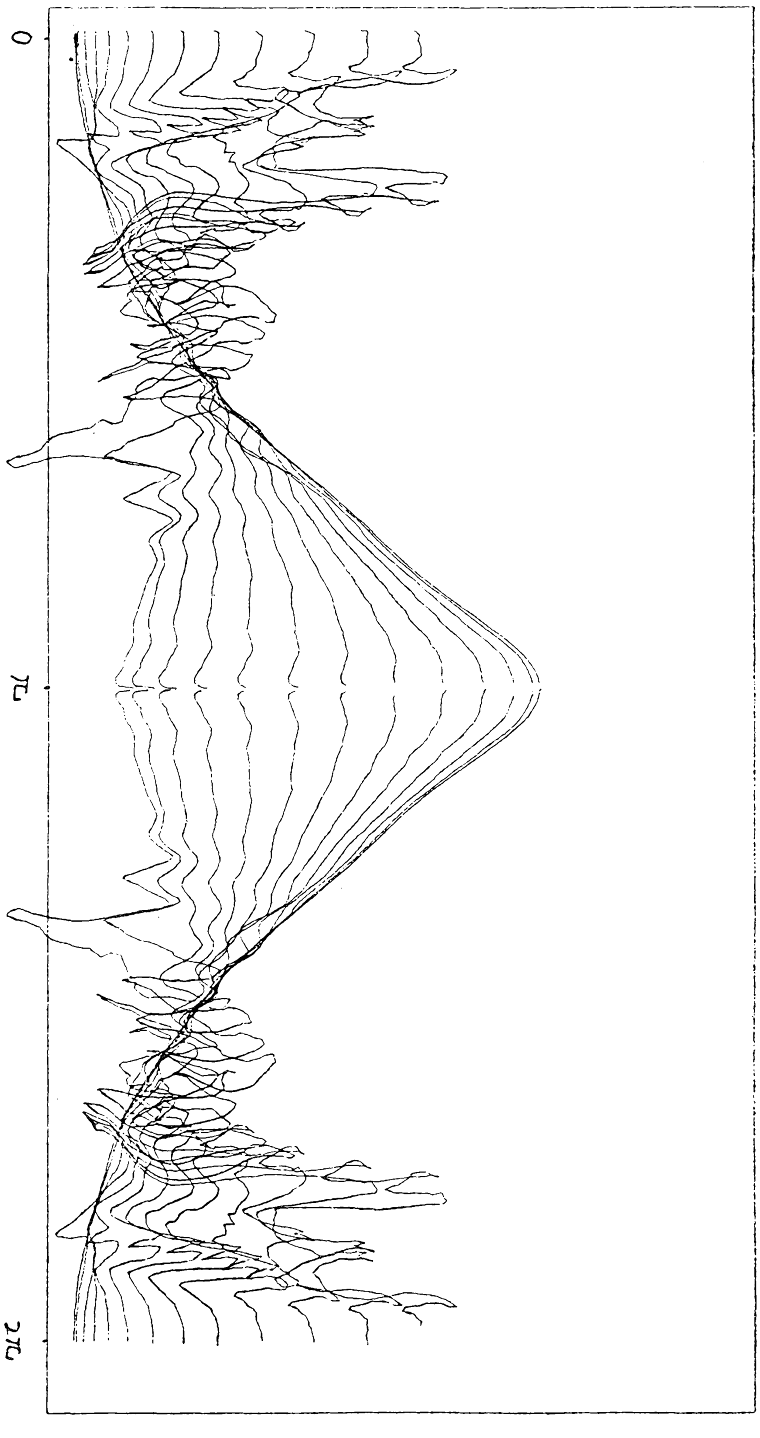


Fig. 6





A = 0.58
ν = 0.001

Fig. 7

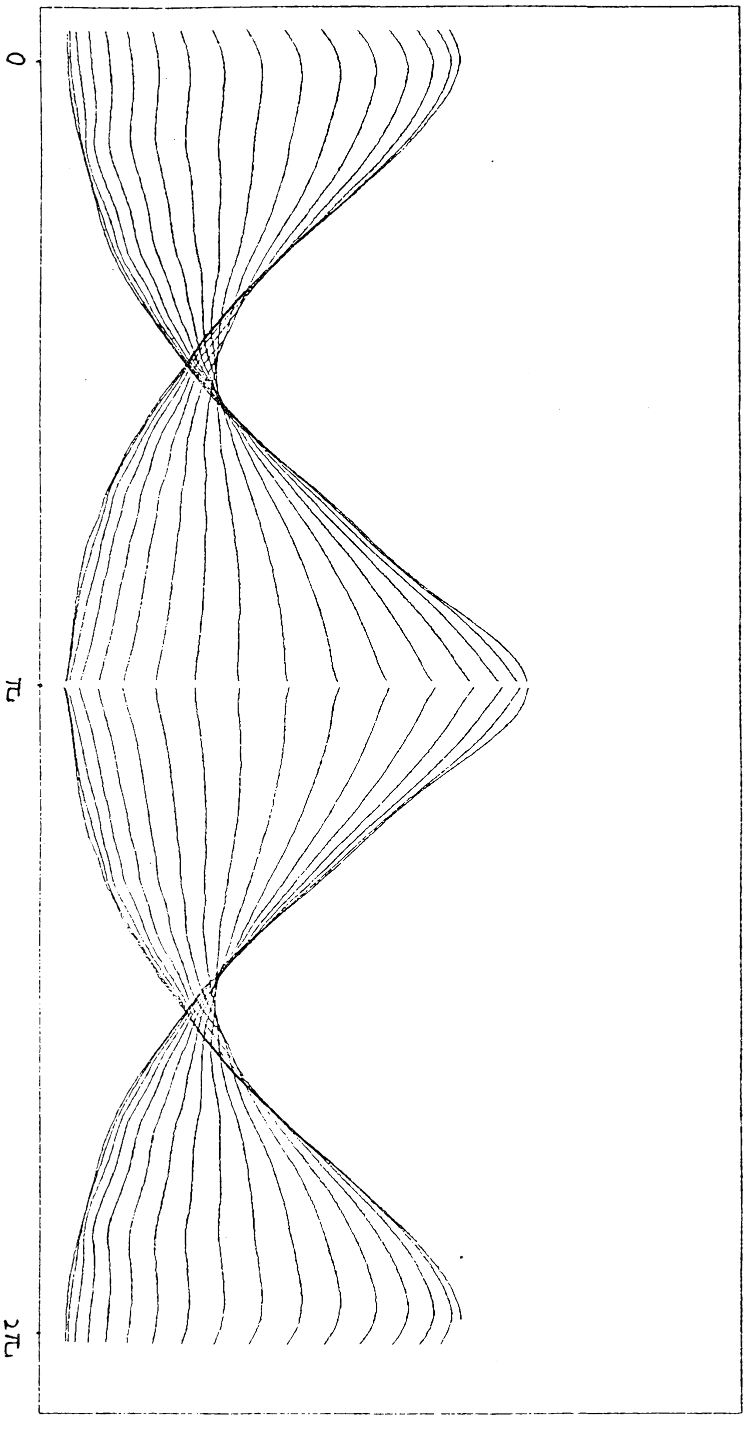


Fig. 8

A = 0.58

ν = 0.001