

Problems

[1] Let $f(s)$ be a Dirichlet series absolutely convergent in some half-plane $\sigma > \sigma_a \geq 0$ and satisfy the functional equation

$$f(s) = f(-s)$$

($f(s)$ is defined in some manner in the strip $0 \leq \sigma \leq \sigma_a$). Can one prove that $f(s)$ equals identically a constant under some suitable conditions?

[2] Let $\Phi(z) = \sum_{n=1}^{\infty} a_n \exp(2\pi i n z)$ be a cusp form of weight k , and let $L_{\Phi}(s, \chi)$ be the Dirichlet-Hecke series associated with Φ . Does there exist a Dirichlet character χ for which $L_{\Phi}(\frac{k}{2}, \chi)$ does not vanish? There are many known results about the value $L_{\Phi}(\frac{k}{2} + it, \chi)$ when $|t|$ is large (see, e.g. A. Good, J. Number Theory 13 (1980), 18-65).

[3] We may stress again the question of determining $g(4)$ in 柴林 problem. It should be noted that R. Balasubramanian has recently proved (Hardy-Ramanujan J. 2 (1979), 1-34) that

$$(19 \leq) g(4) \leq 21,$$

where the first inequality has been known.

[4] Let $\sigma_r(n) = \sum_{d|n} d^r$ and for $b > 0, \rho > 0$ put

$$P^{\rho}(x, r, b) := \frac{1}{\Gamma(\rho + 1)} \sum_{n \leq x} (x^b - n^b)^{\rho} \sigma_{-r}(n) - S^{\rho}((\pi x)^b, r, b),$$

where the prime on the summation sign means that if $\rho = 0$ and

$n = x$, then the term $\sigma_{-r}(n)$ is to be halved, and

$$S^0(x, r, b) = \sum_{\xi} \operatorname{Res}_{s=\xi} \frac{\Gamma(s) \pi^{-bs} \zeta(bs) \zeta(bs+r)}{\Gamma(s+\rho+1)} x^{s+r}.$$

Also for $a \in \mathbb{R}$, $k \in \mathbb{N}$ define

$$G_{a,k}(x) := \sum_{n \leq \sqrt{x}} n^a B_k\left(\frac{x}{n} - \left[\frac{x}{n}\right]\right),$$

where $B_k(x)$ is the k -th Bernoulli polynomial. Is there any explicit relation between $P^0(x, r, b)$ and $G_{a,k}(x)$? (There is an asymptotic relation between $P^1(x, r, r)$ and $G_{1-r,2}(x)$ if $-\frac{1}{2} \leq r < 3$, $r \neq 0, 1$.)

(以上 金光 滋)