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GENERALIZED PARENTHESES LANGUAGES AND
MINIMALIZATION OF THEIR PARENTHESES PARTS
(extended abstract)

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1. INTRODUCTION

The parenthesis grammar defined by McNaughton [2] is a context-free grammar $G = (N,K,P,S)$ such that the terminal alphabet $K$ contains a pair of parentheses, say $<$ and $>$, and the production rules are of form

$$A \rightarrow <u>$$

where $A$ is a nonterminal symbol, and $u$ is a word not containing the parentheses $<$ and $>$. Then for parenthesis grammars the equivalence problem was proved to be decidable [2].

The generalized parenthesis language is defined [3] by extending the spirit of parenthesis languages so that it reflects the block structure prevalent in modern programming languages, while preserving the mathematical wealth.

Let $K$ be an alphabet that includes a set

$$\hat{I} = \{ a, \bar{a} \mid a \text{ is in } I \}$$

of parentheses, and $G = (N,K,P,S)$ be a context-free grammar (cfg, for short) such that the production rules in $P$ are of form

$$A \rightarrow au\bar{a}B, \quad A \rightarrow bB, \quad \text{or} \quad A \rightarrow e$$

where $A$ and $B$ are in the nonterminal alphabet $N$, $a$ is in
I, u is a word over $\mathbb{N} \cup K$ not containing symbols in I, and b is in $J = K - \widehat{I}$. (The e stands for the empty word.) Then we call G a generalized parenthesis grammar (gpg, for short), and the language generated thereby a generalized parenthesis language (gpl, for short) over K with the parenthesis part $\widehat{I}$ (or simply, over K[I]).

The class of gpl's so defined has been proved to have nice mathematical features; for example, the equivalence problem for gpg's over K[I] are decidable, and they enjoy various closure properties (under language-theoretic operations in relativized forms, with respect to the 'universal' gpl specified below) [3], [4]. On the other hand, the expressive power of gpl is sufficiently large; for example, it can describe the syntax of ALGOL 60 with five pairs of parentheses, (,), [,], if, then, begin, end, and ',' [5].

In this paper, after a short preliminary in the rest of this section, in section 2 we study relations between regular sets and gpl's, and solve some decision problems affirmatively. In particular, we show that the regularity problem for gpl's is decidable, and that for a given regular set L over K and a set $\widehat{I}$ of parentheses in K, one can decide whether L is a gpl over K[I] or not. In section 3 we apply these results to the study of parenthesis parts of gpl's, resulting in affirmative answers to more general problems. Among others we prove that for a given gpg G over K[I] and a subset I' of I it is decidable whether L(G) is a gpl over K[I'] or not. Thus we can minimalize the parenthesis part of a given gpl. (If the minimalized parenthesis part is empty then the gpl is regular.) In section 4, relations
between gpl's and context-free languages (cfl's, for short) are studied. We give a characterization of cfl's and that of gpl's, both in terms of universal gpl's, regular sets, and projections. We also give a negative answer to the decision problem to ask whether a given cfg generates a gpl or not.

Let \( \hat{I} = \{ a, \bar{a} \mid a \text{ is in } I \} \subseteq K \), and \( J = K - \hat{I} \) as above. Consider the gpg \( G = (\{S\}, K, P, S) \) such that

\[
P = \{ S \to aSaS, S \to bS, S \to e \mid a \text{ is in } I \text{ and } b \text{ in } J \}.
\]

Any gpl over \( K[I] \) is included in the gpl generated by \( G \). We call the language \( L(G) \) the universal gpl over \( K[I] \), and denote it by \( D_{I,J} \). In case of \( J = \phi \), the language equals the Dyck set \( D_I \) over \( \hat{I} \). If \( I = \phi \) then \( D_{I,J} = J^* \). In general, \( D_{I,J} \) is equal to \( \text{Shuffle}(D_I, J^*) \), the shuffle product of \( D_I \) and \( J^* \).

For each element \( w \) of \( D_{I,J} \), the nonnegative integer \( \text{depth}_I(w) \) is defined as follows:

\[
\text{depth}_I(e) = 0,
\]

\[
\text{depth}_I(au\bar{a}v) = \max\{ 1 + \text{depth}_I(u), \text{depth}_I(v) \},
\]

\[
\text{depth}_I(bu) = \text{depth}_I(u).
\]

where \( a \) is in \( I \), \( b \) is in \( J \), and \( u \) and \( v \) are in \( D_{I,J} \).

For a language \( L \) in \( D_{I,J} \), we define

\[
\text{depth}_I(L) = \sup\{ \text{depth}_I(w) \mid w \text{ is in } L \},
\]

which may or may not be finite.

If \( uvw \) is a word in \( D_{I,J} \) then we can write

\[
v = v_0\bar{a}_1v_1\bar{a}_2v_2\ldots\bar{a}_n a_{n+1}v_{n+1}\ldots a_{n+m}v_{n+m}
\]

for some \( a_1, a_2, \ldots, a_{n+m} \) in \( I \), \( v_0, v_1, \ldots, v_{n+m} \) in \( D_{I,J} \), \( n, m \geq 0 \). In this case we will write

\[
|v|_I = \bar{a}_1\bar{a}_2\ldots\bar{a}_n a_{n+1} a_{n+2}\ldots a_{n+m}.
\]

For a language \( L \) in \( D_{I,J} \), we define

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subword \_I \( (L) = \{ v \text{ in } D_{I,J} \mid uvw \text{ is in } L \text{ for some } u,w \} \).

For any word \( w \) in \( D_{I,J} \), we define the word \( \text{surface}_I(w) \) in \( J^* \) as follows: If
\[
w = u_0(a_1v_1\tilde{a}_1)u_1(a_2v_2\tilde{a}_2)u_2...a_nv_n\tilde{a}_nu_n
\]
for some \( n \geq 0, u_0,...,u_n \) in \( J^* \), \( a_1,...,a_n \) in \( I \), and \( v_1,...,v_n \) in \( D_{I,J} \), then
\[
\text{surface}_I(w) = u_0u_1...u_n.
\]

For a language \( L \) in \( D_{I,J} \), we define
\[
\text{surface}_I(L) = \{ \text{surface}_I(w) \mid w \text{ is in } L \}.
\]

We may suppress the suffix \( I \) in these notations when it is clear from the context.

This paper is an extended abstract of [5], and we will omit the proofs of theorems.

2. REGULAR SETS AND GENERALIZED PARENTHESIS LANGUAGES

It has been proved [4] that the class of gpl's over \( K[I] \) is closed under intersection with regular sets, and therefore any regular set included in \( D_{I,J} \) is a gpl over \( K[I] \). In this section we study properties of these regular sets, and give positive answers to some decision problems for gpl's.

Theorem 2.1 If \( L \) is a regular set included in \( D_{I,J} \), then \( \text{depth}(L) \) is finite.

Theorem 2.2 If \( L \) is a gpl over \( K[I] \) and \( \text{depth}(L) \) is finite, then \( L \) is regular.

Corollary 2.3 For a language \( L \) in \( D_{I,J} \) the following three
conditions are equivalent.

(1) \( L \) is a regular set.

(2) \( L \) is a gpl over \( K[I] \), and \( \text{depth}(L) \) is finite.

(3) \( L \) is obtained from subsets of \( J \) by a finite number of applications of regular operations \( U, \cdot, * \), and bracketting by symbols in \( I \) (i.e., \( aXa \) for \( X \), where \( a \) is in \( I \)).

Theorem 2.4 For a given regular expression \( E \) over \( K \) and a set of parentheses \( \hat{I} \) in \( K \), one can decide whether the regular set \( L \) denoted by \( E \) is a gpl over \( K[I] \). If this is the case, one can effectively obtain a gpg over \( K[I] \) to generate the set \( L \).

Note that any regular set in \( K^* \) is a gpl over \( K[\emptyset] \).

Therefore to specify the parenthesis part \( \hat{I} \) in theorem 2.4 is important. From the theorem, for a given regular set \( L \) in \( K^* \), we can effectively list up all the paired subalphabets \( \hat{I} \) of \( K \) such that \( L \) is a gpl with parenthesis part \( \hat{I} \).

Theorem 2.5 Whether a given gpg generates a regular set or not is decidable.

3. ON MINIMALIZATION OF THE PARENTHESIS PART

The regularity problem for gpg's (theorem 2.5) is nothing but to ask whether the parenthesis part of a given gpg can be reduced to the empty set. In this section we consider a more general problem to minimalize the parenthesis part of a given gpg. First
we note a property of the mapping $D_{I,J} \rightarrow J^*$. 

Theorem 3.1 If $L$ is a gpl over $K[I]$, then $\text{surface}(L)$ is a regular set over $K-\hat{I}$.

As a consequence we know that if a gpl $L$ over $K[I]$ is also a gpl over $K[I']$ where $I' \subseteq I$ then $\text{surface}_{I'}(L)$ is a regular subset of $D_{I-I',J}$. The converse of this statement is not true. However we can prove the following.

Theorem 3.2 Let $G = (N,K,P,S)$ be a gpg over $K[I]$, $L_A = \{ w \in K^* \mid A \xrightarrow{*} w \text{ in } G \}$ for each $A$ in $N$, and $I' \subseteq I$. If $\text{surface}_{I'}(L_A)$ is regular for each $A$, then each $L_A$ is a gpl over $K[I']$.

Theorem 3.3 Let $L$ be a gpl over $K[I]$, and $I' \subseteq I$. Then $L$ is a gpl over $K[I']$ if and only if $\text{surface}_{I'}(\text{subword}_{I}(L))$ is regular (i.e., depth$_{I-I'}(\text{surface}_{I'}(\text{subword}_{I}(L)))$ is finite).

Corollary 3.4 Let $G$ be a gpg over $K[I]$, and $I' \subseteq I$. Then it is decidable whether $L(G)$ is a gpl over $K[I']$ or not. If the answer is affirmative, one can effectively obtain a gpg $G'$ over $K[I']$ to generate the language $L(G)$.

As for the expansion of the parenthesis parts of gpl's, we can extend theorem 2.4 as follows.

Theorem 3.5 Let $L$ be a gpl over $K[I]$, $I \subseteq I'$, and $\hat{I}' \subseteq K$. 

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Then $L$ is a gpl over $K[I']$ if and only if $L \subseteq D_{I',J'}$ where $J' = K - \hat{I}'$. For a given gpg $G$ over $K[I]$ and an expansion $I'$ of $I$, one can decide whether the condition is satisfied or not.

If this is the case one can effectively obtain a gpg $G'$ over $K[I']$ to generate the language $L(G)$.

By corollary 3.4 and theorem 3.5, for a given gpg $G$ over $K[I]$ we can list up all restrictions and expansions $I'$ of $I$ such that $L(G)$ is a gpl over $K[I']$. In particular, we can obtain all minimal parenthesis parts for a given gpg $G$ over $K[I]$, i.e., all minimal subsets $I'$ of $I$ such that $L(G)$ is a gpl over $K[I']$.

It is interesting to note that a gpl may have no 'minimum' nor 'maximum' parenthesis part. For instance, consider

$$L = \{ (ab)^i(cd)^i \mid i=0,1,2,\ldots \}.$$ 

The language $L$ is a gpl in various ways; it is a gpl with $\hat{I}_1 \sim \hat{I}_5$ below as the parenthesis part.

$$\begin{align*}
\hat{I}_1 &= \{ a, d \}, \\
\hat{I}_2 &= \{ a, c \}, \\
\hat{I}_3 &= \{ b, c \}, \\
\hat{I}_4 &= \{ b, d \}, \\
\hat{I}_5 &= \{ a, d; b, c \}.
\end{align*}$$

Among these, $\hat{I}_1 \sim \hat{I}_4$ are minimal, while $\hat{I}_2$, $\hat{I}_4$ and $\hat{I}_5$ are maximal. But none of them is the minimum, nor the maximum. At present we do not know any algorithm to get all possible parenthesis parts of a given gpl, or whether one can expect it at all or not.
4. CONTEXT-FREE LANGUAGES AND GENERALIZED PARENTHESIS LANGUAGES

First we note that any gpl is a deterministic cfl, indeed Greibach and Friedman [1] have shown a stronger result that any gpl is a superdeterministic language.

We prove that a language \( L \) is a cfl (or gpl, respectively) if and only if \( L = h(D_{I,J} \cap R) \) for a regular set \( R \) and a projection (or pair preserving projection) \( h \). Finally we prove the undecidability of whether a given cfl is a gpl or not.

Let \( K \) and \( K' \) be alphabets. A homomorphism \( h : K^* \to K'^* \) is said to be a projection if \( h(K) \subseteq K' \). A projection \( h : (\hat{I} \cup J)^* \to (\hat{I}' \cup J')^* \) is said to be pair preserving if \( h(I) \subseteq I' \), \( h(J) \subseteq J' \) and \( h(\bar{a}) = \overline{h(a)} \) for any \( a \) in \( I \).

Theorem 4.1 A language \( L \) is a gpl over \( K[I] \) if and only if \( L = h(D_{I,J} \cap R) \) for some alphabets \( I' \) and \( J' \), a pair preserving projection \( h \), and a regular set \( R \) over \( \hat{I}' \cup J' \).

Theorem 4.2 A language \( L \) is a cfl if and only if \( L = h(L') \) for a gpl \( L' \) over some alphabet \( K[I] \), and a projection \( h \).

Corollary 4.3 A language \( L \) is a cfl if and only if \( L = h(D_{I,J} \cap R) \) for a projection \( h \) and a regular set \( R \) over some alphabet \( K = \hat{I} \cup J \).

Theorem 4.4 For a given cfg whether it generates a gpl or not is undecidable.
REFERENCES


