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楕円形の開孔をもつ平板に沿うShear Flow

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§0 はじめに

隔壁によって境された粘性流体の運動が、隔壁の孔をとおして，他方の領域におよぼす影響、あるいはその反作用としての自身に対する影響は多くの点から興味のあることであるが未だにはっきりしない点が多い。十分に低いレイノルズ数の場合に話を限っても、両側の圧力差による流れ[1-4]以外はくわしい研究は少ない。これまで2次元のばあいについては本講究録においても片側におかれた微少物体の運動[1]やShear Flowに対するスリットの効果,[1,5,12]特に後者では非線型効果が論じられている。また，3次元流では円孔の影響についてストークス方程式の厳密解による研究[11,13-16]が行なわれて来たが、今回はこれを一般化して、楕円型の孔のShear flowに対する影響を論じて見たい。楕円座標をもちいてストークス方程式の厳密解を求め，特に誘導流が上流から孔に向う半径流として入射し、孔では壁に平行に進み，下流では放射流として流出して行く特長がこの場合も現われることを示す。

Low Reynolds Number Shear Flow along an Elliptic Hole in a Wall

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The influence of an elliptic hole (of semi-axes a_1 and a_2, $t = a_2/a_1 \leqq 1$) on the shear flow along a thin solid plane wall is investigated on the basis of the Stokes equation. Exact solution is obtained in terms of ellipsoidal coordinates. If the flow is parallel to the major or minor axis, a radial incoming flow induced upstream along $\pm\pi/4$ direction to the wall moves almost parallel to the axis in the neighbourhood of the hall and is reflected into a radial outgoing flow. The total flux of this flow is found to be $\frac{1}{3}\alpha k^2 t^2 a_1{}^3/c$, where $c = t^2K + (1-2t^2)E$ or $(2-t^2)E - t^2K$, K and E being the complete elliptic integrals with the modulus $k = (1-t^2)^{1/2}$.

§1. Introduction

The effect of a hole in a wall upon the stream along the wall is of interest from many view points. However, even with the case of solid thin screen and with the low Reynolds number approximation, details seem to have missed study except the flows due to pressure difference[1-4] across a circular hole,[1,2] an elliptic hole and general holes.[3,4]

In a previous paper the author[5] investigated the shear flow along a circular hole on the basis of the exact solution of the Stokes equation. Essential feature is the appearance of a radial incoming flow of flux $4\alpha a^3/(9\pi)$ induced upstream reflecting into a radial outgoing flow of the same flux, where α is the rate of shear and a is the radius of the hole. In this paper this study is extended to the case of an elliptic hole in order to study the effect of ellipticity of the hole.

§2. Fundamental Equations and Solutions

We consider a viscous fluid separated by the plane x_3 with an elliptic hole of semi axes a_1 and a_2 directed parallel to the x_1 and x_2 axes in the cartesian coordinates (x_1, x_2, x_3) whose origin is at the centre of the hole. The unit vector parallel to each axis is denoted by $\hat{x}_j (j = 1, 2, 3)$ respectively. (Fig. 1)

Let us assume a uniform shear flow $V = \alpha x_3 \hat{x}_1$ far from the hole in the upper domain $x_3 > 0$ in contrast to the lower one $(x_3 < 0)$ where the fluid is at rest far from the hole. The pres-

sure difference as well as the flux through the hole is assumed to be zero, since the solution is already known[4] and is additive to ours owing to the linearity, on the assumption that the Stokes approximation is valid in the domain under study. Then, it is convenient to subtract the uniform shear flow in the whole field

$$v_0 = \frac{1}{2}\alpha x_3 \hat{x}_1 , \tag{2.1}$$

and solve the boundary value problem for the Stokes equation which is symmetric with respect to $x_3 = 0$ (Fig. 2) :

$$v = V - v_0 \to \frac{1}{2}\alpha |x_3| \hat{x} , \quad \text{as} \quad r \to \infty , \tag{2.2}$$

and

$$v = 0 \quad \text{on the wall} \quad z = 0 \quad \text{and} \quad \sigma < 0 \tag{2.3}$$

where r is the distance from the origin and

$$\sigma = 1 - \frac{x_1^2}{a_1^2} - \frac{x_2^2}{a_2^2} . \tag{2.4}$$

The pressure p may be assumed to be 0 at infinity. If we take into account the symmetry with respect to $x_3 = 0$ and continuity of the velocity and pressure on the hole, we have only to solve the problem only in the upper half plane $x_3 > 0$.

Owing to the symmetry, the velocity v and the pressure p are represented in terms of three harmonic functions Φ_1, Φ_2 and Φ_0 as follows :[6,7]

$$v/\alpha = x_3 \operatorname{grad} \Phi_3 - \Phi_3 \hat{x}_3 + x_1 \operatorname{grad} \Phi_1 - \Phi_1 \hat{x}_1 + \operatorname{grad} \Phi_0 , \tag{2.5}$$

$$\frac{p}{2\mu\alpha} = \frac{\partial\Phi_1}{\partial x_1} + \frac{\partial\Phi_3}{\partial x_3} \quad , \tag{2.6}$$

with

$$\Delta\Phi_j = 0 \quad , \tag{2.7}$$

where Φ_3 is antisymmetric with respect to x_3 in contrast to Φ_1 and Φ_0 which are symmetric :

$$\Phi_3(x_1, x_2, x_3) = -\Phi_3(x_1, x_2, -x_3) \ ,$$

$$\Phi_1(x_1, x_2, x_3) = \Phi_1(x_1, x_2, -x_3) \ ,$$

$$\Phi_0(x_1, x_2, x_3) = \Phi_0(x_1, x_2, -x_3) \ . \tag{2.8}$$

These functions can be determined from the boundary conditions at infinity in the upper half domain :

$$\Phi_3 \to A_3 x_1 \ , \quad \Phi_1 \to A_1 x_3 \ , \quad \Phi_0 \to A_0 x_1 x_3 \quad \text{as} \quad r \to \infty \ ,$$

$$\text{i.e.} \quad A_3 = A_1 + A_0 \ , \quad A_0 = \frac{1}{4} \ , \tag{2.9}$$

as well as the condition at $x_3 = 0$:

$$\Phi_3 = \partial\Phi_1/\partial x_3 = \partial\Phi_0/\partial x_3 = 0$$

$$\text{on the hole} \quad x_3 = 0 \ , \quad \sigma > 0 \ , \tag{2.10}$$

$$\Phi_1 = \Phi_0 = 0 \quad \text{on the wall} \quad x_3 = 0 \ , \quad \sigma < 0 \tag{2.11}$$

and

$$\Phi_3 = x_1 \frac{\partial\Phi_1}{\partial x_3} + \frac{\partial\Phi_0}{\partial x_3} \quad \text{on the wall} \quad x_3 = 0 \ , \quad \sigma < 0 \ , \tag{2.12}$$

where the constant A_3 is to be determined later. The condition (2.9) yields the shear flow (2.2) at infinity, (2.10) guarantees the symmetry and continuity on the hole and (2.11) as well as (2.12) corresponds to vanishing of the tangential and the normal velocity on the wall.

§3. Determination of Unknown Functions

Let us introduce the ellipsoidal coordinates $(\lambda_1, \lambda_2, \lambda_3)$ (Fig. 2) : 後出

$$x_i^2 = \prod_{j=1}^{3}(\lambda_j + a_i^2)/\prod_{j\neq i}'(a_i^2 - a_j^2)$$

$$\text{with} \quad a_3 = 0 \ , \quad \text{and} \quad \lambda_1 \geqq 0 \geqq \lambda_2 \geqq -a_2^2 \geqq \lambda_3 \geqq -a_1^2 \ . \tag{3.1}$$

Here the hole $(\sigma > 0)$ and the wall $\sigma < 0$ at $x_3 = 0$ correspond respectively to $\lambda_1 = 0$ and $\lambda_2 = 0$; the surfaces $\lambda_1, \lambda_2, \lambda_3$ (= const.) being ellipsoids, hyperboloids of one sheet and hyperboloids of two sheets, respectively. We may also note the relations

$$r^2 = \sum_{i=1}^{3} x_i^2 = \sum_{j=1}^{3}(\lambda_j + a_j^2) , \tag{3.2}$$

$$\frac{\partial x_i}{\partial \lambda_j} = \frac{1}{2}\frac{x_i}{a_i^2 + \lambda_j} \tag{3.3}$$

$$\frac{\partial \lambda_j}{\partial x_i} = h_j^2 \frac{\partial x_i}{\partial \lambda_j} \tag{3.4}$$

with

$$h_j^{\,2} = 4[\Delta(\lambda_j)]^2 / \prod_{i \neq j}{}' (\lambda_j - \lambda_i) \tag{3.5}$$

and

$$\Delta(\lambda_j) = \prod_{i=1}^{3} (\lambda_j + a_i)^{\frac{1}{2}} \tag{3.6}$$

which yield

$$\lambda_1^{-1/2} \frac{\partial \lambda_1}{\partial x_3} = 2\sigma^{-\frac{1}{2}} \quad \text{for} \quad \lambda_1 = 0 \tag{3.7}$$

i.e. on the hole and

$$\frac{1}{x_3} \frac{\partial \lambda_1}{\partial x_3} = 2 \frac{\delta(\lambda_1)}{\lambda_1 - \lambda_3} \quad \text{for} \quad \lambda_2 = 0 \tag{3.8}$$

i.e. on the wall, where

$$\delta(\lambda) = (\lambda + a_1^2)^{\frac{1}{2}} (\lambda + a_2^2)^{\frac{1}{2}} \quad . \tag{3.9}$$

The result of the previous paper with reference to Lamb's treatise[8] suggests the following fact :

i) Φ_3 is the electric potential due to the conducting elliptic disk $\lambda_1 = 0$ placed in the uniform field parallel to the x_1 axis.

ii) Φ_1 is the velocity potential due to the disk $\lambda_1 = 0$ placed in the uniform flow of a perfect fluid parallel to the x_3 axis.

iii) Φ_0 is related to the velocity potential due to the disk $\lambda_1 = 0$ rotating about the x_2 axis.

It is fortunate to notice that such harmonic functions are given in Lamb's treatise.[8] Rearranging them so as to satisfy the conditions (2.8)-(2.11), we have

$$\Phi_3 = A_3 x_1 [1 - \beta_3 \phi_1(\lambda_1)] \tag{3.10}$$

$$\Phi_1 = A_1 x_3 [1 - \beta_1 \phi_3(\lambda_1)] \tag{3.11}$$

$$\Phi_0 = A_0 x_1 x_3 [1 - \beta_0 \{\phi_1(\lambda_1) - \phi_3(\lambda_1)\}] \tag{3.12}$$

where

$$\phi_j(\lambda) = \int_\lambda^\infty \frac{d\lambda}{(a_j^2 + \lambda)\Delta(\lambda)} \tag{3.13}$$

and β_j $(j = 3, 1, 0)$ are constants:

$$\beta_3 = \frac{1}{\gamma_1}, \qquad \beta_1 = -\frac{1}{\gamma_1 + \gamma_2}, \qquad \beta_0 = \frac{1}{2\gamma_1 + \gamma_2} \tag{3.14}$$

with

$$\gamma_j = \phi_j(0) \ . \tag{3.15}$$

We have made use of the relations at $\lambda_1 = 0$:

$$x_3 \phi_3(\lambda_1) = 2\sigma^{1/2}/\delta(0) = 2\sigma^{1/2}/(a_1 a_2)$$

$$\text{and} \quad \partial(x_3\phi_3)/\partial x_3 = -\gamma_1 - \gamma_2 \tag{3.16}$$

as well as (3.7). It should be noted that γ_j are reduced to the complete elliptic integrals :

$$\gamma_1 = \frac{2}{a_1^3} \frac{1}{k^2} [K(k) - E(k)] \tag{3.17}$$

and

$$\gamma_2 = \frac{2}{a_1^3} \frac{1}{k^2 k'^2} [E(k) - k'^2 K(k)] \tag{3.18}$$

with

$$k^2 = 1 - t^2 , \quad k'^2 = t^2 \tag{3.19}$$

where t is the thickness ratio of the ellipse :

$$t = a_2/a_1 . \tag{3.20}$$

The constant A_1 can be determined by the remaining boundary condition (2.12) for $\lambda_2 = 0$ by noting (3.8). We have

$$A_3(1 - \beta_3\phi_1) = A_1(1 - \beta_1\phi_3) + A_0[1 - \beta_0(\phi_1 - \phi_3)] \tag{3.21}$$

with (2.9), which yields

$$A_3 = \frac{\beta_0}{\beta_3} A_0 = \frac{\gamma_1}{2\gamma_1 + \gamma_2} A_0$$

and

$$A_1 = \frac{\beta_0}{\beta_1} A_0 = - \frac{\gamma_1 + \gamma_2}{2\gamma_1 + \gamma_2} . \tag{3.22}$$

Introducing into (3.10)-(3.12), we have finally :

$$\Phi_3 = B[\gamma_1 - \phi_1(\lambda_1)]x_1$$

$$\Phi_1 = - B[\gamma_1 + \gamma_2 + \phi_3(\lambda_1)]x_3$$

and

$$\Phi_0 = B[2\gamma_1 + \gamma_2 - \{\phi_1(\lambda_1) - \phi_3(\lambda_1)\}]x_1x_3$$

$$\text{with} \quad B = \frac{A_0}{2\gamma_1 + \gamma_2} \quad \text{and} \quad A_0 = \frac{1}{4} \tag{3.23}$$

The velocity $V = \frac{1}{2}\alpha x_3 \hat{x}_1 + v$, in conjunction with (2.5), (2.8) and (3.23) forms the solution of our problem. We may notice that the expression for the pressure (2.6) is simple ; we find

$$\frac{p}{4\mu\alpha} = \frac{\partial\Phi_3}{\partial x_3} = \frac{\partial\Phi_1}{\partial x_1} = \frac{2B\,\Delta(\lambda_1)}{\lambda_1(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}\,\frac{x_1|x_3|}{\lambda_1+a_1^{\,2}} \tag{3.24}$$

which is positive for $x_1 > 0$ and is negative for $x_1 < 0$ (see (3.1), (3.6) and (3.23)).

§4. Several Features of the Flow

Let us consider the behaviour of the induced velocity v and pressure for $z = 0$ and $r \to \infty$.

i) Velocity and pressure on the hole

Putting $\lambda_1 = 0$ in (2.5), (3.23) and (3.24), we have on the hole ($z = 0$, $\sigma > 0$)

$$V = x_1 \operatorname{grad}\Phi_1 - \Phi_1\hat{x}_1 + \operatorname{grad}\Phi_0 = (4B\sigma^{1/2}/a_1a_2)\hat{x}_1 \tag{4.1}$$

$$p/8\mu\alpha = B\,x_1/(a_1^{\,3}a_2\,\sigma^{1/2}) \tag{4.2}$$

where use has been made of (3.13), (3.16) and (3.1).

The velocity on the hole is parallel to the direction $\hat{x}$ of the main flow and is maximum at the centre where $\sigma = 1$, the isovels being similar ellipses to the hole. The pressure is infinite but integrable on the rim where $\sigma = 0$. The isobars are ellipses inscribed in the hole with the common major axis

along $x_1 = 0$. The tagential stress $\mu\,\partial\mathbf{v}/\partial z$ is found to be zero except the stress $\mu\alpha\hat{x}$ due to the unperturbed flow.

ii) Shear stress on the wall

On the wall $\lambda_2 = 0$, $\sigma \leqq 0$ we have (3.8) and

$$\frac{\partial\lambda_i}{\partial x_j} = 2\,\frac{\lambda_1 + a_i^2}{\lambda_1 - \lambda_3}\,x_j \quad (j = 1,\ 2;\ i \neq j,\ 3), \qquad \frac{\partial\lambda_1}{\partial x_3} = 0 \tag{4.3}$$

and

$$\partial\Phi_1/\partial x_3 = -B[\gamma_1 + \gamma_2 + \phi_3(\lambda_1)] \tag{4.4}$$

The velocity and the pressure (3.24) are zero there. The tangential stress $\mu\,\partial\mathbf{v}/\partial x_3$ expressed by

$$2\mu\left(\frac{\partial\Phi_3}{\partial x_1} - \frac{\partial\Phi_1}{\partial x_3},\ \frac{\partial\Phi_3}{\partial x_2},\ 0\right) \tag{4.5}$$

yields a usual singularity $2\mu B(-\sigma)^{-1/2}$ on the rim $\sigma = 0$ and tends monotonically to $\pm\mu\alpha/2$ due to the external flow $\alpha|x_3|\hat{x}_1/2$.

iii) The field at infinity

Letting $\lambda_1 \to \infty$ we can find the asymptotic flow at infinity $\gamma \to \infty$(see (3.1) and (3.2)). For this purpose, the following expressions are useful

$$\phi_1(\lambda) = \frac{2}{3}\lambda^{-\frac{3}{2}} - \frac{2}{5}(2a_1^2 + a_2^2)\lambda^{-\frac{5}{2}} + \cdots$$

$$\phi_3(\lambda) = \frac{2}{3}\lambda^{-\frac{3}{2}} - \frac{2}{5}(a_1^2 + a_2^2)\lambda^{-\frac{5}{2}} + \cdots$$

and

$$\lambda_1 = r^2 + o(1) \tag{4.6}$$

Introducing (4.6) into (3.23), (2.5) and (3.24) we have

$$v = \frac{1}{2}\alpha|z|\hat{x} + \tilde{v} ,$$

with

$$\frac{\tilde{v}}{\alpha} = 4B\frac{x_1|x_3|}{r^4}\hat{r} + o(\frac{1}{r^4}) \doteq 2B\frac{|\sin 2\theta|\cos\omega}{r^2}\hat{r} , \tag{4.7}$$

and

$$\frac{p}{\mu\alpha} = 8B\frac{x|x_3|}{r^5} + o(\frac{1}{r^5}) , \tag{4.8}$$

where $\hat{r}$ is the radial unit vector from the origin, and (r, θ, ω) are the polar coordinates with x_3 as the polar axis.

It is interest to notice that (4.7) and (4.8) are in perfect accordnace to those in the case of a circular hole of radius a. Except the external flow $v = \frac{1}{2}\alpha|x_3|\hat{x}_1$, the induced velocity $\tilde{v}$ is radial and its magnitude is just $2rp/\mu$. As to the angular dependence we have an incoming flow with maximum along $\theta = \pm\pi/4$, $\omega = \pi$ (i.e. $x_2 = 0$, $x_1 = -|x_3|$) and an outgoing one with maximum along $\theta = \pm\pi/4$, $\omega = o$ ($x_2 = 0$, $x_1 = |x_3|$) (Fig. 4).

These two flows are connected by the flow parallel to $\hat{x}_1$ near the hole as shown by (4.1) and the velocity $\tilde{v} = \tilde{u}\hat{x}_1$ ($\tilde{u} > 0$) at the plane $x_1 = 0$ ($\lambda_3 = -a_1^2$) where

$$\tilde{u} = 2B[\phi_3(\lambda_1) - \phi_1(\lambda_1)]x_3 \tag{4.9}$$

and

$$x_2^{\,2} = (\lambda_1 + a_2^{\,2})(\lambda_2 + a_2^{\,2})/(a_1^{\,2} - a_2^{\,2})$$

$$x_3^{\,2} = \lambda_1(-\lambda_2)/a_1^{\,2}a_2^{\,2}$$

$$(\lambda_1 \geqq 0 \geqq \lambda_2 \geqq -a_2^{\,2})$$

The total flux of induced flow $\tilde{u}$ through the upper x_2x_3 plane is given by

$$Q = 2\alpha \int_0^\infty\!\!\int_0^\infty \tilde{u}\, dx_2\, dx_3 = \frac{8}{3} B\alpha \tag{4.10}$$

which is in accordance with the total incoming (outgoing) flux in the upper ($x_3 > 0$) domain calculated from (4.7) for $\tilde{\mathbf{v}}\cdot\hat{r}$.[5)]

Fig. 3 shows the value of important parameter $B/a_1^{\,3}$ as the function of thickness t(upper curve).

§5. The Case for the Flow Parallel to the Miner Axis

The case for the flow parallel to $\hat{x}_2$ can be obtained perfectly in the same manner. We have only to adopt Φ_2 in place of Φ_1 in (2.5) :

$$\mathbf{v}/\alpha = x_3 \operatorname{grad}\Phi_3 - \Phi_3\hat{x}_3 + x_2 \operatorname{grad}\Phi_2 - \Phi_2\hat{x}_2 + \operatorname{grad}\Phi_0 \tag{5.1}$$

corresponding to the undisturbed flow

$$\mathbf{v}_0 = \frac{1}{2}\alpha x_3\hat{x}_2 \tag{5.2}$$

We have finally

$$\Phi_3 = B_2[\gamma_2 - \phi_2(\lambda_1)]x_2$$

$$\Phi_2 = -B_2[\gamma_1 + \gamma_2 + \phi_3(\lambda_1)]x_3$$

and

$$\Phi_0 = B_2[\gamma_1 + 2\gamma_2 - \{\phi_2(\lambda_1) - \phi_3(\lambda_1)\}]x_2x_3$$

with

$$B_2 = \frac{A_0}{\gamma_1 + 2\gamma_2} \tag{5.3}$$

instead of (3.23).

For the pressure in (3.24) we have only to replace $\partial\Phi_1/\partial x_1$, x_1, a_1^2, and a_2^2 by $\partial\Phi_2/\partial x_2$, x_2, a_2^2 and a_1^2 respectively.

Other interesting values corresponding to those in §4 are given as follows

$$V = x_2 \operatorname{grad}\Phi_2 - \Phi_2\hat{x}_2 + \operatorname{grad}\Phi_0 = (4B_2\sigma^{1/2}/a_1a_2)\hat{x}_2 , \tag{4.1}'$$

$$p/8\mu\alpha = B_2x_2/(a_1 a_2^3 \sigma^{1/2}) \tag{4.2}'$$

$$\phi_2(\lambda) = \frac{2}{3}\lambda^{-\frac{3}{2}} - \frac{2}{5}(a_1^2 + 2a_2^2)\lambda^{-\frac{5}{2}} + \tag{4.6}'$$

$$\frac{\tilde{\mathbf{v}}}{\alpha} = 4B_2 \frac{x_2|x_3|}{r^4}\hat{r} + O(\frac{1}{r^4}) \doteq \frac{2B_2|\sin 2\theta|\sin\omega}{r^2}\hat{r} \tag{4.7}'$$

$$\frac{p}{\mu\alpha} = 8B_2 \frac{x_2|x_3|}{r^3} + O(\frac{1}{r^3}) \tag{4.8}'$$

$$\tilde{\mathbf{v}} = 2B_2[\phi_3(\lambda_1) - \phi_2(\lambda_1)]\, x_3\hat{x}_2 \quad \text{for} \quad x_2 = 0 \quad \text{i.e.} \tag{4.9}'$$

It is easily seen that the essential features are the same as those in §4, within the numerical factor B_2 given by (5.3). The isovels in the hole are ellipses whose major axes coincide with that of the hole in this case. The values of B_2/a_1^3 as a function of t are given in Fig.3 (lower curve), showing that the general effect of the hole is smaller than that of (B/a_1^3) in th previous case parrallel to the major axis.

§6. General and Special Cases

On account of the linearity of the problem, the general cas of the shear flow parallel to the plane can be derived easily by superposition. If we let $k^2 \to 0$ and make use of the expansion

$$K = \frac{\pi}{2}(1 + \frac{1}{4}k^2 + \frac{9}{64}k^4 + \cdots) ,$$
$$E = \frac{\pi}{2}(1 - \frac{1}{4}k^2 - \frac{3}{64}k^4 + \cdots) , \tag{6.1}$$

we have from (3.17) and (3.18)

$$\gamma_1 a_1^3 = \frac{\pi}{2}(1 + \frac{3}{8}k^2 + \cdots)$$
$$\text{and} \quad \gamma_2 a_1^3 = \frac{\pi}{2}(1 + \frac{9}{8}k^2 + \cdots) , \tag{6.2}$$

which yield for B in (3.23) and (4.1)

$$B/a_1^3 = \frac{1}{6\pi}(1 - \frac{5}{8}k^2 + \cdots) ,$$
$$\text{and} \tag{6.3}$$
$$B_2/a_1^3 = \frac{1}{6\pi}(1 - \frac{7}{8}k^2 + \cdots) .$$

If $k = 0$ we have the value for a circular hole of radius $a_1 = a$:

$$B = B_2 = \frac{a^3}{6\pi} \quad , \quad \text{or} \quad Q = \frac{4\alpha}{9\pi} a^3 \quad . \tag{6.4}$$

in accordance with the previous paper.[5] If we let $k^2 \to 1$, i.e. $k'^2 = t^2 \to 0$ and make use of the expansion

$$K = L + \frac{1}{4}(L - 1)t^2 + \cdots$$

and (6.5)

$$L = 1 + \frac{1}{2}(L - \frac{1}{2})t^2 + \cdots$$

with

$$L = \log(4/t)$$

we have

$$B = \frac{a_1^3 t^2}{8} [1 - \frac{1}{2}(3L - \frac{5}{2})t^2 + \cdots]$$

$$B_2 = \frac{a_1^3 t^2}{16} [1 - \frac{1}{4} t^2 + \cdots] \quad . \tag{6.6}$$

In the extreme case of narrow hole B_2 is found to be just half of B.

The value of limiting value of the flux in the second case divided by the transversal length $2a_1$ tends to

$$\bar{Q}_2 = \frac{4 B_2 \alpha}{3a_1} = \frac{a_2^2}{12}$$

which is not in accordance with the corresponding value $a_2^2/8$ for the case of a narrow transversal slit of width $2a_2$ to be reported in a subsequent paper or calculated by use of Wakiya's

study[9] for the shear flow past a ditch in a plane wall. It may be interesting to notice that the same ratio 2/3 is also found in the case of the flux due to the pressure difference thought an elliptic hole and that through a slit in a plane wall.

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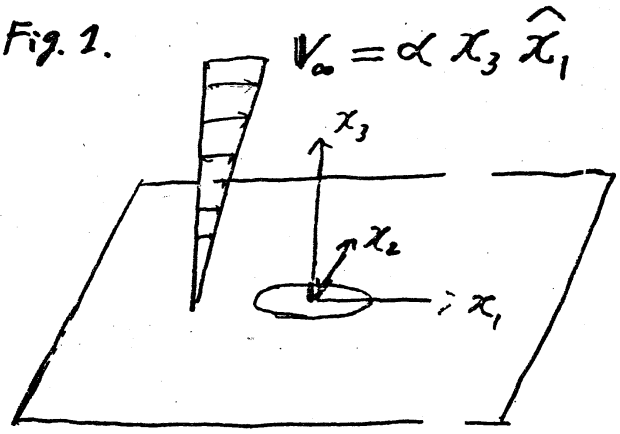
Fig. 1.



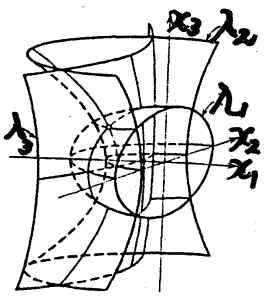


Fig. 2.

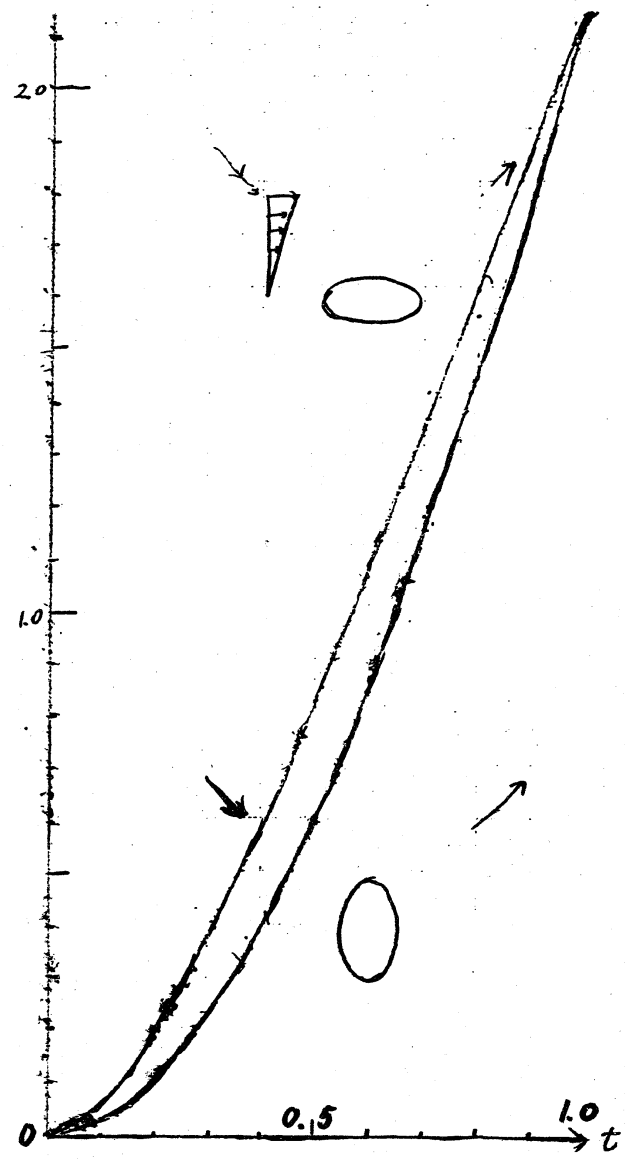


Fig. 3

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