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HOUSING AS AN ASSET AND THE EFFECTS OF PROPERTY TAXES ON THE RESIDENTIAL DEVELOPMENT PROCESS

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Introduction

Although there is extensive literature on the housing market and property taxes, most of the analyses have been carried out in static frameworks and the aspect of housing as an asset has not been fully analyzed.[1] Shoup (1969), Bahl (1968), Arnott and Lewis (1979), and others examined the individual behaviour of a land investor, recognizing the asset side of housing and land. Their analyses, however, are partial in the sense that they simply assumed that investors take the future price path as given and did not close the model to determine the equilibrium price path.

Weiss (1978) analyzed the effect of capital gains tax on the choice between renting and owning a house, but he also assumed the future price path to be exogenous in most part of his paper. Though he suggested a steady state model with the endogenous price path in section 3, his analysis there remains heuristic.

Assuming perfect foresight, Markusen and Scheffman (1978) constructed a simple two-period model of the land development process and analyzed the effect on the equilibrium price path of a land tax and a capital gains tax.[2] This paper extends their model and offers more detailed analyses of a captial gains tax and a property tax.

Major differences from the Markusen-Scheffman model are as follows. First, the two-period model is extended to an infinite-horizon/continuous-time framework. This enables us to separate long-run, or steady-state, effects from short-run effects and makes clear the relationship with the traditional static analysis of property taxes. Second, substitutability

between land and capital is introduced into housing production, where capital here includes both improvements and structures. This extension allows us to analyze the distortionary effects on captial-land ratio which have been one of the major issues in the analysis of property taxes.

Markusen and Scheffman obtained the result that a realized capital gains tax speeds up the land development process. It is shown that their result crucially depends on their implicit assumption that a change in tax rate is anticipated beforehand. In their model, the capital gains tax is levied only in period 2 and the tax rate is known in period 1. If a rise in tax rate is anticipated, there is an incentive to sell land before the rise occurs and it is natural that the development process is accelerated.

In this paper both unanticipated and anticipated changes in the capital gains tax are considered. It is shown that an unanticipated rise in tax rate causes decreases in residential land and the housing stock at each instant of time. An unanticipated rise therefore slows down the residential development process.

If, however, a rise in tax rate is anticipated, then the situation is different and the perfect foresight equilibrium path has an upward jump in the amount of residential land right before the rise occurs. The jump is followed by a period in which residential development stops, and after residential development starts again, the amount of residential land follows a path lower than that with no change in tax rate.

The result in Markusen and Scheffman corresponds to the case in which the future rise in tax rate becomes larger. In that case, the jump in residential land becomes larger, but the path that the residential land eventually follows becomes lower. Thus, there is speeding of

development at the moment when the tax rate is changed but in the long run the amount of residential land decreases. An increase in the jump size occurs since a larger tax increase gives more incentive to develop land before the tax increase.

The effect of the property tax, or a tax on property values, is similar to that obtained by Markusen and Scheffman and is also a generalization of the traditional static analysis to a dynamic framework.

A model of the residential development process with a capital gains tax is constructed in Section 1, and the perfect foresight equilibrium path is characterized in Section 2. Section 3 examines the effect of an unanticipated rise in the tax rate on the equilibrium paths of residential land, housing capital, housing stock, and land price. An anticipated change in the capital gains tax is analyzed in Section 4. In Section 5, a tax on property values is introduced instead of a capital gains tax and the effect of the property tax on the perfect foresight equilibrium path is analyzed. Finally, Section 6 contains concluding remarks. All the proofs are relegated to the Appendix.

1. The Model

A representative landowner initially owns $\bar{L}$ units of homogeneous land and uses the land for, say, an agricultural purpose. The land is gradually developed and converted into residential use. It is assumed that there is no rental market for land and housing so that the development is possible only if the landowner sells land. Between time t and t+Δt, the agricultural landowner sells $I^L(t)\Delta t$ units of land to a representative consumer. The consumer prepares the land for residential use, for example, by installing sewerage and building roads, and build houses.

The production function of housing is denoted by H(L,K), where L and K are respectively the amounts of land and housing capital (including land servicing costs). The production function is assumed to be concave, homogeneous of degree one, twice continuously differentiable, and to satisfy $H_{LL}(L,K)<0$, $H_{KK}(L,K)<0$, where $H_{LL}\equiv\partial^2H/\partial L^2$ and $H_{KK}\equiv\partial^2H/\partial K^2$. The amount of the housing stock produced between t and t+Δt is then $H(I^L(t)\Delta t, I^K(t)\Delta t)=H(I^L(t), I^K(t))\Delta t$, where $I^K(t)\Delta t$ is the amount of housing capital bought by the consumer between t and t+Δt.

The prices of land and capital at time t are denoted by $p^L(t)$ and $p^K(t)$ respectively. The price of land is determined endogenously, but the price of housing capital is exogenous and supply of capital is infinitely elastic at that price. The price path of housing capital is assumed to be differentiable and increasing: $\dot{p}^K(t)>0$.

The total amount of residential land at time t, L(t), satisfies

$$L(t) = \int_0^t I^L(s)ds, \tag{1.1}$$

where L(t) is allowed to have jumps. It is assumed that the conversion of land into residential use is irreversible: $I^L(t) \geqq 0$ for any t.

At time t, the landowner has $\bar{L}-L(t)$ units of agricultural land which yields the agricultural income of $R(\bar{L}-L(t),t)$. The agricultural land rent is then $r(\bar{L}-L(t),t)=\partial R(\bar{L}-L(t),t)/\partial(\bar{L}-L(t))$ and we assume[3] that $r_L(\bar{L}-L,t) \equiv \partial r(\bar{L}-L,t)/\partial(\bar{L}-L)<0$ and $r_t(\bar{L}-L,t) \equiv \partial r(\bar{L}-L,t)/\partial t>0$.

When the landowner sells land, a tax is levied on the capital gains that the land has earned. Specifically, the value of land sales in excess of the value of that land evaluated at a certain base price, b, (which may be the price at time 0) is subject to an ad valorem tax with tax rate τ. The tax at time t is then $\tau(p^L(t)-b)I^L(t)$ and the net income from land sales is $[(1-\tau)p^L(t)+\tau b]I^L(t)$. It is assumed that the base price is low enough to satisfy $b<r(\bar{L}-L,t)/i$ in the relevant range of L.

The tax revenue is redistributed to the landowner and the consumer as lump-sum subsidies. The subsidies at time t are $S^A(t)$ for the landowner and $S^C(t)$ for the consumer, where $S^A(t)+S^C(t) =\tau(p^L(t)-b)I^L(t)$.

In addition to land and housing, there is another asset, called the bond, whose rate of return is i and fixed. The landowner forms expectations of the land price path and maximizes the discounted sum of future earnings plus the lump-sum subsidy, $\int_0^\infty\{R(\bar{L}-L(t),t) +[(1-\tau)p^L(t)+\tau b]I^L(t)+S^A(t)\}\exp(-it)dt$, where the discount rate is the rate of return on the bond, i. The maximization is carried out under constraint (1.1) and the nonnegativity constraint for $I^L(t)$.

First order conditions for maximization can be easily obtained and yield supply of land, $I_s^L(t)$, by the landowner: $I_s^L(t)=\infty$ if $p^L(t)>$

$\bar{p}^L(t,\tau)$, $I_s^L(t)$=indeterminate if $p^L(t)=\bar{p}^L(t,\tau)$, and $I_s^L(t)=0$ if $p^L(t)<\bar{p}^L(t,\tau)$, where $\bar{p}^L(t,\tau)\equiv\int_t^\infty r^L(L(s),s,\tau)\exp(-i(s-t))ds$ and $r^L(L,t,\tau)\equiv\frac{1}{1-\tau}[r(\bar{L}-L,t)-\tau ib]$. These conditions can be interpreted as follows. If land is sold at time t, then the price of land net of the tax is $(1-\tau)p^L(t)+\tau b$. If land is not sold, then it earns agricultural rent and the discounted sum of future earnings is $\int_t^\infty r(\bar{L}-L(s),s)\exp(-i(s-t))ds$. If the former is higher than the latter, or if $p^L(t)>\bar{p}^L(t,\tau)$, then it pays the landowner to sell land. The rate at which land is sold, $I^L(t)$, is infinite since both the objective function and the constraints are linear with respect to $I^L(t)$. If the former equals the latter, or if $p^L(t)=\bar{p}^L(t,\tau)$, then the landowner is indifferent between selling and keeping land and $I^L(t)$ is indeterminate. Finally, if the former is lower than the latter, or if $p^L(t)<\bar{p}^L(t,\tau)$, then the landowner has no incentive to sell land and $I^L(t)=0$.

The representative consumer buys land and housing capital and build houses. The instantaneous utility function of the consumer is $U(c,H,t)$, where c and H are the consumption of the composite consumer good and the amount of housing stock respectively, and $U(c,H,t)$ is assumed to be concave in c and H. The dependence of the utility function on time may be interpreted as reflecting population increases and changes in taste.

Forming expectations on future land price and capital price, the consumer maximizes the discounted sum of instantaneous utilities[4], $\int_0^\infty U[c(t),H(L(t),K(t)),t]\exp(-it)dt$, subject to the intertemporal budget constraint[5], $W+\int_0^\infty[y(t)+S^C(t)-c(t)-p^L(t)I^L(t)-p^K(t)I^K(t)]\exp(-it)dt=0$, constraints (1.1) and

$$K(t)=\int_t^\infty I^K(s)ds, \qquad (1.2)$$

and nonnegativity constraints, $I^L(t) \geqq 0$ and $I^K(t) \geqq 0$ for any t, where W_0 and y(t) are respectively the initial asset holding and the income at time t, and it is assumed that the consumer cannot resell housing capital. Note that implicit in (1.2) is the assumption that housing capital is malleable: the capital-land ratio of houses built in the past can be changed freely and housing capital and land bought in the past can be recombined to obtain the homogeneous housing stock H(L(t), K(t)).

First order conditions for the consumer's maximization problem can be easily obtained. First, marginal utility of the consumer good is constant over time: $U_c(c(t),H(L(t),K(t)),t)=\alpha$. From this relationship, consumption of the consumer good satisfies $c(t)=\gamma(H(L(t),K(t)),t,\alpha)$, where $U_c[\gamma(H,t,\alpha),H,t]\equiv\alpha$. Using $\gamma(H,t,\alpha)$, the marginal rate of substitution between housing and the consumer good, U_H/U_c, can be written $q(H,t,\alpha)\equiv \frac{1}{\alpha}U_H[\gamma(H,t,\alpha),H,t]$, where $q_H(H,t,\alpha)\leqq 0$ by concavity of U(c,H,t) in c and H and it is assumed that $q_t(H,t,\alpha)>0$. q can be interpreted as the shadow rent of housing services.

Next, define the shadow rent of residential land, $q^L(L,K,t,\alpha)\equiv q(H(L,K),t,\alpha)H_L(L,K)$, the shadow rent of housing capital, $q^K(L,K,t,\alpha)\equiv q(H(L,K),t,\alpha)H_K(L,K)$, the shadow price of residential land, $\underline{p}^L(t)\equiv \int_t^\infty q^L(L(s),K(s),s,\alpha)\exp(-i(s-t))ds$, and the shadow price of housing capital, $\underline{p}^K(t)\equiv\int_t^\infty q^K(L(s),K(s),s,\alpha)\exp(-i(s-t))ds$. Then, first order conditions for the consumer's problem yield demand for residential land and housing capital, $I_d^L(t)$ and $I_d^K(t)$:

(i) $I_d^L(t)=0$ if $p^L(t)>\underline{p}^L(t)$, $I_d^L(t)$=indeterminate if $p^L(t)=\underline{p}^L(t)$, and $I_d^L(t)=\infty$ if $p^L(t)<\underline{p}^L(t)$.

(ii) $I_d^K(t)=0$ if $p^K(t)>\underline{p}^K(t)$, $I_d^K(t)=$indeterminate if $p^K(t)=\underline{p}^K(t)$, and $I_d^K(t)=\infty$ if $p^K(t)<\underline{p}^K(t)$.

Thus, if the market price of land (capital) is higher than the shadow price of residential land (capital), then demand for land (capital) is zero; if the market price equals the shadow price, then demand for land (capital) is indeterminate; and if the market price is lower than the shadow price, then demand for land (capital) is infinite.

2. The Perfect Foresight Equilibrium Path

As described in Section 1, the landowner and the consumer determine demand and supply of land and capital, given the expectations of price paths, $p^L(t)$ and $p^K(t)$. Along the perfect foresight equilibrium path, the price expectations are correct and the expected price paths coincide with the equilibrium price paths. Then, the landowner and the consumer have the same expectations and given the price expectations the land market is in equilibrium at each instant of time. The equilibrium condition is: (i) $I_d^L(t)=I_s^L(t)$, or (ii) $I_d^L(t)>I_s^L(t)$ and $L(t)=\bar{L}$, or (iii) $I_d^L(t)<I_s^L(t)$ and $p^L(t)=0$. Note that capital market equilibrium is always guaranteed since supply of housing capital is perfectly elastic.

It is assumed that the shadow rent of housing, q, rises fast enough to yield the following conditions.

Assumption 1. The shadow rent of housing, $q(H,t,\alpha)$, the shadow rent of agricultural land, $r^L(L,t,\tau)$, and the rental price of capital, $r^K(t)\equiv ip^K(t)-\dot{p}^K(t)$, satisfy the inequalities:

$$q_t/q \geqq r_t^L/r^L$$

$$q_t/q \geqq r_t^K/r^K$$

$$\eta[(q_t/q)-\omega(r_t^L/r^L)-(1-\omega)(r_t^K/r^K)]+\sigma(1-\omega)[(r_t^K/r^K)-(r_t^L/r^L)] \geqq 0$$

$$\eta[(q_t/q)-\omega(r_t^L/r^L)-(1-\omega)(r_t^K/r^K)]+\sigma\omega[(r_t^L/r^L)-(r_t^K/r^K)] \geqq 0,$$

where σ, η, and ω are the elasticity of substitution between capital and land in housing production, the shadow rent elasticity of demand for housing services, and the share of land rent in housing production respectively: $\sigma\equiv -L^2H_KH_L/(KHH_{KK})>0$, $\eta\equiv -q/(Hq_H)>0$, $\omega\equiv LH_L/H>0$.

The first two inequalities assume that when L(t) and K(t) are constant, the shadow rent of housing rises faster than or as fast as

the agricultural rent and the rental price of capital. The last two inequalities ensure that L(t) and K(t) are both nondecreasing over time along the equilibrium path. If, for example, $q_t/q > r_t/r = \dot{p}^K/p^K$ and b=0, then all four inequalities are satisfied.

Next, in order to ensure an interior solution, we make the following assumption.

Assumption 2. $q^L(L,K,t,\alpha)$, $r^L(L,t,\tau)$, $q^K(L,K,t,\alpha)$ and $r^K(t)$ satisfy

$$q^L(0,K,t,\alpha) > r^L(0,t,\tau)$$

$$q^K(L,0,t,\alpha) > r^K(t)$$

$$q^L(\bar{L},K,t,\alpha) < r^L(\bar{L},t,\tau)$$

$$\lim_{K\to\infty} q^K(L,K,t,\alpha) < r^K(t) \qquad \text{for any relevant } L,\ K,\ t,\ \alpha,\ \tau.$$

Under Assumptions 1 and 2, the perfect foresight equilibrium path is characterized by the following Proposition.

Proposition 1. The perfect foresight equilibrium path satisfies $\underline{p}^L(t)=p^L(t)=\bar{p}^L(t,\tau)$ and $\underline{p}^K(t)=p^K(t)$ for any t in $[0,\infty)$. Hence, the equilibrium path follows the differential equations:

$$\dot{p}^L(t) = ip^L(t)-q^L(L(t),K(t),t,\alpha)$$

$$= ip^L(t)-r^L(L(t),t,\tau), \qquad 0 \leq t < \infty,$$

$$\dot{p}^K(t) = ip^K(t)-q^K(L(t),K(t),t,\alpha), \qquad 0 \leq t < \infty,$$

$$\lim_{t\to\infty} p^L(t)\exp(-it) = 0.$$

Although the proof of the Proposition is tedious, the Proposition itself is simple and easy to understand. Along the perfect foresight equilibrium path, the shadow price of residential land, the market

price of land, and the shadow price of agricultural land are all equal, and the shadow price of housing capital equals the market price of housing capital. This condition implies that the shadow rent of residential land, q^L, equals the shadow rent of agricultural land, r^L, and that the shadow rent of housing capital, q^K, equals its rental price, $r^K(t)=ip^K(t)-\dot{p}^K(t)$.

3. An Unanticipated Rise in the Realized Captial Gains Tax

Next, consider the effect of an unanticipated change in the tax rate, τ. The effect depends on the way in which the tax revenue is redistributed between the landowner and the consumer. We assume the simplest case where $S^A(t)$ and $S^C(t)$ are determined in such a way to keep unchanged the consumer's marginal utility of the consumer good, α. The effect of a change in α is the pure income effect and its nature is well known. Henceforth, we suppress α's in $q()$, $q^L()$, and $q^K()$.

Proposition 2. Consider two perfect foresight equilibrium paths corresponding to two tax rates, one higher than the other. Then, the equilibrium path with the higher tax rate has

(a) the smaller amount of residential land, $L(t)$,

(b) the larger (smaller) amount of housing capital, $K(t)$, if the elasticity of substitution between capital and land, σ, is larger (smaller) than the shadow rent elasticity of demand for housing, η,

(c) the smaller amount of housing stock, $H(t)$, and

(d) the higher price of land, $p^L(t)$, but the lower price net of the tax, $p^L(t)-\tau[p^L(t)-b]$,

at each instant of time.

Since the realized capital gains tax is levied when agricultural land is converted to residential use, it discourages residential development. Proposition 1 yields $(1-\tau)qH_L+\tau ib=r$ and $qH_K=r^K$. Hence, if $b=0$, then the capital gains tax is equivalent to a tax on residential land with no tax on agricultural land and housing capital and causes a fall in the amount of residential land. If $b>0$, then the term with b

works in the opposite direction but this effect is weaker than the above effect under our assumption that $b<r/i$. Thus, the capital gains tax always reduces the amount of residential land as shown in condition (a) of the Proposition.

The effect on the amount of housing capital depends on the relative magnitudes of two forces. On one hand, the tax causes a rise in the cost of housing and induces a fall in demand for housing. This effect tends to decrease the amount of housing capital. On the other hand, the price of land rises relative to the price of capital and the resultant substitution from land to capital tends to increase housing capital. Condition (b) shows that if the elasticity of substitution is larger (smaller) than the rent elasticity of housing demand, then the latter effect is stronger (weaker) than the former and the capital gains tax increases (decreases) the amount of housing capital.

As in condition (c), even if housing capital increases, the increase is offset by a decrease in residential land and the amount of the housing stock always falls.

Condition (d) results from condition (a). In order to induce a fall in the amount of residential land the gross price of land that the consumer pays must be raised, and the price net of the tax that the landowner receives must be lowered to increase the amount of agricultural land.

The Proposition compares two equilibrium paths corresponding to two tax rates. If a tax rate, τ_0, is maintained up to time T and the tax rate is suddenly raised to $\tau_1 (>\tau_0)$, then the equilibrium path shifts from that with tax rate τ_0 to that with tax rate τ_1. However, since

$L(t)$ and $K(t)$ cannot fall over time by our assumption, there is a period in which $I^L(t)=0$ (and $I^K(t)=0$ if $\sigma<\eta$) after time T and it may take a while for them to reach the new equilibrium path.

It should be noted that the price of land jumps up at time T. This is a consequence of the assumption of long-run perfect foresight, that is, the assumption that both the landowner and the consumer have the correct expectations of the entire future paths of land and capital prices. If they face stable environment, the assumption would be a reasonable approximation of reality, but if the environment is changing rapidly, it is unlikely that the long-run perfect foresight assumption is satisfied. In such a case, the myopic perfect foresight assumption may be a better one. Under the myopic perfect foresight assumption, only the rates of change of land and capital prices at the present moment are expected correctly. Demand for and supply of land are determined in such a way that only the short-run optimality conditions are satisfied: for the consumer (the landowner), the return on the alternative asset equals the residential (agricultural) shadow rent plus capital gains from a rise in the price of land, or $ip^L(t)=q^L+\dot{p}^L(t)=r^L+\dot{p}^L(t)$. If myopic foresight is assumed and the present price is taken as given, then the price path is continuous and it can be seen that the price path becomes flatter at the time when the tax is raised: $\partial\dot{p}^L/\partial\tau=-\partial r^L/\partial L$ $\partial L/\partial\tau<0$. Therefore, the tax lowers the land price as in Fig. 1 in sharp contrast to the case of long-run perfect foresight. After a while, however, it may be realized that their expectations are wrong and a jump in the price path to the long-run perfect foresight path may occur.

Since the consumer does not sell a house, there is no market price of housing. Although the representative consumer may be interpreted as consisting of many identical consumers and trading of houses among them may be considered, they never sell their houses if they have to pay the capital gains tax. Only when the tax rate is zero, the market price of housing can de defined since they are indifferent between selling their houses and buying others' houses. The shadow price of housing, $\int_t^\infty q(H(L(s),K(s)),s)\exp(-i(s-t))ds$, however, can be defined, and it is easy to see that the shadow price rises when the tax rate is raised.

4. An Anticipated Change in the Capital Gains Tax

Markusen and Scheffman (1978) analyzed a tax on realized capital gains in a two period model and obtained the result that "a capital gains tax will speed the conversion of undeveloped land into final use". This contradicts our result in Section 3 that a capital gains tax reduces residential land at each instant of time. The difference is caused by the fact that Markusen and Scheffman assumed an anticipated change in tax rate, while an unanticipated change is assumed in Section 3. They assumed that there is no tax in the first period, that capital gains from the first period to the second period are taxed, and that the tax rate is known in the first period. In such a case, there is an incentive to develop land in the first period to avoid the capital gains tax in the second period.

In this section, we consider an anticipated rise in tax rate at time T from τ_0 to τ_1. The tax rate is then τ_0 in period $[0,T]$ and τ_1 in period (T,∞), where $\tau_1>\tau_0$. The only difference from Section 2 is that the objective function of the landowner becomes $\int_0^T\{R(\bar{L}-L(t),t)+[(1-\tau_0)p^L(t)+\tau_0 b]I^L(t)+S^A(t)\}\exp(-it)dt + \int_T^\infty\{R(\bar{L}-L(t),t)+[(1-\tau_1)p^L(t)+\tau_1 b]I^L(t)+S^A(t)\}\exp(-it)dt$. Supply of land by the landowner satisfies the same condition as that in Section 2 if we substitute τ_0 for τ in period $[0,T]$ and τ_1 in period (T,∞).

Proposition 3 characterizes the perfect foresight equilibrium path when a rise in the tax rate is anticipated.

Proposition 3. Define $\hat{L}(t,\tau)$ and $\hat{K}(t,\tau)$ by $q^L(\hat{L}(t,\tau),\hat{K}(t,\tau),t)\equiv r^L(\hat{L}(t,\tau),t,\tau)$ and $q^K(\hat{L}(t,\tau),\hat{K}(t,\tau),t)\equiv r^K(t)$; and $\Phi(K,t,\tau)$ and $\Psi(L,t)$ by $q^L(\Phi(K,t,\tau),K,t)\equiv r^L(\Phi(K,t,\tau),t,\tau)$ and $q^K(L,\Psi(L,t),t)\equiv r^K(t)$. Then, the perfect foresight equilibrium path with an anticipated rise in the capital gains

tax from τ_0 to τ_1 at time T satisfies the following conditions.

(i) If the elasticity of substitution is not larger than the rent elasticity of housing demand, or $\sigma \leqq \eta$, then

(a) in period $[0,T)$,

$$\underline{p}^L(t)=p^L(t)=\bar{p}^L(t,\tau_0) \quad \text{and} \quad \underline{p}^K(t)=p^K(t);$$

$$L(t)=\hat{L}(t,\tau_0) \text{ and } K(t)=\hat{K}(t,\tau_0);$$

(b) in period $[T,T_1)$, $T<T_1 \leqq \infty$,

$$\underline{p}^L(t) \leqq p^L(t) \leqq \bar{p}^L(t,\tau_1) \text{ with } \underline{p}^L(t)<\bar{p}^L(t,\tau_1) \text{ and } \underline{p}^K(t)=p^K(t);$$

$$L(t)=L_1 \geqq \hat{L}(T,\tau_0) \text{ and } K(t)=\Psi(L_1,t),$$

where $L_1>\hat{L}(T,\tau_0)$ if $T_1<\infty$; and

(c) in period $[T_1,\infty)$,

$$\underline{p}^L(t)=p^L(t)=\bar{p}^L(t,\tau_1) \text{ and } \underline{p}^K(t)=p^K(t);$$

$$L(t)=\hat{L}(t,\tau_1) \text{ and } K(t)=\hat{K}(t,\tau_1)$$

where $L_1=\hat{L}(T_1,\tau_1)$.

(ii) If $\sigma>\eta$, then

(a) in period $[0,t_0]$, $0 \leqq t_0 \leqq T$,

$$\underline{p}^L(t)=p^L(t)=\bar{p}^L(t,\tau_0) \text{ and } \underline{p}^K(t)=p^K(t);$$

$$L(t)=\hat{L}(t,\tau_0) \text{ and } K(t)=\hat{K}(t,\tau_0);$$

(b) in period (t_0,T),

$$\underline{p}^L(t)=p^L(t)=\bar{p}^L(t,\tau_0) \text{ and } \underline{p}^K(t)<p^K(t);$$

$$L(t)=\Phi(K(t_0),t,\tau_0) \text{ and } K(t)=K(t_0)=\hat{K}(t_0,\tau_0);$$

(c) in period $[T,t_1)$, $T \leqq t_1 \leqq T_1$,

$$\underline{p}^L(t) \leqq p^L(t) \leqq \bar{p}^L(t,\tau_1) \text{ with } \underline{p}^L(t)<\bar{p}^L(t,\tau_1) \text{ and } \underline{p}^K(t)<p^K(t);$$

$$L(t)=L_1 \geqq \Phi(K(t_0),T,\tau_0) \text{ and } K(t)=K(t_0),$$

where $L_1>\Phi(K(t_0),T,\tau_0)$ if $T_1<\infty$;

(d) in period $[t_1, T_1)$,

$\underline{p}^L(t) \leq p^L(t) \leq \bar{p}^L(t, \tau_1)$ with $\underline{p}^L(t) < \bar{p}^L(t, \tau_1)$ and $\underline{p}^K(t) = p^K(t)$;

$L(t) = L_1$ and $K(t) = \Psi(L_1, t)$,

where $K(t_0) = \Psi(L_1, t_1)$; and

(e) in period $[T_1, \infty)$,

$\underline{p}^L(t) = p^L(t) = \bar{p}^L(t, \tau_1)$ and $\underline{p}^K(t) = p^K(t)$;

$L(t) = \hat{L}(t, \tau_1)$ and $K(t) = \hat{K}(t, \tau_1)$,

where $\hat{L}(T_1, \tau_1) = L_1$ and $\hat{K}(T_1, \tau_1) = \Psi(L_1, T_1)$.

The Proposition is illustrated in Figs. 2 and 3. If the elasticity of substitution is smaller than or equal to the rent elasticity of housing demand, then, up to time T, L(t) follows $\hat{L}(t, \tau_0)$ which is the equilibrium path when the tax rate is τ_0 and does not change; jumps up to L_1 at time T; remains constant until time T_1 when $\hat{L}(t, \tau_1)$ becomes equal to L_1; and follows $\hat{L}(t, \tau_1)$ from that time on. Thus, there is a sudden increase in L(t) at time T but L(t) eventually follows the path, $\hat{L}(t, \tau_1)$ which is lower than the path without a change in tax rate, $\hat{L}(t, \tau_0)$. The amount of housing capital also experiences an upward jump at time T. From time T it moves in such a way to equate the shadow rent and the rental price of housing capital and eventually follows $\hat{K}(t, \tau_1)$ which is lower than $\hat{K}(t, \tau_0)$.

If the elasticity of substitution is larger than the rent elasticity of housing demand, the situation is more complicated since the phase in which K(t) remains constant appears. In this case there is no jump in K(t) and K(t) eventually becomes larger than $\hat{K}(t, \tau_0)$ since $\hat{K}(t, \tau_1) > \hat{K}(t, \tau_0)$.

Note that T_1 may be infinite, in which case $L(t)=L_1$ for any t in $[T,\infty)$. It can be seen that if T_1 is infinite, L_1 is not smaller than the steady state value of $\hat{L}(t,\tau_1)$, i.e., $L_1 \geqq \hat{L}(\infty,\tau_1)$.

If T_1 is finite, then L(t) always has a jump at time T, but if T_1 is infinite, then L(t) may not jump. If, for example, $r(\bar{L}-L,t)=r^*(\bar{L}-L)\exp(nt)$, $p^K(t)=p^{K*}\exp(nt)$, $q(H,t)=q^*(H)\exp(nt)$, and b=0, then it is not diffcult to see that L(t) and K(t) are constant over the entire time horizon.

Next, Proposition 4 yields the effect of an anticipated rise in the tax rate, τ_1.

<u>Proposition 4</u>. If $T_1<\infty$, then an expected rise in τ_1 has no effect on $\hat{L}(t,\tau_0)$, but raises L_1 and lowers $\hat{L}(t,\tau_1)$. Hence, an infinitesimal rise in τ_1 keeps L(t) unchanged in [0,T); but raises it in $[T,T_1)$; and lowers it in (T_1,∞).

This Proposition corresponds to the result of Markusen and Scheffman. An anticipated rise in τ_1 speeds up development at time T when the size of a jump in L(t) becomes larger, but eventually reduces the amount of residential land since $\hat{L}(t,\tau_1)$ becomes smaller.

If a change in τ_1 is discrete, then the Proposition should be modified slightly: L(t) rises in $[T,T_2)$ and falls in (T_2,∞) for some T_2 which is larger than T_1.

5. A Tax on Property Values

Next, consider an ad valorem tax on property values with tax rate τ. Land is evaluated at market price of land, and housing is evaluated at factor costs, that is, the value of $H(L(t),K(t))$ at time t is $p^L(t)L(t)+p^K(t)K(t)$.

The landowner owns $\bar{L}-L(t)$ units of land at time t and the property tax he has to pay is $\tau p^L(t)(\bar{L}-L(t))$. His objective function is then $\int_0^\infty\{R(\bar{L}-L(t),t)+p^L(t)I^L(t)-\tau p^L(t)(\bar{L}-L(t))+S^A(t)\}\exp(-it)dt$. Supply of land is given by the same condition as that in Section 2 if we modify the definition of $\bar{p}^L(t,\tau)$ to $\bar{p}^L(t,\tau)=\int_t^\infty[r(\bar{L}-L(s),s)-\tau p^L(s)]\exp(-i(s-t))ds$. The shadow price of agricultural land, $\bar{p}^L$, is the discounted sum of the shadow rent of agricultural land, where the shadow rent equals the agricultural rent minus the property tax per unit land, $r-\tau p^L$.

The consumer pays the property tax, $\tau[p^L(t)L(t)+p^K(t)K(t)]$, on the housing stock, $H(L(t),K(t))$, that he owns. The budget constraint is modified to $W_0+\int_0^\infty\{y(t)+S^C(t)-c(t)-p^L(t)I^L(t)-p^K(t)I^K(t)-\tau[p^L(t)L(t)+p^K(t)K(t)]\}\exp(-it)dt=0$. Demand for land, $I_d^L(t)$, and demand for housing capital, $I_d^K(t)$, satisfy the same conditions as those in Section 2 if $\underline{p}^L(t)$ and $\underline{p}^K(t)$ are replaced by $\underline{p}^L(t,\tau)=\int_t^\infty[q(H(L(s),K(s)),s,\alpha)H_L(L(s),K(s))-\tau p^L(s)]\exp(-i(s-t))ds$ and $\underline{p}^K(t,\tau)=\int_t^\infty[q(H(L(s),K(s)),s,\alpha)H_K(L(s),K(s))-\tau p^K(s)]\exp(-i(s-t))ds$.

If Assumptions 1 and 2 in Section 2 are maintained with modifications, $r^L(L,t,\alpha)=r(\bar{L}-L,t)$, $r^K(t)=(i+\tau)p^K(t)-\dot{p}^K(t)$, $q^L(L,K,t,\alpha)=q(H(L,K),t,\alpha)H_L(L,K)$, and $q^K(L,K,t,\alpha)=q(H(L,K),t,\alpha)H_K(L,K)$, then the following Proposition is obtained.

Proposition 5. The perfect foresight equilibrium path with a tax on property values satisfies $\underline{p}^L(t,\tau)=p^L(t)=\bar{p}^L(t,\tau)$ and $\underline{p}^K(t,\tau)=p^K(t)$ for any t. Hence,

$$\dot{p}^L(t)=(i+\tau)p^L(t)-q(H(L(t),K(t)),t,\alpha)H_L(L(t),K(t))$$
$$=(i+\tau)p^L(t)-r(\bar{L}-L(t),t)$$
$$\dot{p}^K(t)=(i+\tau)p^K(t)-q(H(L(t),K(t)),t,\alpha)H_K(L(t),K(t))$$
$$\lim_{t\to\infty} p^L(t)\exp(-it)=0$$

Next, if the tax revenue is redistributed in such a way that α remains the same, then the effect of a change in the tax rate is:

Proposition 6. The equilibrium path with a higher tax rate has

(a) a larger (smaller) amount of residential land, L(t), if the elasticity of substitution between capital and land is larger (smaller) than the rent elasticity of demand for housing.

(b) a smaller amount of housing capital, K(t),

(c) a smaller amount of housing stock, H(t),

at each instant of time, and

(d) a lower price of land at the steady state.

From Proposition 5, the equilibrium path satisfies $qH_L=r$ and $qH_K=(i+\tau)p^K-\dot{p}^K$, and the effect of the property tax is to raise the effective rental price of capital from $ip^K-\dot{p}^K$ to $(i+\tau)p^K-\dot{p}^K$. A tax on housing is equivalent to taxes on land and capital at the same rate. A tax on capital has a real effect but a tax on land does not cause distortion in resource allocation since supply of land is fixed. In effect, therefore, the property tax is equivalent to a tax on capital only.

Being equivalent to a tax on capital, the property tax reduces the amount of capital as in (b) of Proposition 6. The effect on the amount of residential land is ambiguous, since a rise in capital price causes substitution from capital to land as well as a decline in housing demand. If the elasticity of substitution is larger than the rent elasticity of housing demand, then the property tax increases residential land, and vice versa. Although residential land may decrease, it is offset by an increase in capital and the amount of housing stock always falls.

It is unlikely that the price of land rises when the property tax is raised, but I have not been able to prove that the land price always falls. As in Proposition 6, however, the price of land at the steady state falls. It can also be seen that if $\sigma<\eta$, then the price falls at each point in time.

Our results in this section are similar to those obtained by Markusen and Scheffman. The results also extend the traditional static analysis of property taxes to a dynamic framework. Proposition 6 yields the steady state effect as a special case and the steady state results correspond to the static results in the traditional literature. Although this paper does not deal with the question of the incidence of the property tax, or who ultimately bears the tax burden, the analysis of the incidence can be carried out by assuming that the subsidies, $S^A(t)$ and $S^C(t)$, are zero and examining the effects of the tax on the welfare levels of the landowner and the consumer.

6. Concluding Remarks

Constructing a perfect foresight equilibrium model of the land development process, this paper has analyzed the effects of the capital gains tax, both unanticipated and anticipated, and the property tax. The analysis can be extended in several directions.

First, housing subsidies and other forms of taxes such as a tax on accrued capital gains can be easily incorporated. In a slightly different formulation, Kanemoto (1981) showed that a subsidy on interest payments of housing loans tends to raise the prices of land and housing and to increase the amounts of residential land and housing stock. A tax on accrued capital gains can be seen to have effects similar to a tax on property values.

Second, the rental market can be introduced quite easily in addition to the property market.[6] If rental housing and owner-occupied housing are perfect substitutes for the consumer, the analysis is especially simple. In the case of no taxes, both suppliers and demanders are indifferent between rental and owner-occupied houses and their relative shares are indeterminate. A tax on property values does not change the situation if the tax is levied on rental housing as well as owner-occupied housing and agricultural land. If a realized capital gains tax is imposed, however, the property market collapses and there is only rental housing in equilibrium.

Third, housing capital is assumed to be malleable and durability of housing is ignored. If this assumption is not satisfied, the steady state may not be unique and may depend on initial conditions. In such a case, the analysis becomes much more complicated, but it is not likely that the comparative-dynamic properties that are derived in this paper are radically changed.

Finally, there is no uncertainty in our model and the financial market is complete so that everyone can lend or borrow at the same interest rate, i. Extending the model to include uncertainty and incompleteness of the financial market would be the most fruitful direction.

Appendix

Proof of Proposition 1: This follows from Lemmas 1 through 5.

Lemma 1. For any $t \geqq 0$,

$$q_t^L(L,K,t,\alpha) \geqq r_t^L(L,t,\tau)$$
$$q_t^K(L,K,t,\alpha) \geqq r_t^K(t)$$
$$q_K^L(L,\Psi(L,t),t,\alpha)\Psi_t(L,t)+q_t^L(L,\Psi(L,t),t,\alpha) \geqq r_t^L(L,t,\tau)$$
$$q_L^K(\Phi(K,t,\tau),K,t,\alpha)\Phi_t(K,t,\tau)+q_t^K(\Phi(K,t,\tau),K,t,\alpha) \geqq r_t^K(t)$$

Proof: The inequalities follow directly from Assumption 1.

Lemma 2. The perfect foresight equilibrium path satisfies $\bar{p}^L(t,\tau) \geqq \underline{p}^L(t)$ and $p^K(t) \geqq \underline{p}^K(t)$ for any t.

Proof: If $L(t)=\bar{L}$, then $L(s)=\bar{L}$ for any $s \geqq t$. Hence, by Assumption 2, $\bar{p}^L(t,\tau) > \underline{p}^L(t)$.

Suppose that, for some t_0, $L(t_0)<\bar{L}$ and $\underline{p}^L(t_0)>\bar{p}^L(t_0,\tau)$. Then $I_d^L(t_0)=I_s^L(t_0)$, or $I_d^L(t_0)<I_s^L(t_0)$ and $p^L(t_0)=0$. Furthermore, by continuity of $\underline{p}^L(t)$ and $\bar{p}^L(t,\tau)$, there exists $\varepsilon>0$ such that $\underline{p}^L(t)>\bar{p}^L(t,\tau)$ for any t in $[t_0,t_0+\varepsilon)$. If $\underline{p}^L(t)>\bar{p}^L(t,\tau)$, then either $\underline{p}^L(t)>p^L(t)$ or $p^L(t)>\bar{p}^L(t,\tau)$. In the former case, $I_d^L(t)=\infty$ and hence $I_s^L(t) \geqq I_d^L(t)=I^L(t)=\infty$, where $I^L(t)$ denotes the equilibrium path. In the latter case, $I_s^L(t)=\infty$ and since $p^L(t)>\bar{p}^L(t,\tau) \geqq 0$, we obtain $I_d^L(t)=I_s^L(t)=I^L(t)=\infty$. Thus, in both cases, $I^L(t)=\infty$ in $[t_0,t_0+\varepsilon)$ and $L(t_0+\varepsilon)=\infty$, which contradicts the assumption that $L(t) \leqq \bar{L}$. This proves the first inequality in the Lemma.

If $\underline{p}^K(t_0)>p^K(t_0)$ for some t_0, then $\underline{p}^K(t)>p^K(t)$ in $[t_0,t_0+\varepsilon)$ for some $\varepsilon>0$. Hence $K(t_0+\varepsilon)=\infty$ and, by Assumption 2, $\underline{p}^K(t_0+\varepsilon)<p^K(t_0+\varepsilon)$, which is a contradiction since by continuity $\underline{p}^K(t_0+\varepsilon) \geqq p^K(t_0+\varepsilon)$. This proves the second inequality.

Lemma 3. The perfect foresight equilibrium path satisfies $\bar{p}^L(0,\tau)=\underline{p}^L(0)$ and $p^K(0)=\underline{p}^K(0)$.

Proof: We prove the first equality only. The second equality can be proven in a similar way.

From Lemma 2, it suffices to prove that $\bar{p}^L(0,\tau)>\underline{p}^L(0)$ cannot hold. Otherwise, either $p^L(0)<\bar{p}^L(0,\tau)$ or $p^L(0)>\underline{p}^L(0)$. Hence, by the equilibrium condition, $I^L(0)=0$ and $L(0)=0$.

Now, by continuity of $\bar{p}^L$ and $\underline{p}^L$, either one of the following cases must occur: (i) there exists some t_1 such that $\bar{p}^L(t_1,\tau)=\underline{p}^L(t_1)$, and $\bar{p}^L(t,\tau)>\underline{p}^L(t)$ with $L(t)=0$ for any t in $[0,t_1)$, or (ii) $\bar{p}^L(t,\tau)>\underline{p}^L(t)$ with $L(t)=0$ for any t in $[0,\infty)$. By Assumption 2, $q^L(t)>r^L(t)$ if $L(t)=0$. Hence, in case (i), $\underline{p}^L(0)=\int_0^{t_1}q^L(t)\exp(-it)dt+\exp(-it_1)\underline{p}^L(t_1)$ $>\int_0^{t_1}r^L(t)\exp(-it)dt+\exp(-it_1)\bar{p}^L(t_1,\tau)=\bar{p}^L(0,\tau)$, and in case (ii), $\underline{p}^L(0)=\int_0^{\infty}q^L(t)\exp(-it)dt>\int_0^{\infty}r^L(t)\exp(-it)dt=\bar{p}^L(0,\tau)$. Thus, a contradiction is reached in both cases.

Lemma 4. If $\bar{p}^L(t_0,\tau)=\underline{p}^L(t_0)$ and $\bar{p}^L(t,\tau)>\underline{p}^L(t)$ for any t in (t_0,t_1), then $q^L(L(t),K(t),t,\alpha)\geqq r^L(L(t),t,\tau)$ for any t in (t_0,t_1) along the perfect foresight equilibrium path. If $p^K(t_0)=\underline{p}^K(t_0)$ and $p^K(t)>\underline{p}^K(t)$ for any t in (t_0,t_1), then $q^K(L(t),K(t),t,\alpha)\geqq r^K(t)$ for any t in (t_0,t_1) along the equilibrium path.

Proof: We prove only the first half of the Lemma, since the latter half can be proven in a similar way.

The assumption implies that $\dot{\bar{p}}^L(t_0^+,\tau)\geqq\dot{\underline{p}}^L(t_0^+)$, where superscript $^+$ denotes the right-hand limit. Hence, $q^L\geqq r^L$ at $t=t_0^+$.

Since $L(t)=L(t_0)$ at any t in (t_0,t_1), we have $\dot{\underline{p}}^K(t)=i\underline{p}^K(t)-$

$q^K(L(t_0),K(t),t,\alpha)$. Now, suppose that $\underline{p}^K(t)=p^K(t)$. Then, by Lemma 2, $\dot{\underline{p}}^K(t^-) \geqq \dot{p}^K(t) \geqq \dot{\underline{p}}^K(t^+)$ and hence $q^K(L(t_0),K(t^-),t,\alpha) \leqq r^K(t) \leqq q^K(L(t_0),K(t^+),t,\alpha)$, where superscript $^-$ denotes the left-hand limit. But, since $K(t^+) \geqq K(t^-)$ and $q^K_K=q_H(H_K)^2+qH_{KK}<0$, these inequalities hold only when $K(t^+)=K(t^-)$. It follows that $K(t)=\Psi(L(t_0),t)$ for any t in (t_0,t_1) if $\underline{p}^K(t)=p^K(t)$.

Next, if $p^K(t)>\underline{p}^K(t)$, then $\dot{K}(t)=0$. Hence, either $K(t)=\Psi(L(t_0),t)$ or $\dot{K}(t)=0$, which implies that K(t) is nondecreasing since $\dot{\Psi}(L(t_0),t)=\Psi_t>0$. Hence, by Lemma 1, q^L-r^L is nondecreasing at any t in (t_0,t_1). Combining this result with $q^L \geqq r^L$ at $t=t_0^+$ yields $q^L \geqq r^L$ at any t in (t_0,t_1).

<u>Lemma 5</u>. Along the perfect foresight equilibrium path, $\bar{p}^L(t,\tau)=p^L(t)=\underline{p}^L(t)$ and $p^K(t)=\underline{p}^K(t)$ for any t in $[0,\infty)$.

<u>Proof</u>: We prove the first equality. The second equality can be proven in a similar way.

It is first shown that $\bar{p}^L(t,\tau)>\underline{p}^L(t)$ cannot hold for any $t>0$. Suppose, on the contrary, that, for some $\tilde{t}>0$, $\bar{p}^L(\tilde{t},\tau)>\underline{p}^L(\tilde{t})$. Then, by Lemma 2 and continuity of $\bar{p}^L$ and $\underline{p}^L$, there exists some $t_0(<\tilde{t})$ such that $\bar{p}^L(t_0,\tau)=\underline{p}^L(t_0)$ and $\bar{p}^L(t,\tau)>\underline{p}^L(t)$ for any t in $(t_0,\tilde{t}]$.

Now, either one of the following two cases occurs: (i) there exists some $t_1(>\tilde{t})$ such that $\bar{p}^L(t,\tau)>\underline{p}^L(t)$ for any t in (t_0,t_1) and $\bar{p}^L(t_1,\tau)=\underline{p}^L(t_1)$, or (ii) $\bar{p}^L(t,\tau)>\underline{p}^L(t)$ for any t in (t_0,∞). In the first case, Lemma 4 implies that $q^L \geqq r^L$ for any t in (t_0,t_1). Hence, $\bar{p}^L(\tilde{t},\tau)=\int_{\tilde{t}}^{t_1} r^L\exp(-i(t-\tilde{t}))dt+\bar{p}^L(t_1,\tau)\exp(-i(t_1-\tilde{t})) \leqq \int_{\tilde{t}}^{t_1} q^L\exp(-i(t-\tilde{t}))dt+\underline{p}^L(t_1)\exp(-i(t_1-\tilde{t}))=\underline{p}^L(\tilde{t})$, which is a contradiction. The same contradiction can be derived in the second case, too. Thus, $\bar{p}^L(t,\tau)$

$=\underline{p}^L(t)$ for any $t \geqq 0$.

Next, we show that $p^L(t)=\bar{p}^L(t,\tau)=\underline{p}^L(t)$ for any t. Otherwise, either $p^L(t)>\bar{p}^L(t,\tau)=\underline{p}^L(t)$ or $p^L(t)<\bar{p}^L(t,\tau)=\underline{p}^L(t)$ for some t. In the first case, $I_d^L(t)=0$ and $I_s^L(t)=\infty$. Hence, by the equilibrium condition, $p^L(t)=0$. But, this implies $\bar{p}^L(t,\tau)=\underline{p}^L(t)<0$, which is a contradiction. In the second case, $I_d^L(t)=\infty$ and $I_s^L(t)=0$. Hence, $L(t)=\bar{L}$ and, by Assumption 2, $\bar{p}^L(t,\tau)=\int_t^\infty r^L(\bar{L},s,\tau)\exp(-i(s-t))ds<\int_t^\infty q^L(\bar{L},K(s),s,\alpha)\exp(-i(s-t))ds=\underline{p}^L(t)$, which is also a contradiction. Thus, $\bar{p}^L(t,\tau)=p^L(t)=\underline{p}^L(t)$ for any $t \geqq 0$.

<u>Proof of Proposition 2</u>: From Proposition 1, the equilibrium paths of $L(t)$ and $K(t)$ satisfy $q^L(L(t),K(t),t)=r^L(L(t),t,\tau)$ and $q^K(L(t),K(t),t)=r^K(t)$. Solving these two equations with respect to $L(t)$ and $K(t)$ yields $L(t)=\hat{L}(t,\tau)$ and $K(t)=\hat{K}(t,\tau)$ with

$$\partial\hat{L}/\partial\tau = [(r-ib)/D(1-\tau)^2][q_H(H_K)^2+qH_{KK}] < 0$$

$$\partial\hat{K}/\partial\tau = [(r-ib)/D(1-\tau)^2][q_H KHH_{KK}/L^2][\sigma-\eta] \gtrless 0 \quad \text{as } \sigma \gtrless \eta,$$

where $D=qq_H H^2 H_{KK}/L^2-r_L^L[q_H(H_K)^2+qH_{KK}]>0$. Then, $H(t)=H(\hat{L}(t,\tau),\hat{K}(t,\tau)) \equiv \hat{H}(t,\tau)$ satisfies

$$\partial\hat{H}/\partial\tau = [(r-ib)/D(1-\tau)^2]qHH_{KK}/L < 0.$$

Next, the price of land is given by $\hat{p}^L(t,\tau)=\int_t^\infty q^L(\hat{L}(s,\tau),\hat{K}(s,\tau),s)\exp(-i(s-t))ds$ and we obtain $\partial\hat{p}^L/\partial\tau>0$, since $\partial q^L/\partial\tau=qq_H H^2 H_{KK}/L^2>0$. The price net of the tax is $\hat{p}^L(t,\tau)(1-\tau)+\tau b=\int_t^\infty r(\bar{L}-\hat{L}(s,\tau)$ and $\partial r(L-\hat{L}(t,\tau),t)/\partial\tau=-r_L\partial\hat{L}/\partial\tau<0$ yields $\partial[\hat{p}^L(t,\tau)(1-\tau)+\tau b]/\partial\tau<0$.

<u>Proof of Proposition 3</u>: This follows immediately from Lemmas 6, 7, and 8.

Lemma 6. The perfect foresight equilibrium path satisfies

$$\bar{p}^L(t,\tau_0)=\underline{p}^L(t),\ L(t)=\Phi(K(t),t,\tau_0) \quad \text{in} \quad [0,T]$$

$$\bar{p}^L(t,\tau_1)>\underline{p}^L(t),\ L(t)=L_1 \geqq \Phi(K(T),T,\tau_0) \quad \text{in} \quad [T,T_1)$$

$$\bar{p}^L(t,\tau_1)=\underline{p}^L(t),\ L(t)=\Phi(K(t),t,\tau_1) \quad \text{in} \quad [T_1,\infty),$$

where T_1 may be infinite.

Proof: Applying the same argument as in Lemma 2 yields $p^K(t)\geqq\underline{p}^K(t)$ in $[0,\infty)$; $\bar{p}^L(t,\tau_0)\geqq\underline{p}^L(t)$ in $[0,T]$ and $\bar{p}^L(t,\tau_1)\geqq\underline{p}^L(t)$ in $[T,\infty)$.

Next, it is shown that $\bar{p}^L(0,\tau_0)=\underline{p}^L(0)$. Suppose the contrary. Then $\bar{p}^L(0,\tau_0)>\underline{p}^L(0)$ and $L(0)=0$. Applying the same argument as in Lemma 3 shows that there is no t_1 in $[0,T]$ such that $\bar{p}^L(t_1,\tau_0)=\underline{p}^L(t_1)$. Hence, $\bar{p}^L(t,\tau_0)>\underline{p}^L(t)$ for any t in $[0,T]$. Since $\bar{p}^L(T,\tau_0)<\bar{p}^L(T,\tau_1)$, we have $\bar{p}^L(T,\tau_1)>\underline{p}^L(T)$. Applying the argument in Lemma 3 again yields a contradiction. Hence, $\bar{p}^L(0,\tau_0)=\underline{p}^L(0)$.

Now, suppose that there exists $\tilde{t}$ in $(0,T]$ such that $\bar{p}^L(\tilde{t},\tau_0)>\underline{p}^L(\tilde{t})$. Then, we can use arguments similar to those in Lemma 4 and Lemma 5 to obtain $\bar{p}^L(t,\tau_0)>\underline{p}^L(t)$ for any t in $(t_0,T]$, where $t_0(<\tilde{t})$ satisfies $\bar{p}^L(t_0,\tau_0)=\underline{p}^L(t_0)$. Then, $\bar{p}^L(T,\tau_1)>\bar{p}^L(T,\tau_0)>\underline{p}^L(T)$ and the argument in Lemma 5 yields a contradiction. Thus, $\bar{p}^L(t,\tau_0)=\underline{p}^L(t)$ for any t in $[0,T]$.

Since $\bar{p}^L(T,\tau_1)>\bar{p}^L(T,\tau_0)=\underline{p}^L(T)$ and both $\bar{p}^L$ and $\underline{p}^L$ are continuous in t, we have $\bar{p}^L(t,\tau_1)>\underline{p}^L(t)$ for any t in $[T,T_1)$ for some T_1 in $(T,\infty]$. Note that T_1 may be infinite.

Applying the argument in Lemma 5 also yields that, if $\bar{p}^L(T_1,\tau_1)=\underline{p}^L(T_1)$ for some $T_1\geqq T$, then $\bar{p}^L(t,\tau_1)=p^L(t)$ for any t in $[T_1,\infty)$

Lemma 7. Suppose $\sigma\leqq\eta$. Then, $p^K(t)=\underline{p}^K(t)$ for any t in $[0,\infty)$, and, if $T_1<\infty$, then $L(T^-)=\hat{L}(T,\tau_0)<L_1$ and $L_1=\hat{L}(T_1,\tau_1)$.

Proof: The same argument as in the proof of Lemma 3 proves $p^K(0)=\underline{p}^K(0)$.

The arguments in Lemmas 4 and 5 cannot be applied directly to prove that $p^K(t)=\underline{p}^K(t)$ for any t in $[0,\infty)$, since L(t) may have a jump at T. If $\sigma\leqq\eta$, however, $\partial q^K(K,L,t)/\partial L\geqq 0$ and an upward jump in L(t) cannot cause a fall in q^K. Hence, a slight modification of the arguments in the proof of Lemmas 4 and 5 proves the equality.

Next, since $\underline{p}^L(T)=\bar{p}^L(T,\tau_0)$ and $\bar{p}^L(T_1,\tau_1)=\underline{p}^L(T_1)>\bar{p}^L(T_1,\tau_0)$, we have $\int_T^{T_1} q^L\exp(-i(t-T))dt<\int_T^{T_1} r^L\exp(-i(t-T)dt$. From Assumption 1, $\dot{q}^L\geqq\dot{r}^L$ and hence $q^L(L_1,K(T),T)<r^L(L_1,T,\tau_0)$. But, since $q^L_L-r^L_L<0$ and $q^L(L(T^-),K(T),T)=r^L(L(T^-),T,\tau_0)$, we have $L_1>L(T^-)=L(T,\tau_0)$.

Finally, we show that $L_1=\hat{L}(T_1,\tau_1)$. Suppose the contrary. Then, $L_1<\hat{L}(T_1,\tau_1)$. Since $q^L(\hat{L}(T_1,\tau_1),\hat{K}(T_1,\tau_1),T_1)=r^L(\hat{L}(T_1,\tau_1),T_1,\tau_1)$ and $q^L_L(L,\Psi(L,T_1),T_1)+q^L_K\Psi_L-r^L_L(L,T_1,\tau_1)<0$, it follows that $q^L(L_1,\Psi(L_1,T_1),T_1)-r^L(L_1,T_1,\tau_1)>q^L(\hat{L}(T_1,\tau_1),\hat{K}(T_1,\tau_1),T_1)-r^L(\hat{L}(T_1,\tau_1),T_1,\tau_1)=0$. Hence, $\dot{\underline{p}}^L(T_1^-)<\dot{\bar{p}}^L(T_1^-,\tau_1)$, which contradicts the fact that $\bar{p}^L(t,\tau_1)>\underline{p}^L(t)$ in (T,T_1) and $\bar{p}^L(T_1,\tau_1)=\underline{p}^L(T_1)$.

<u>Lemma 8</u>. Suppose $\sigma>\eta$. Then $\underline{p}^K(t)=p^K(t)$ in $[0,t_0]$ for some t_0 in $[0,T]$, $\underline{p}^K(t)<p^K(t)$ in $[t_0,t_1)$ for some t_1 in $[T,T_1)$, and $\underline{p}^K(t)=p^K(t)$ in $[t_1,\infty)$, where if $t_0=t_1=T$, then $\underline{p}^K(t)=p^K(t)$ for any t. If $T_1<\infty$, then $L(T^-)=\Phi(K(t_0),T,\tau_0)<L_1$ and $L_1=\hat{L}(T_1,\tau_1)$.

<u>Proof</u>: The arguments in Lemmas 2 and 3 can be applied to yield $p^K(t)\geqq\underline{p}^K(t)$ for any t and $p^K(0)=\underline{p}^K(0)$.

If $L_1=L(T^-)$, then arguments similar to those in the proofs of Lemmas 4 and 5 can be applied to show that, if $p^K(\tilde{t})>\underline{p}^K(\tilde{t})$ for some $\tilde{t}$ in $(0,T_1]$, then $p^K(t_0)=\underline{p}^K(t_0)$ for some $t_0<\tilde{t}$ and $p^K(t)>\underline{p}^K(t)$ for any t in $(t_0,T_1]$. Hence, $K(T_1)=K(t_0)$. Now, it is shown that $L_1=L(T_1^+)=\Phi(K(t_0),T_1,\tau_1)$. Otherwise, $L_1<L(T_1^+)$. But, since $\dot{\underline{p}}^L(T_1^-)\geqq\dot{\bar{p}}^L(T_1^-,\tau_1)$,

we have $q^L(L_1,K(t_0),T_1)\leqq r^L(L_1,T_1,t_1)$ and hence $q^L(L(T^+),K(t_0),T_1)$ $<q^L(L_1,K(t_0),T_1)\leqq r^L(L_1,T_1,\tau_1)<r^L(L(T_1^+),T_1,\tau_1)$, which implies $\dot{\underline{p}}^L(T_1^+)$ $>\dot{\bar{p}}^L(T_1^+,\tau_1)$. This contradicts inequality, $\bar{p}^L(t,\tau_1)\geqq \underline{p}^L(t)$ for any t, obtained in Lemma 6.

Thus, there is no jump in L(t) at $t=T_1$ and arguments similar to those in Lemmas 4 and 5 yield a contradiction. Hence $p^K(t)=\underline{p}^K(t)$ for any t in $[0,T_1]$.

Next, consider the case of $L_1>L(T^-)$. It is first shown that $p^K(T)$ $>\underline{p}^K(T)$. Otherwise, $p^K(T)=\underline{p}^K(T)$ and $\dot{\underline{p}}^K(T^-)\geqq\dot{\underline{p}}^K(T)\geqq\dot{\underline{p}}^K(T^+)$. This implies that $q^K(L(T^-),K(T^-),T)\leqq r^K(T)\leqq q^K(L_1,K(T^+),T)$. But, since $\partial q^K/\partial L<0$ and $\partial q^K/\partial K<0$, it follows that $q^K(L(T^-),K(T^-),T)>q^K(L_1,K(T^-),T)\geqq q^K(L_1,K,T)$ for any $K\geqq K(T^-)$, which contradicts the above inequality.

Second, arguments similar to those in Lemmas 4 and 5 show that there exists t_0 in $(0,T]$ such that $p^K(t)=\underline{p}^K(t)$ for any t in $[0,t_0]$ and $p^K(t)>\underline{p}^K(t)$ for any t in $(t_0,T]$.

Third, it is shown that $q^K(K(t_0),\Phi(K(t_0),t,\tau_0),t)>r^K(t)$ in $(t_0,T]$, $q^K(K(t_0),L_1,t)<r^K(t)$ in $[T,t_1)$, and $q^K(K(t_0),L_1,t)>r^K(t)$ in (t_1,∞). From $\dot{p}^K(t_0)\geqq\dot{\underline{p}}^K(t_0)$, we obtain $q^K(K(t_0),L(t_0),t_0)\geqq r^K(t_0)$. Hence, by Lemma 1, $q^K(K(t_0),\Phi(K(t_0),t,\ _0),t)\geqq r^K(t)$ in $(t_0,T]$.

Next, we show that $q^K(K(t_0),L_1,T)<r^K(T)$. Otherwise, $\dot{p}^K(T^+)\geqq\dot{\underline{p}}^K(T^+)$ and hence, by Lemma 1 and $p^K(T)>\underline{p}^K(T)$, we obtain $p^K(t)>\underline{p}^K(t)$ in $(T,T_1]$. Then the argument used in the case of $L_1=L(T^-)$ can be applied to derive $L_1=L(T_1^+)=\Phi(K(t_0),T,\tau_1)$. Hence, $q^K(t)>r^K(t)$ in (T,∞) and a contradiction is obtained in the same way as in Lemma 5.

Thus, $q^K(K(t_0),L_1,T)<r^K(T)$. Since $dq^K/dt\geqq dr^K/dt$, it follows that $q^K(K(t_0),L_1,t)<r^K(t)$ in $[T,t_1)$ and $q^K(K(t_0),L_1,t)>r^K(t)$ in (t_1,∞) for

some t_1, where t_1 may be infinite.

Fourth, it is shown that, if $\tilde{t}$ is defined by $p^K(\tilde{t})=\underline{p}^K(\tilde{t})$ and $p^K(t)>\underline{p}^K(t)$ in $(t_0,\tilde{t})$, then $\tilde{t}<T_1$. Suppose the contrary: $\tilde{t}\geqq T_1$. At $t=\tilde{t}$, $\dot{p}^K(\tilde{t}^-)\leqq\dot{\underline{p}}^K(\tilde{t}^-)$ and hence $q^K(K(t_0),\Phi(K(t_0),\tilde{t},\tau_1),\tilde{t})\leqq r^K(\tilde{t})$. But, $q^K(\hat{K}(\tilde{t},\tau_1),\Phi(\hat{K}(\tilde{t},\tau_1),\tilde{t},\tau_1),\tilde{t})=q^K(\hat{K}(\tilde{t},\tau_1),\hat{L}(\tilde{t},\tau_1),\tilde{t})=r^K(\tilde{t})$ and $(\partial/\partial K)q^K(K,\Phi(K,t,\tau_1),t)\leqq 0$ by concavity of $H(L,K)$. Hence, $K(t_0)\geqq\hat{K}(\tilde{t},\tau_1)$. But we have $K(t_0)=\hat{K}(t_0,\tau_0)<\hat{K}(t,\tau_0)<\hat{K}(t,\tau_1)$, where the last inequality results from Proposition 2, and a contradiction is derived.

Fifth, it is shown that $\tilde{t}=t_1$. If $\tilde{t}>t_1$, then $\underline{p}^K(t_1)=\int_{t_1}^{\tilde{t}} q^K\exp(-i(t-t_1)d+\underline{p}^K(\tilde{t})\exp(-i(\tilde{t}-t_1))\geqq\int_{t_1}^{\tilde{t}} r^K\exp(-i(t-t_1)dt+\underline{p}^K(\tilde{t})\exp(-i(\tilde{t}-t_1))=\underline{p}^K(t_1)$, which is a contradiction. If $\tilde{t}<t_1$, then $p^K(\tilde{t})=\underline{p}^K(\tilde{t})$ and $\dot{p}^K(\tilde{t})<\dot{\underline{p}}^K(\tilde{t})$. This contradicts $p^K(t)\geqq\underline{p}^K(t)$.

Sixth, we show that $p^K(t)=\underline{p}^K(t)$ for any t in $[t_1,T_1]$. If $p^K(\tilde{t})>\underline{p}^K(\tilde{t})$ for some $\tilde{t}$ in $[t_1,T_1]$, then arguments similar to those in Lemmas 4 and 5 yield $p^K(t)>\underline{p}^K(t)$ for any t in $[\tilde{t},T_1]$. Hence, the argument used above can be applied to obtain continuity of $L(t)$ at T. Then, arguments similar to those in Lemmas 4 and 5 can be used again to derive a contradiction. Thus, $p^K(t)=\underline{p}^K(t)$ for any t in $[t_1,T_1]$.

Regardless of $L(t)$ having a jump at T_1, an argument similar to that in the proof of Lemma 7 proves that if $T_1<\infty$, then $L(T^-)<L_1$.

Next, we show that $L_1=\hat{L}(T_1,\tau_1)$ and $K(T_1)=\hat{K}(T_1,\tau_1)$. Otherwise, $L_1<\hat{L}(T_1,\tau_1)=L(T_1^+)$ and/or $K(T_1^-)<\hat{K}(T_1,\tau_1)=K(T_1^+)$. Since $\underline{p}^K(t)=p^K(t)$ in $[t_1,T_1]$, we have $q^K(L_1,K(T_1^-),T_1)=r^K(T_1)$. Hence, $q^K(L(T_1^+),K(T_1^+),T_1)\leqq q^K(L_1,K(T_1^+),T_1)\leqq q^K(L_1,K(T_1^-),T_1)=r^K(T_1)$, where one of the inequalities is strict. Hence, $\dot{\underline{p}}^K(T^+)>\dot{p}^K(T^+)$, which is a contradiction.

Finally, arguments similar to those in the proofs of Lemmas 4 and 5 yield $p^K(t)=\underline{p}^K(t)$ in $[T_1,\infty)$.

Proof of Proposition 4: Since $\underline{p}^L(T)=\bar{p}^L(T,\tau_0)$, $\underline{p}^L(T_1)=\bar{p}^L(T_1,\tau_1)$, and $\underline{p}^L(T)=\int_T^{T_1} q^L\exp(-i(t-T))dt+\underline{p}^L(T_1)\exp(-i(T_1-T))=\bar{p}^L(T,\tau_0)=\int_T^{T_1} r^L \exp(-i(t-T))dt+\bar{p}^L(T_1,\tau_0)\exp(-i(T_1-T))$, we obtain $\int_T^{T_1}[q^L(L_1,K(t),t) -r^L(L_1,t,\tau_0)]\exp(-i(t-T))dt=-[(\tau_1-\tau_0)/(1-\tau_0)(1-\tau_1)]\{\int_{T_1}^{\infty}[r(\bar{L}-\hat{L}(t,\tau_1),t) -ib]\exp(-i(t-T))dt$. Hence, if $\sigma\leqq\eta$, then $K(t)=\Psi(L_1,t)$ in $[T,T_1)$ and

$$\frac{\partial L_1}{\partial \tau_1} = \frac{-1}{\Delta(1-\tau_1)^2(1-\tau_0)}\int_{T_1}^{\infty}\frac{r-ib}{D}[(1-\tau_0)qq_H(H/L)^2H_{KK}+(q_HL(H_K)^2+qH_{KK})r_L]$$
$$\exp(-i(t-T))dt > 0 \qquad (*)$$

where it is easy to see that $\Delta=\int_T^{T_1}[q_L^L-r_L^L+q_K^L\Psi_L]\exp(-i(t-T))dt<0$.

If $\sigma>\eta$, then $K(t)=K(t_0)$ in $[T,t_1)$ and $K(t)=\Psi(L_1,t)$ in $[t_1,T_1)$. Since $\underline{p}^K(t_0)=p^K(t_0)$ and $\underline{p}^K(t_1)=p^K(t_1)$, $K(t_0)$ satisfies $\int_{t_0}^{T} q^K(\Phi(K(t_0),t,\tau_0), K(t_0),t)\exp(-i(t-t_0)dt+\int_T^{t_1} q^K(L_1,K(t_0),t)\exp(-i(t-t_0))dt= \int_{t_0}^{t_1} r^K(t)\exp(-i(t-t_0))dt$. Totally differentiating the above equality and noting that $q^K=r^K$ at $t=t_0$ and $t=t_1$, we obtain $dK(t_0)/dL_1=-[\int_T^{t_1} q_L^K\exp(-i(t-t_0))dt]/[\int_{t_0}^{T} (q_L^K\Phi_K+q_K^K) \exp(-i(t-t_0))dt+\int_T^{t_1} q_K^K\exp(-i(t-t_0))dt]$. Since $q_L^K<0$, $q_K^K<0$, and $q_L^K\Phi_K+q_K^K<0$, we obtain $0>dK(t_0)/dL_1>-[\int_T^{t_1} q_L^K\exp(-i(t-t_0))dt/\int_T^{t_1} q_K^K\exp(-i(t-t_0))dt]$.

Now, in the same way as in the case of $\sigma\leqq\eta$, it can be seen that $\partial L_1/\partial\tau_1$ satisfies the equality in (*), where $\Delta=\int_T^{t_1}[q_L^L-r_L^L+q_K^L dK(t_0)/dL_1] \exp(-i(t-T))dt+\int_{t_1}^{T_1}[q_L^L-r_L^L+q_K^L\Psi_L]\exp(-i(t-T))dt$. The second integral on the right hand side is negative as in the previous case. Next, we show that the first integral is negative. Define $Q_H=\int_T^{t_1} q_H\exp(-i(t-T))dt$ and $Q=\int_T^{t_1} q\exp(-i(t-T))dt$. Then, since $L(t)=L_1$ and $K(t)=K(t_0)$ in $[T,t_1)$ and $q_L^L=q_H(H_L)^2+qH_{LL}$, $q_K^L=q_HH_KH_L+qH_{LK}$, $q_K^K=q_H(H_K)^2+qH_{KK}$, the first integral satisfies

$$\int_T^{t_1}[q_L^L-r_L^L+q_K^L dK(t_0)/dL_1]\exp(-i(t-T))dt$$
$$<\int_T^{t_1}[q_L^L-r_L^L-q_K^L(\int_T^{t_1} q_L^K\exp(-i(t-T))dt/\int_T^{t_1} q_K^K\exp(-i(t-T))dt)]\exp(-i(t-T))dt$$
$$<(H_L)^2Q_H+H_{LL}Q-(H_KH_LQ_H+H_{LK}Q)(H_KH_LQ_H+H_{LK}Q)/((H_K)^2Q_H+H_{KK}Q)$$

$$= \frac{1}{(H\ )^2 Q_H + H_{KK} Q} \{Q^2[H_{KK}H_{LL}-(H_{LK})^2]+Q_H Q[(H_L(-H_{KK})^{1/2}-H_K(-H_{LL})^{1/2})^2$$

$$+2H_K H_L((-H_{KK})^{1/2}(-H_{LL})^{1/2}-H_{LK})]\} \leqq 0.$$

Hence, $\Delta<0$ and $\partial L_1/\partial\tau_1>0$. The Proposition now follows.

Proof of Proposition 5: Similar to the proof of Proposition 1 and omitted.

Proof of Proposition 6: From Proposition 5, we have $q(H(L(t),K(t)),t)$ $H_L(L(t),K(t))=r(\bar{L}-L(t),t)$ and $q(H(L(t),K(t)),t)H_K(L(t),K(t))=(i+\tau)p^K(t)$ $-\dot{p}^K(t)$, which can solved to yield $L(t)=\hat{L}(t,\tau)$ and $K(t)=\hat{K}(t,\tau)$, where

$$\partial\hat{L}/\partial\tau = [p^K(t)/D]q_H(HK/L)H_{KK}[\sigma-\eta] \gtrless 0 \quad \text{as} \quad \sigma \gtrless \eta$$

$$\partial\hat{K}/\partial\tau = [p^K(t)/D][q_H(H_L)^2+qH_{LL}+r_L] < 0$$

and D is the same as that in the proof of Proposition 2. Hence, $\hat{H}(t,\tau)$ $=H(\hat{L}(t,\tau),\hat{K}(t,\tau))$ satisfies

$$\partial\hat{H}/\partial\tau = [p^K(t)/D][q(HK/L^2)H_{KK}+LH_K r_L] < 0.$$

Next, define $\hat{p}^L(t,\tau)=\int_t^\infty r(\bar{L}-\hat{L}(s,\tau),s)\exp(-(i+\tau)(s-t))ds$. Then

$$\partial\hat{p}^L/\partial\tau = \int_t^\infty[-r_L(\partial\hat{L}/\partial\tau)+(t-s)r]\exp(-(i+\tau)(s-t))ds$$

$$= -\int_t^\infty[r_L(\partial\hat{L}/\partial\tau)+p^L(s)]\exp(-(i+\tau)(s-t))ds,$$

where the second equality follows from integration by parts. The integrand is

$$r_L(\partial\hat{L}/\partial\tau)+p^L = \frac{1}{D}[p^L q q_H(H/L)^2 H_{KK}+r_L(p^K q(K/L)H_{KK}+p^L qH_{KK})$$
$$+r_L q_H LH_K(p^L H_K-p^K H_L)].$$

At the steady state, $\dot{p}^L/p^L=\dot{p}^K/p^K\equiv n$. Then, $qH_L=(i+\tau-n)p^L$ and $qH_K=(i+\tau-n)p^K$. Hence

$$r_L(\partial\hat{L}/\partial\tau)+p^L = \frac{1}{D}[p^L q q_H(H/L)^2 H_{KK}+r_L(p^K q(K/L)H_{KK}+p^L qH_{KK})] > 0$$

and $\partial\hat{p}^L/\partial\tau<0$.

Footnotes

1. See, for example, Henderson (1977) and Kanemoto (1980) for the static analysis of the residential market in a spatial model, and Mieszkowski (1972), Courant (1977), Sonstelie (1979), and Ch.9 of Henderson (1977) for the static analysis of the property tax.
2. They also analyzed the monopolistic behaviour of developers and compared the monopolistic solution with the competitive solution.
3. The assumption of $r_t>0$ and the later assumption of $q_t>0$ are made to ensure that the equilibrium path of land price rises over time. Otherwise, the analysis of the capital gains tax is meaningless since there are no capital gains.
4. No generality is lost by assuming that the discount rate is i, since the utility function is time dependent.
5. This budget constraint implicitly assumes that the consumer can borrow money at the same interest rate as the rate of return on the bond. Because of this assumption, the equilibrium path does not depend on the time profile of income, y(t), as long as the discounted sum remains the same.
6. As mentioned in the Introduction, Weiss (1978) analyzed the effect of capital gains taxes on the choice between renting and owning a house.

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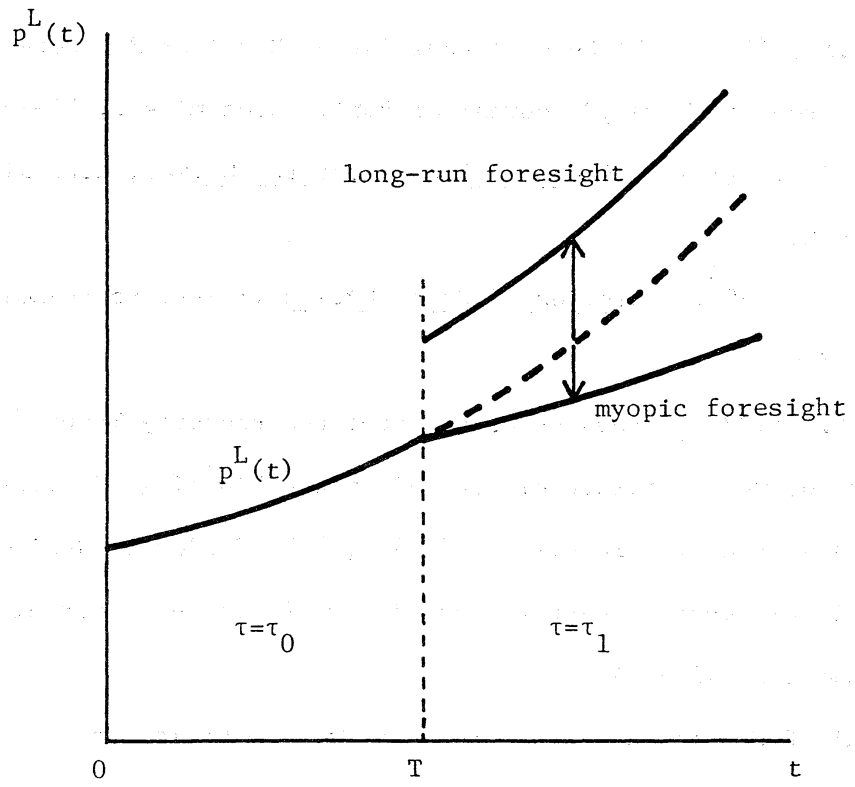

Fig. 1. Long-run and mypic foresight cases

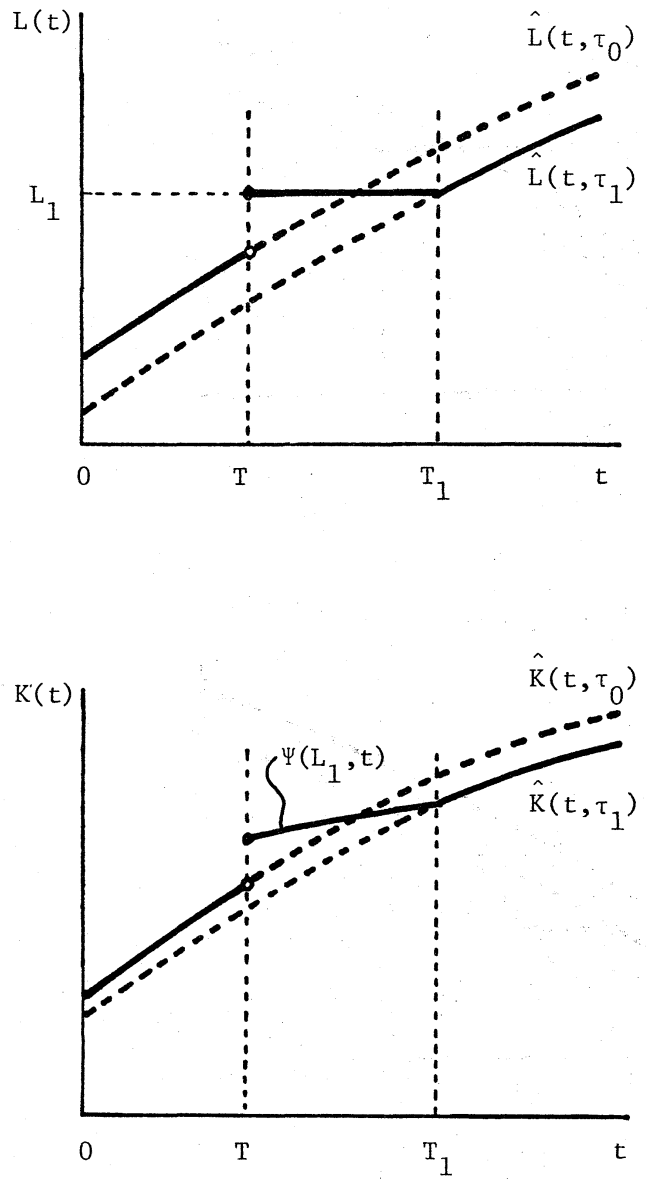

Fig. 2. An anticipated rise in the capital gains tax: the case of $\sigma \leqq \eta$

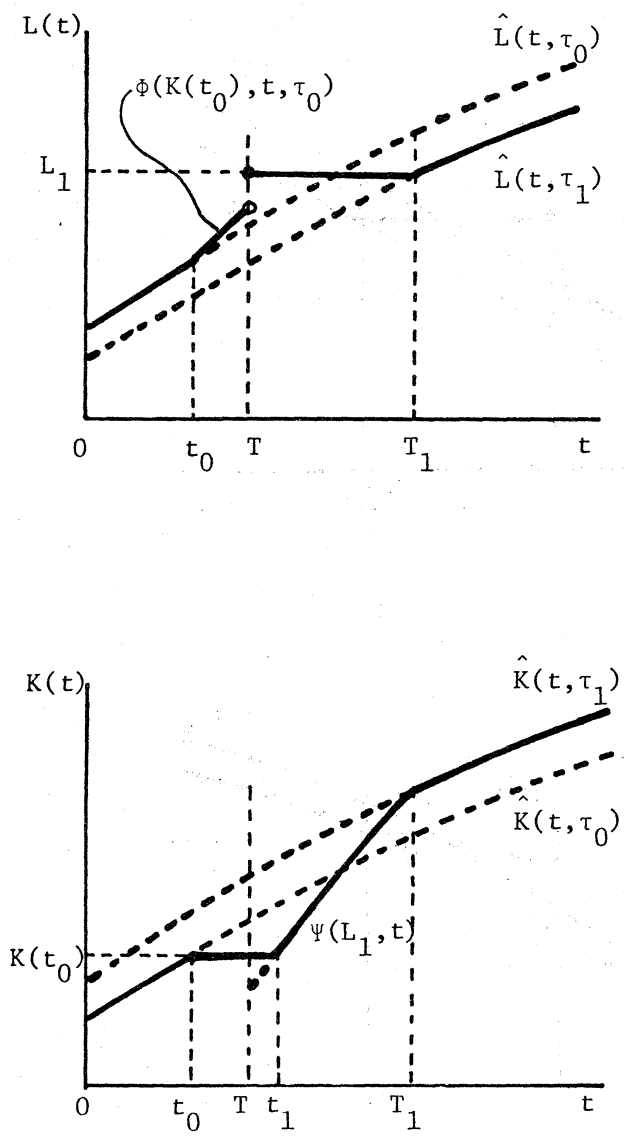

Fig. 3. An anticipated rise in the capital gains tax: the case of $\sigma > \eta$