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OPTIMAL GROWTH MODELS UNDER ALTERNATIVE CRITERIA

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0. INTRODUCTION

"What is the most reasonable criterion which can be applied to an intergenerational resource allocation problem especially in a growing economy?" The question has a unique aspect. That is, a generation can do something for later generations, but cannot be compensated by them because of the irreversible nature of time. The paper is devoted to a search for such a criterion.

Much work has been done concerning the optimal capital accumulation and consumption paths since F. Ramsey's celebrated paper in 1928. It is now well known that the optimal paths crucially depend upon the formulation of a social welfare function. It was the standard procedure until the early 1970s to adopt the utilitarian criterion as the objective funciton. In the 1960s we saw many variants of the Ramsey model, e.g., Cass (1965), von Weizsäcker (1965), and a series of papers by Koopmans. Their models are all based on the utilitarian criterion.

This attitude had to change as a result of J. Rawls's influential book, A Theory of Justice, published in 1971. In the book, Rawls, criticizing the utilitarian criterion as inappropriate on the ground that it neglects the aspect of justice, and basing his theory on the social contract of justice, proposes an alternative optimality criterion that initiated economists to consider the fairness or just allocation in the optimal economic growth model. Both Arrow (1973a) and Dasgupta (1974) were the first papers to capture the Rawls idea in their formulation of the criterion.

However, as we will see later, the consequence of the Rawlsian criterion is not appealing either, especially in a growing economy. Roughly speaking, it demands each geneation zero saving in order to assure just allocation. Having observed these results, we came to realize the necessity of a third criterion incorporating the spirit of both criteria. We shall call the third criterion the "utilitarian-Rawls criterion" and explore its implications.

In this paper we shall first compare the consequences of the three criteria numerically using a simplified economy, and see the advantages of our criterion (Section I). However, the optimal consumption path under the criterion shows "inconsistency" of the plan. In order to justify the criterion, therefore, we reinterpret the concept of a social welfare functional. This new interpretation will give a sound basis for the validity of our criterion. The amount of utility information which each criterion uses is also clarified. It is pointed out that our criterion reflects both "unit" and "level" comparabilities (Section II). Finally, in Section III, the optimal paths in a Solow type economy are considered. Also, as the limiting case (infinite time horizon economy) of our criterion, the "Modified Rawlsian criterion" is proposed.

I. THE OPTIMAL PATHS IN THE ARROW-DASGUPTA TYPE ECONOMY

1. The Arrow-Dasgupta Type Economy

We shall make the following assumptions concerning the economy discussed in this section.

A-1 Time is measured in discrete intervals of unit length, and the horizon is finite. Time is denoted by i. $(i = 0,1,2, \ldots ,T)$

A-2 There exists a single, non-deteriorating, homogeneous commodity, the stock of which at time i is denoted as K_i.

A-3 The population size in each generation is constant and is denoted by L. Each generation lives for precisely one period and is replaced by the following generation at the beginning of the next period.

A-4 Aggregate consumption at time i is denoted by c_i, and c_i is divided equally among all persons of generation i.

A-5 The preference of each generation is identical, and is represented by a utility function $u(c_i)$ that is concave, increasing, and differentiable in c_i.

Following Arrow (1973a) and Dasgupta (1974), we describe the technology of the economy as follows. At the beginning of time period i, the part of K_i is consumed, and the remainder, $K_i - c_i$, is used in production In the next time period the accumulated capital will be the saving times λ, the production coefficient. Therefore, the capital accumulation path is described by the basic difference equation

$$K_{i+1} = \lambda(K_i - c_i),$$

where λ is assumed to be greater than one.

2. The Three Criteria

Let us summarize the three crieria below.

The utilitarian criterion is formulated by a sum of welfare of each generation,

$$\text{(I)} \quad W_I = \sum_{i=0}^{T} u(c_i).$$

The planner's problem is, therefore, to locate a consumption profile $\{c_i\}$ which will maximize (I) subject to certain feasibility conditions.

As an alternate to the utilitarian criterion, the Rawlsian criterion will take a form of

$$\text{(II)} \quad W_{II} = \text{Min}\,\{u(c_0),\, u(c_1),\, \ldots\,,\, u(c_T)\},$$

which means that the society should be particularly concerned with the least advantaged generation.

Now, we shall propose a third criterion, the utilitarian-Rawls criterion, which embodies both the utilitarian and Rawlsian spirit. It will take the form

$$\text{(III)} \quad W_{III} = \sum_{i=0}^{t} u(c_i) + (T - t)\text{Min}\{u(c_{t+1}),\, u(c_{t+2}),\, \ldots\,,\, u(c_T)\}.$$

A brief comment on the criterion follows. The entire generations are divided into two groups. In the first group of $t + 1$ generations the criterion takes the utilitarian form. On the other hand, the Rawlsian form is taken by the remaining $T - t$ generations. The "t" may be interpreted as the degree of natural concern that the present generation bears directly for the following t generations. After t periods, we assume, the Rawls "behind the veil of ignorance" becomes dominant and that portion of the criterion is reduced to the Rawlsian form.

3. The Optimal Consumption Paths

Let us calculate the optimal consumption path under each criterion in the Arrow-Dasgupta type economy. The general problem may be formulated as

$$\max_{\{c_i\}} W = W\{u(c_0), u(c_1), \ldots, u(c_T)\}$$

$$\text{s.t. } K_{i+1} = \lambda(K_i - c_i)$$

$$K_0 \text{ and } K_{T+1} \text{ given, } K_i \geqq 0 \text{ all } i \in [0, T+1].$$

The optimal consumption path under W_I is known to be a monotone increasing over time. The later generations can enjoy a fruit which is made possible by the sacrifice of the earlier generations. This is a straightforward consequence of the utilitarian criterion in a growing economy.

The optimal path under W_{II} is given by an equal consumption over the generations the level of which depends on the initial and terminal capital stocks, K_0 and K_{T+1}, the production coefficient, and the periods of time, T.

The optimal path under W_{III} exhibits a jump at $i = t$. The jump becomes greater and greater as "t" approaches to T, which is equivalent to saying that the criterion becomes closer and closer to W_I. We also note that it passes through intermediate between the above two paths. These results are depicted in Figure 1.

The figure indicates that if the priority of the policy objective is given to the social justice in the Arrow-Dasgupta economy, the utilitarian criterion must be rejected on the basis of negligence of intergenerational fairness. On the other hand, as we have already seen, the Rawlsian criterion results in a stationary consumption path even in the potentially growth-oriented economy. Our approach, however, reconciles the growth-oriented utilitarian spirit and the fairness-oriented Rawlsian spirit, and incorporate both of them.

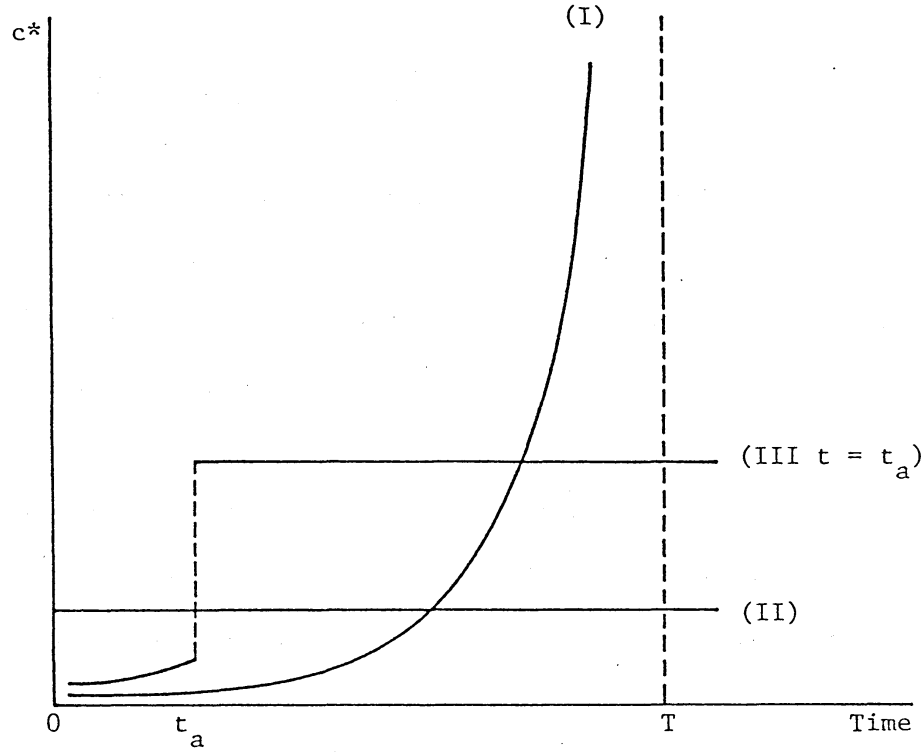

Figure 1 Three optimal consumption paths in the Arrow Dasgupta type economy for a certain terminal capital stock. The figure is based on the data that $K_0 = 1$, $T = 10$, $K_{T+1} = 0$, $\lambda = 2$, $t_a = 2$, and $u = \sqrt{c_i}$.

II. THE SOCIAL WELFARE FUNCITONAL AND INFORMATIONAL BASIS

1. The Intergenerational Welfare Functional (IGWFL)

Some may argue that a decision made at time i for the remaining T-i periods must be invariant if the decision is reevaluated at time i+j for $0 < j \leqq T-i$. As is seen from Figure 1, the criterion W_{III} leads to intertemporal inconsistency in this sense. On the other hand, the paths under the criteria W_I and W_{II} guarantee consistency. This is a natural consequence of an implicit assumption behind the last two criteria.

Let us recall the definition of a social welfare functional (SWFL). Consider a society consisting of n individuals with alternatives x, y,.. . Let

$N = \{1, 2, \ldots, n\}$ be a set of individuals, the number of which is finite, and

$X = \{x, y, z, \ldots\}$ the set of every conceivable alternatives, the number of which is at least three and can be infinite.

Individual i's preference ordering, R_i, can be represented by a numerically bounded utility function $u(\cdot, i)$ defined on X x N.

Definition [Sen 1970]

Let $\mathcal{U}$ be the set of all such real-valed functions. Then the SWFL is defined to be a mapping f from a set $\mathfrak{D} \subseteq \mathcal{U}$ to the set $\mathcal{R}$, i.e., $f: \mathfrak{D} \to \mathcal{R}$, where $\mathcal{R}$ is the set of social ordering on X.

If the decision problem is about contemporaries, then it might be possible to conceive of someone who collects each individual's preference orderings for aggregation. In the intergenerational decision problem, however, how can he collect information of the later generations who are not yet born? The two criteria, as members of the social welfare functional, implicitly assume the existence of a so-called ethical observer who is thought to know everything. By this device consistency is required to every optimal plan based on the SWFL.

We believe that an assumption of the existence of such imaginary ethical observer does not fit for an intergenerational planning problem. Therefore, we would like to negate him, and propose a new interpretation of the collective decision rule. That is, eliminating a rather artificial notion of ethical observer, we interpret the SWFL as the criterion viewed from the present generation. Harsanyi (1977)'s exposition is very instructive in this aspect. The present generation, say generation 0, puts itself as the i-th generation's position and imagines the i-th generation's utility. In this moral judgment generation 0 must be impersonal and impartial. Generation 0's moral judgments go up to the t-th generation. However, it is still known that the entire time periods is T + 1. Thus generation 0 is assemed to place the remaining T-t generations, upon whom it has not the slightest conceptualization, in the Rawls's "original position."

Because of this new interpretation, it does not matter whether the second generation follows the original plan made by the present generation or it makes its own plan in a different way. The important point is that the present generation should follow the current plan which reflects also the welfare of future generations.

Since it is not appropriate to use a term the SWFL by the above stated reason, we denote it as the "intergenerational welfare functional (IGWFL)" by emphasizing that our criterion is a result of moral judgments conducted by the present generation. Every generation, therefore, may have a different IGWFL in our non-overlapping society model. However, only the present generation's IGWFL is implemented for a present planning problem.

2. Informational Basis

Let us compare the three criteria by the amount of welfare information which each criterion uses. For this purpose we shall introduce a measurement condition that partitions $\mathcal{U}$ into equivalent classes under some equivalence relation $\simeq$ such that for all u, $u' \varepsilon \mathcal{U}$, $u \simeq u' \Longrightarrow f(u) = f(u')$. (See, e.g., D'Aspremont-Gevers 1977)

Cardinal Unit Comparability (CUC)

For every u, $u' \varepsilon \mathcal{U}$, $u \simeq u'$ if there exist n + 1 numbers $a_1, a_2, \ldots, a_n$ and $b > 0$ such that, $\forall i \varepsilon N$, $\forall x \varepsilon X$, $u(x, i) = a_i + b\, u'(x, i)$.

Ordinal Level Comparability (OLC)

For every u, $u' \varepsilon \mathcal{U}$, $u \simeq u'$ if there exists a strictly increasing numerical function ϕ such that, $\forall i \varepsilon N, \forall x \varepsilon X$, $u(x, i) = \phi(u'(x, i))$.

Cardinal Full Comparability (CFC)

For every u, $u' \varepsilon \mathcal{U}$, $u \simeq u'$ if there exist 2 numbers a and $b > 0$ such that, $\forall i \varepsilon N$, $\forall x \varepsilon X$, $u(x, i) = a + bu'(x, i)$.

It is easily verified that the utilitarian criterion is based on CUC and is sensitive only to the welfare gains and losses. The Rawlsian criterion shows the other extreme, i.e., it is based on OLC and responds only to the welfare levels of generations. Sen states:

> . . . in making ethical judgments on distributional issues one is typically concerned both with comaprisons of levels of welfare as well as comparisons of welfare gains and losses. It is not surprising that the utilitarian approach and the maximin approach both run into some fairly straightforward difficulties since each leaves out completely one of the two parts of the total picture. . . But a more complete theory is yet to emerge (Sen 1974, p. 292).

Our criterion has an advantage in this respect since it employs both types of comparisons. That is, in the first group of t+1 generations welfare gains and losses are compared, and in the second group of remining T-t generations, welfare levels are compared. Furthermore, welfare gains and losses are compared between two groups. Thus the utilitarian-Rawls criterion commands CFC for its informational basis. In this sense, it can be said that it is an informationally richer criterion.

III. THE OPTIMAL PATHS IN THE SOLOW TYPE ECONOMY

0. From the Arrow-Dasgupta to Solow Type Economy

In the previous section, we have studied various optimal consumption paths resulting from three different criteria numerically. Although the results help us contrast the differences among the criteria, the economy considered there is oversimplified in the following aspects:

i) The technology is represented simply by a constant marginal productivity.

ii) The population is stationary.

iii) The finite time horizon is taken.

Thus we shall instead employ a standard Solow type economy in this section, where the technology is represented by the well-behaved neo-classical production function, and trace the consequences of the utilitarian-Rawls criterion, especially in an infinite time horizon with growing population.

It is now well known that the optimal path recommended by a planner using the utilitarian criterion forces the earlier generations to accumulate capital heavily so that the economy approaches to the (modified) golden rule path asymptotically if the economy's initial capital stock is below the (M)GR level (e.g., Ramsey 1928 , Cass 1965). On the path the sum of generations' utilities is maximized.

On the other hand, the optimal path under the Rawlsian criterion results in a constant consumption level over the generations provided that the marginal rate of transformation between generations is positive (Solow 1974).

1 . Problem

The utilitarian-Rawls criterion in a continuous version with growing population may be formulated as:

$$\int_0^t e^{ni}u(c(i))di + (e^{nT} - e^{nt}) \inf_{i\varepsilon[t,T]} \{u(c(i))\}, \tag{1}$$

where n denotes the rate of population growth and the number of people at initial time $L(0) = L_0 \equiv 1$ for normalization. Note that $e^{nT} - e^{nt}$ represents the number of people belonging to the generations t through T.

The constraints and the initial condition are given by

$$(2) \qquad \begin{aligned} &\dot{k} = f(k) - nk - c \\ &k(0) = k_0 \text{ given and } k(i) \geq 0 \text{ for all } i \,\varepsilon\, [0, T]. \end{aligned}$$

The assumptions on u(c) and f(k) include:

A.1 The utility function $u = u(c)$ is concave with $u'(c) > 0$, $u''(c) < 0$ for $c > 0$, and $u'(0) = \infty$.

A.2 The production function $y = f(k)$ is concave with $f'(k) > 0$, $f''(k) < 0$, $\lim_{k\to 0} f'(k) = \infty$ and $\lim_{k\to\infty} f'(k) = 0$.

Under these assumptions the problem is to find a path which maximizes (1) subject to (2). That is,

$$(3) \qquad \text{Sup } [\int_0^t e^{ni}u(f(k) - \dot{k} - nk)di + (e^{nT} - e^{nt}) \inf_{i\varepsilon[t,T]} \{u(f(k) - \dot{k} - nk)\}] \text{ for } k(0) = k_0 \text{ and } k(i) \geq 0 \text{ for all } i \,\varepsilon\, [0, T].$$

2. The Optimal Path

An approach to the problem may be summarized in the following way. First, we define a value function $\tilde{V}_I(k_t, t)$ of the problem:

$$\text{Sup} \int_0^t e^{ni} u(f(k) - \dot{k} - nk)\,di$$

for $k(0) = k_0$, $k(t) = k_t$ given, and $k(i) \geqslant 0$ for all $i \varepsilon [0, t]$.

Second, we define a value function $V_{II}(k_t, t)$ of the problem:

$$\text{Sup} \inf_{i\varepsilon[t, T]} \{u(f(k) - \dot{k} - nk)\}.$$

Finally, combining these two value functions, we have

$$\tilde{V}(k_t, t) = \tilde{V}_I(k_t, t) + (e^{nT} - e^{nt})V_{II}(k_t, t). \tag{4}$$

Dividing both sides of (4) by total population of the society over time, e^{nT}, and denoting $V_I = \frac{1}{e^{nt}} \tilde{V}_I$ and $V = \frac{1}{e^{nT}} \tilde{V}$, we have

$$V(k_t, t) = \frac{e^{nt}}{e^{nT}} V_I(k_t, t) + (1 - \frac{e^{nt}}{e^{nT}})V_{II}(k_t, t). \tag{5}$$

Since the value function (5) is differentiable (Berveniste and Sheinkman(1979)), we can characterize the optimal capital stock at time t which maximizes the value function.

The first-order necessary condition for the maximum is that the k_t^* must satisfy

$$\frac{dV(k_t)}{dk_t} = \frac{e^{nt}}{e^{nT}} \frac{dV_I(k_t)}{dk_t} + (1 - \frac{e^{nt}}{nT}) \frac{dV_{II}(k_t)}{dk_t} = 0,$$

which can be rewritten

$$- \frac{e^{nt}}{e^{nT}} \frac{dV_I(k_t)}{dk_t} = (1 - \frac{e^{nt}}{e^{nT}}) \frac{dV_{II}(k_t)}{dk_t} . \tag{6}$$

We may summarize our result as

Theorem

In a finite time horizon, the utilitarian-Rawls criterion prescribes the following optimal policies.

(i) If the economy starts from the modified golden rule capital-labor ratio, i.e., $k_0 = \hat{k}$, it will consume all its output and stay there for the entire planning periods. The result coincides with those under the utilitarian and the Rawlsian criteria.

(ii) If it starts from k_0 below $\hat{k}$, then it will accumulate capital stock during t periods so that the economy approaches to the optimal k_t^* at time t, which is characterized by condition (6). After time t, it will consume all its output except the allowances for population growth and stay there for the remaining T-t planning periods.

3. The Modified Rawlsian Criterion — An Infinite Time Horizon

Let us show that the value funciton (5) is reduced to the value function of group II as a time horizon goes to infinity.

$$\begin{aligned} \lim_{T\to\infty} V(k_t) &= \lim_{T\to\infty} e^{n(t-T)} V_I(k_t) + \lim_{T\to\infty} (1 - e^{n(t-T)})V_{II}(k_t) \\ &= V_{II}(k_t). \end{aligned} \tag{7}$$

That is, in an infinite horizon planning problem, the group II gains all the weight and our model degenerates to the Rawlsian world.

However, (7) is different from the value function of the genuine Rawlsian criterion. The latter will be

(8) $\overline{V}_{II}(k_0, 0) = \sup_{\{k_i\}} \inf_{i\varepsilon[0,T]} \{u(f(k) - nk - \dot{k})\}$

and

$$\lim_{T\to\infty} \overline{V}_{II}(k_0, 0) = \overline{V}_{II}(k_0, 0).$$

The value function (8) states the sup-inf value of individual utility over the entire infinite time horizon.

Under the assumptions A.1 and A.2, the genuine Rawlsian value funciton (8) exclucively depends on a historically given initial capital-labor ratio, k_0, save necessary allowances for population growth.

On the other hand, the value function (7) depends on k_t which depend: on k_0 and t. Let us characterize the optimal k_t^* using the differentiabil condition of $V_{II}(k_t)$. Two cases must be distinguished.

Case (1): $\hat{k}_t$ cannot be attained within time t, i.e., $\int_0^{\hat{k}_t} \frac{di}{f(k) - nk} > t$, or $k_t \notin A(t)$ where $A(t) = \{k(t) \mid 0 \leq k(t) \leq \int_0^t (f(k(i) - nk(i))di\}$.

Let $A(t) = [0, \overline{k}_t]$, where $\overline{k}_t < \hat{k}_t$.

Now $\overline{k}_t < \hat{k}_t$ implies $V_{II}(k_t)$ is strictly increasing on A(t). Thus, $\sup V_{II}(k_t) = V_{II}(\overline{k}_t)$.

The first order characterization will be done as follows.

$$\text{Max } V_{II}(k_t)$$
$$\text{s.t.} \quad 0 \leq k_t \leq \overline{k}_t.$$

Form the Lagrangean: $L = V_{II}(k_t) + \beta(\overline{k}_t - k_t)$, where β is the Lagrangean multiplier. Then,

$$u'\cdot(f'(k_t) - n) - \beta = 0 \text{ at } k_t = \bar{k}_t.$$

Letting $\frac{\beta}{u'} = \mu$, we have $f'(\bar{k}_t) = n + \mu > n$.

In this case, therefore, the criterion demands that the group I accumulate capital as possible as it can so that the group II can start with the highest possible capital-labor raito $\bar{k}_t$.

Case (2): $\hat{k}_t$ can be attained within time t, i.e., $\hat{k}_t \varepsilon A(t)$.

The utilitarian-Rawls criterion in this case lets the group II start with the golden rule capital-labor ratio $\hat{k}_t$. The limiting utilitarian-Rawls criterion does not say how $\hat{k}_t$ is achieved by the group I. In fact, any feasible path leading to $\hat{k}_t$ over the time horizon [0, t] will be optimal. In order to avoid this insensibility with respect to the welfare of the group I, we are led to define a new criterion based on the value funciton (7). We define:

Definition

The Modified Rawlsian criterion is defined to be a lexicographic extension of the limiting Rawlsian criterion in the following sense.

(i) If the golden rule $\hat{k}_t$ cannot be attained within finite t periods, the criterion takes the form:

$$\operatorname{Sup} \inf_{i\varepsilon[t,\infty)} \{u(f(k) - nk - \dot{k})\} \quad \text{for some } k_t \varepsilon A(t).$$

(ii) If $\hat{k}_t$ can be attained within finite t periods, then the criterion first dictates

$$\operatorname{Sup} \inf_{i\,\varepsilon[t,\infty)} \{u(f(k) - nk - \dot{k})\} \text{ for some } k_t \varepsilon A(t), \text{ i.e., } k_t = \hat{k}_t,$$

and then it requires to solve

$$\operatorname{Sup}\int_0^t u(c(i))di$$

$$\text{s.t.} \quad \dot{k} = f(k) - nk - c,\ k_0,\ \hat{k}_t \text{given},\ k(i) \geqslant 0 \text{ for all } i\varepsilon[o,t].$$

Below we shall explore the implications of the Modified Rawlsian criterion using diagrams.

Figure 2 describes the Case (i). The locus AB is every combination of (c,k_t) which the economy can achieve during a finite planning time t. The Modified Rawlsian criterion makes the planner to choose the path BB. Point A is chosen under the genuine Rawlsian criterion. On the other hand, in the infinite planning problem, the path DD will be chosen under the genuine utilitarian criterion for time interval [0,t] on the golden rule path.

If the economy is so productive that it can achieve $\hat{k}_t$ within time t, the locus of k_t will shift to the right as in Fig. 3 from AB to AB". In this case the second portion of the Modified Rawlsian criterion can be applied to the choice of paths leading to an end point on $\hat{k}$E". That is, it dictates that the path in the first group be E"E". Again, the genuine Rawlsian criterion recommends the economy to stay at point A. The genuine utilitarian criterion dictates the path D"D" for the time interval [0,t] as a part of march to the golden rule path. As is evident from the diagram, members of the group I can enjoy higher consumption level under the genuine utilitarian criterion than the Modified Rawlsian criterion because of downward locus of the (c,k_t) combinations.

It should be noted that as the ties of sentiment get strong, i.e., as t becomes larger, the locus of k_t shifts to the right as in Fig. 3. As t becomes infinite, then the path coincides with the golden rule path.

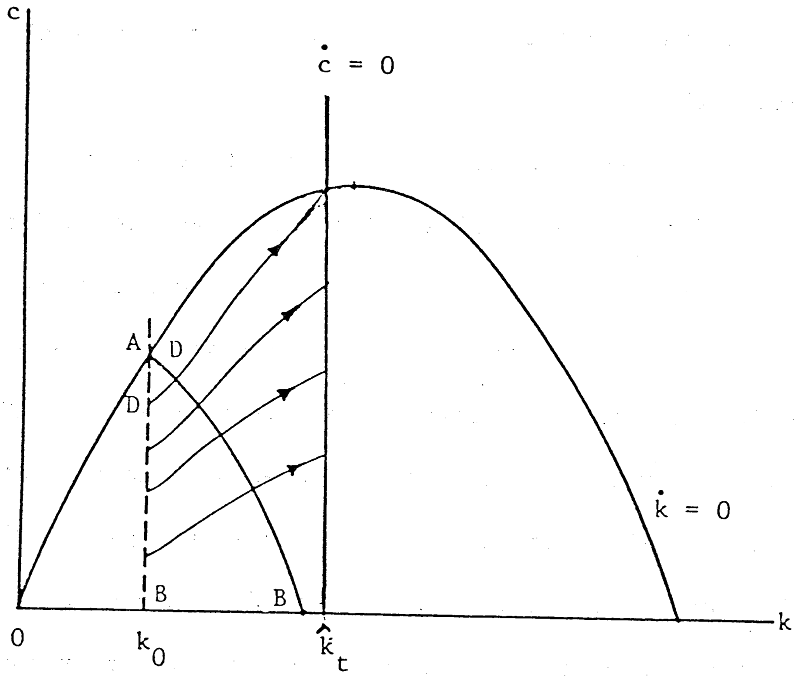

Figure 2. The optimal consumption path under the Modified Rawlsian criterion in the Solow type economy: Case (i).

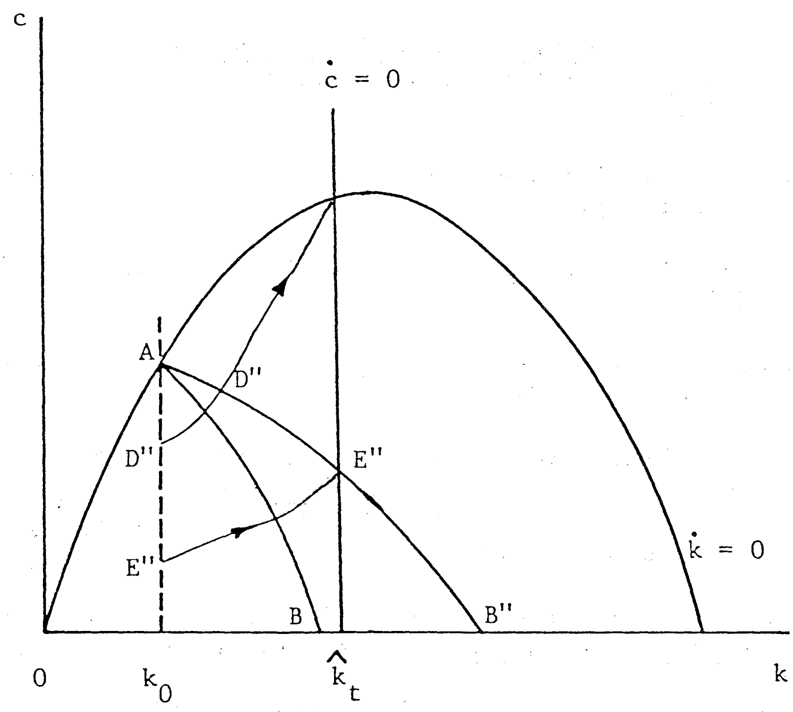

Figure 3. The optimal consumption path under the Modified Rawlsian criterion in the Solow type economy: Case (ii).

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