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THE RATIONAL EXPECTATIONS APPROACH IN GENERAL EQUILIBRIUM ANALYSIS: AN OVERVIEW

by

Koichi Mashiyama
Toyohashi University of Technology

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1. Introduction

The theory of consumer and firm behavior in an Arrow Debreu economy is well understood, and all competitive equilibria are Pareto optimum and any Pareto optimum can be sustained as a competitive equilibrium. Therefore, there exists a clearcut relationship between economic efficiency and competitive equilibria in an Arrow-Debreu world. On the other hand, as pointed out by Radner(1970), it requires that markets are complete in the Arrow-Debreu sense. Namely, the theory requires in principle a complete system of insurance and futures markets. The theory does not take account of at least three important institutional features of modern (capitalist) economies: the existence of money, the stock market, and active spot markets at every date.

These lines of criticism have received serious attention in the literature on economic theory. Two main approaches have been developed: the temporary equilibrium approach and the rational expectations approach.

After all, the information structure hypothesized in the incomplete markets setting is expected to play a crucial role in exploring economic optimality of competitive equilibria. As well known, the relationship between economic efficiency and competitive equilibria is not so simple in economies with incomplete markets, unlike in the Arrow-Debreu economy. To have access to some clear insight into this matter, one is required to formulate a rigorous structure concerning such phenomena as economic agents' gathering and processing information in an economy in terms of economic rationality. This sort of

formulation would be extremely hard to model in detail in the framework of general equilibrium. The rational expectations approach has a considerable advantage in this aspect.

Meanwhile, the rational expectations approach has also recieved considerable attention in the macroeconomic literature and it seems to get a dominant position in macrotheory. The rational expectations approach in macrotheory has been developed in parallel with those developments in general equilirbium theory although the same concept of the rational expectations equilirbium is used. For these developments in macrotheory, one should refer to survey articles by Shiller(1978), McCallum(1980), and Barro(1980). In the general equilibrium setting Gale(1982) explains the usefullness of the rational expectations approach for monetary theory. See also Radner(1982).

The present paper is intended to review how the rational expectations hypothesis works in the general equilibrium setting and related issues concerning the informational role of monetary policy. Important issues in the theory of incomplete markets in general equilibrium models are concerned with economic efficiency and informational efficiency of competitive equilibria, and the relationship between economic and information efficiency.

Informational aspects of competitive prices have been explored by, among others, Grossman, Stiglitz, Radner, and Kihlstrom and Mirman. Questions on economic efficiency of the rational expectations equilibrium(REE) rests on what concept of optimality is utilized since the standard Pareto optimality

is difficult to apply in a model of the rational expectations hypothesis(REH). Several notions represented by social Nash optimum, equal-treatment Pareto optimum, and constrained Pareto optimum have been proposed. See, for example, Grossman(1977) and Muench(1977). Tentative conclusions arising from these studies imply that:(1) under appropriate conditions, "generically", a REE reveals to all traders the information possessed by all of traders taken together(Radner 1979), (2) a REE is a social Nash optimum(Grossman 1977). However, conclusions are not definite. The existence problem of the REE also remains to be an open question.

Even if the existence of the REE is confirmed, naturally arises the question, under what conditions the "learning" process converges to a REE? Under the adaptive expectations hypothesis this corresponds to the stability of a model. This question seems to be hard to answer since an economic model with REH assumes that individual economic agents participate in markets with differential information. See Townsend for a step in this direction.

The informational role of monetary policy is explored by Lucas(1972) in a over-lapping generations model with the REH. If there exists no noise in real variables, competitive prices completely reveal the information about the economic environment and, as a result, money is neutral. However, if real shocks exist, economic agents cannot correctly distinguish monetary shocks from real shocks by observing market signals when monetary policy is noisy. Therefore, the Phillips curve obtains but it

is not structural trade-off. These result in the general equilibrium setting have led quite few economists to the belief that economic policy is in general ineffective. However, this is a dangerous jump from a general equilibrium model to a policy practice. Recently Muench(1977) advanced our understanding of economic properties of the Lucas model. He shows that the Lucas equilibrium can be "Pareto dominated" by economic policy other than the constant rule policy.

In subsequent pages we shall discuss the issues mentioned above, which are theoretically important in recent developments of general equilibrium theory coupled with the REH.

2. The Rational Expectations Approach in General Equilibrium Models.

It is expected that when traders come to a market with differential information about economic conditions, the resulting market prices might reveal to traders something about economic conditions in aggregates. In other words, rational traders may make inference from market signals (prices) about the information possessed by other traders. This possibility depends on individual traders' own (subjective) models or beliefs of how market prices are determined. On the other hand, the determination of market prices is endogenous to the market system. If traders have any opportunity to compare the operation of the market system with traders' own models, the economic rationality must imply that the traders' subjective model is consistent with their observations of the market outcomes in the Bayesian sense. This theoretical belief motivates one to utilize the rational expectations hypothesis.[1]

The rational expectations hypothesis is defined by the assumption that the public can know the price function endogenous to the market system. By the rational expectations hypothesis(REH), macroeconomists mean that the public's subjective expectations equal the objective mathematical expectations conditional on data available when the expectation was formed. Gale(1982) suggests that the REH works as an "organizing principle" in the same way as the utility maximization and the profit maximization. The "utility" of using the REH arises from preserving the logical consistency in the general

equilibrium model, not from its literal accuracy.

To understand precisely what this hypothesis means in a model of equilibrium, we consider a pure exchange economy.[2] Markets are not complete in the Arrow-Debreu sense. Namely, trading takes place in a sequence of spot markets. It is not possible for each trader to store commodities from one date to the next. There are 2n agents at each date; n of these agents are in the first half of their lives and n in the second. Each agent lives two periods. The young agent at period t is indexed by i=1,2,..,n and the old agent by i=n+1,..,2n. There is a monetary authority, indexed by 0. The monetary authority issues fiat money. An agent is given his endowment e of commodities only the first period of his two period life and has to hold money at the end of the first period to have consumption in the second.

Uncertainty is introduced by the states of nature Ω. In the present model, uncertainty arises from the randomness of initial endowments. The noise in the monetary policy can be introduced in the model.[3] There are assumed to be c goods and every trader chooses to consume a nonnegative bundle of goods at each date, i.e., his consumption set is $\mathbb{R}^C_+ \times \mathbb{R}^C_+$. Let X denote the set of plans. Then each trader's preferences can be represented by a preference relation $\prec$ on X. He has also an endowment e in $\mathbb{R}^C_+$ in his first period. An information structure is a partition F of the set Ω. Hence the i-th agent is completely described by the ordered triple $(\prec_i, e_i, F_i)$.

At each period every agent observes a current price vector $p \in \mathbb{R}^C_+$. For the young agent to infer anything from this observatio he must know which vector goes with which state. This suggests that information is not revealed by a price vector alone but by a price function which associates a price vector with each state of nature. Let the pair of price vector (p,q) denote price vectors in period 0 and period 1. Then, a price system is a function $\phi = (\phi^0, \phi^1)$ from Ω to $\mathbb{R}^C_+ \times \mathbb{R}^C_+$. The ordered pair (ϕ^0, ϕ^1) defined on Ω represents price systems at period 0 and 1, respectively.

The rational expectations hypothesis(REH) assumes that every agent knows the price function ϕ. Roughly speaking, the REH means that every agent knows the correct statistical relationship between market prices and exogenous noises.

Since the i-th agent possesses information F_i, the total information available to the agents collectively can be represent ed by $F = \bigcup_{i=0}^{2n} F_i$. It is reasonable to assume that the price function ϕ^0 is F-measurable. The information structure generated by a price vector can be by $F[\phi^0]$.[4] The i-th agent initially knows F_i and observes $F[\phi^0]$, so the information available to him is $F_i \vee F[\phi^0]$, the coarsest partition of Ω which is finer than both F_i and $F[\phi^0]$. Denote it by $F_i[\phi^0]$. The monetary policy $\bar{m}$ is said admissible for the authority at the price system ϕ if $\bar{m}$ is $F_0[\phi^0]$ - measurable.

The exchange economy is defined to be an (2n + 1)-tuple $\{F_0, (\prec_i, e_i, F_i)\}$ consisting of the authority's information F_0, preference relation $\prec_i$, and endowments $e_i (i=1,\ldots,2n)$. Before we define a REE, we need to define each agent's budget set $\beta_i(\phi)$.

Let $x_i = (x_i^0, x_i^1)$ denote a consumption plan of the i-th agent. Consider the young agent at period 0. In the first period the value of his sales of goods to the old generation must pay for the money he demands:

(1) $\quad px_i^0 + m_i \leq pe_i.$

In the second period, the money carried from the first plus the transfer from the authority are suppllied inellastically in exchange for goods:

(2) $\quad qx_i^1 \leq m_i + z_i,$

where z_i is the transfer in the second. The future price q and the future transfer z_i are unknown to him at the first period.

Under the REH, every agent knows what will happen in each state of nature. At date 1, if $A \in \Omega$ has occured, each young agent's budget constraints are

(1') $\quad \phi^0(\omega)x_i^0(\omega) + m_i(\omega) \leq \phi^0(\omega)e_i(\omega), \ (\forall \omega \in A),$

(2') $\quad \phi^1(\omega)x_i^1(\omega) \leq m_i(\omega) + z_i(\omega), \ (\forall \omega \in A).$

Given the price system $\phi(\omega)$, let $\beta_i(\phi)$ be the set of consumption plans and money holding plans (x_i, m_i) for the i-th agent which satisfy (1') and (2').

The old agent inellastically supplies his money holding $m_i + z_i$, $i=n+1,..,2n$, which have already determined in his first period.

Definition

A rational expectations equilibrium(REE) for the exchange economy defined above is an array $\{(x_1,m_1),..,(x_{2N},m_{2N})\}$ of consumption and money holding plans, and a monetary policy $\bar{m}$ and the price system ϕ, such that

(i) for each $i = 1,2,..,N$, (x_i,m_i) is maximal with respect to $\prec_i$ in the budget set $\beta_i(\phi)$ and (x_i,m_i) is $F_i[\phi^0]$- measurable.

(ii) $\sum_{i=1}^{N} (x_i^0 + x_{N+i}^1) \leq \sum_{i=1}^{N} e_i$,

(iii) $\sum_{i=1}^{N} m_i = \sum_{i=1}^{N} (m_{N+1} + z_{N+i}) = \bar{m}$,

(iv) $\bar{m}$ is admissible. [5]

The equilibrium thus defined is characterized by the REH that each agent knows the prices $\phi(\omega)$ in each state ω. However, agents have very limited information about the true state of nature. Each agent cannot correctly infer which state will occur.

Since this definition is somewhat strong, we may relax the condition (i) above. Suppose that the preferences $\prec_i$ are represented by a utility function $u_i(x_i^0,x_i^1)$. The condition (i) can be relaxed by :

(i') for each $i=1,..,N$, (x_i,m_i) maximizes $E[u_i(x_i^0,x_i^1)|F_i[\phi]]$ subject to $px_i^0 + m_i \leq pe_i$, $qx_i^1 = m_i + z_i$.

This modified definition is utilized by Lucas, Radner and others.

The economic properties of the REE defined in this way can be investigated. The first issue is whether a REE is revealing or not. Call a pair (ω_1,ω_2) of states of nature

confounding if there is a solution (π_1, π_2) of the price equation

$$\phi(\omega_1) = \pi_1,$$

$$\phi(\omega_2) = \pi_2,$$

$$\pi_1 = \pi_2, \quad \omega_1 \neq \omega_2.$$ [6]

If the price system is confounding, the REE is not revealing. Otherwise, it is a revealing equilibrium. In the present formulation, a REE is fully revealing if $F_i[\phi^0] = F$, for all i. A brief summary of recent investigations in this matter is presented in Section 3.

The informational aspect of monetary policy is also worthwhile to examine since the non-neutrality of money is a crucial issue in monetary theory. [7] In a REE, the monetary policy can influence economic agents' real behavior only through changes in the informational content of prices. Therefore, a monetary policy that cannot change the informational content of prices is neutral. Suppose that λ is a function from Ω to $\mathbb{R}$ such that $\lambda(\omega) > 0$ for every $\omega \in \Omega$ and $F_i[\lambda \times \phi^0] = F_i[\phi^0]$ for every i, where $\lambda \times \phi^0$ refers to the function which maps $\omega \in \Omega$ to $\lambda(\omega)\phi^0(\omega)$. Then the consumption plans for every agent remain the same when a policy $\bar{m}$ is replaced by $\lambda \times \bar{m}$. It follows that equilibrium is invariant to positive proportional changes in the policy function, $\lambda\bar{m}$, $\lambda > 0$, $\lambda \in \mathbb{R}$. However, this does not in general claim the neutrality of money.

3. The Rational Expectations Equilibrium and Informational Efficiency.

3.1. The Informational Role of Competitive Price Systems.

The informational effeciency of competitive price systems is made clear in a series of studies by Grossman and Stiglitz. The starting point of investigations is to examine the "efficient market" hypothesis, which states that the price in competitive (stock) markets reflect and reveal all the relevant information. However, this is a classical conundrum. Since equilibrium must entail perfect abritrage and no profit can be gained from arbitrage activity, no trader has any motives to engage in collecting exogenous information and therefore, there is no exogenous information that prices reveal to traders.

Grossman(1976,1978), Grossman and Stiglitz(1976, 1980), and Kihlstrom and Mirman(1975) attempted to resolve this classsical paradox by utilizing the notion of rational expectations equilibrium and by introducing exogenous random shocks specific to markets. (A set of propositions established by them can be called the "weak version" of the "efficient market" hypothesis.)

We utilize a variant of Grossman's model. There are n agents indexed by i=1,..,n. Each agent i allocates his initial wealth w_{0i} between a riskless and risky asset. The riskless asset yields per unit r units of a single consumption good and the risky asset $\tilde{x}$ units. r is predetermined but $\tilde{x}$ is a random variable. Let p be the price of the risky asset by taking the riskless asset as numeraire. If agent i holds z_i units of the risky asset, his portfolio yields the return $w_i = \tilde{x}z_i + rb_i$, where b_i is

agent i's holding of the riskless asset.

Some agent i can have exogenous information y_i by observing the noisy realization of a random variable $\tilde{x}$, which may costs agents some consumption goods.

$$y_i = \tilde{x} + \eta_i,$$

where η_i is a random variable. The noise η_i can be interpreted as an observation noise. Some agents, on the other hand, do not engage in collecting exogenous information. Following grossman, call the former agent the informed and the latter agent the uninformed. For simplicity, it is assumed that $\eta_i = \eta$, for all informed agents i. A weaker assumption is used in Grossman(1978) and Hellwig(1980). The supply of the risky asset $\tilde{z}$ is introduced as a random variable.

Three important assumptions are in order.

(1)(Expected Utility)

The informed agent i maximizes the expected utility of consumption $E[u_i(w_i)|y, \tilde{p} = p]$.

The uninformed agent i maximizes $E[u_i(w_i)|\tilde{p} = p]$.

(2)(Normality)

(x,z,η) has a joint normal distribution with mean $(\bar{x},\bar{z},0)$ and nonsingular covariance matrix Σ.

$$\Sigma = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_z^2 & 0 \\ 0 & 0 & \sigma_\eta^2 \end{bmatrix}$$

(3)(Rational Expectations)

Each agent i knows the actual joint distribution of the 4-tuple $(\tilde{x},\tilde{y},\tilde{z},\tilde{p})$.

Let $z_I(p|\tilde{y} = y, \tilde{p} = p)$ and $z_u(p|\tilde{p} = p)$ denote the demand for the risky asset of the informed and uninformed, respectively. Then the market clearing is

(1) $\lambda z_I(p|\ \tilde{y} = y, \tilde{p} = p) + (1-\lambda)z_u(p|\tilde{p} = p) = z,$

where λ is a fraction of the informed traders. Our question is how well market-clearing prices reveal to the uninformed traders the information needed to predict the realization $\tilde{x}=x$ (and/or $\tilde{z} = z$). The present model implies that , when no supply shocks are present, competitive prices fully reveal the information possessed by the informed traders. That is, p is a sufficient statistic for $\tilde{x}$. On the other hand, under the presence of supply shocks, if uninformed traders do not observe z, uninformed traders are prevented from learning x via observations of the market prices because they cannot distinguish variations in price due to changes in the informed trader's information from variations due to vhanges in aggregate supply. In general the price system does not reveal all the information about the "true" state of the risky asset. This conclusion seems to be intuitively reasonable.

A question arises then, what factors affect the informativeness of the competitive price system? The informativeness is expected to depend on the cost of information, the quality of information the informed traders have, and the risk preferences. This issue is explored by Grossman and Stiglitz(1980) and Hellwig(1980).

Grossman and Stiglitz showed that when the "strong" efficient market hypothesis holds and information is costly,

competitive markets break down.[8] This is so, because prices fully reflect all the information when there is no supply shock,and informed traders stop paying for information since they can not take "better" positions than those of the uninformed. Even when information is costless, competitive markets are to "thin". It is expected that, as a fraction of informed traders increases, competitive prices will be more influenced by the informed and, as a result, the price system becomes more informative. It is shown that a decrease in the cost of information increases the informativeness of the price system since it increases the percentage of informed traders. On the other hand, the marginal benefit arising from being informed decreases as the ratio of informed to uninformed traders increases. Consequently, competitive markets where prices fully reflect the information possessed by informed traders and information is costless are likely to be "thin".

Grossman's agents are slightly schizophrenic in the sense that each agent exerts a nonnegligible influence on the price while they are supposed to be price-takers. Hellwig shows in a large economy model that the relative importance of the exogenous information possessed by each agent depends on his risk preferences and the equilibrium price reflects only those elements of information that are common to a large number of agents.[9] The "strength" of agents' reactions to their signals, i.e., y_i's, is inversely related to both their degree of risk aversion and the level of noise in signals σ_η^2.[10] y_i is relatively the more important, the less risk averse agent i is,

provided the noise level being fixed. It is worthwhile to draw information from their own information as well as the price. Thus competitive equilibrium with the property that each agent is informed after observing equilibrium prices but prices alone reveal only part of exogenous information are economically more viable than those in which price alone conveyes all the relevant information to traders.

The existence of futures markets is not explicitly introduced in the models. It is reasonable to expect that an observation of futures price would provide an additional information to traders. A qeustion to raised is whether an observation of the current spot price together with futures price reveals all the relevant information possessed by traders. The answer will in general depend on the dimension of exogenous noises. This issue is explored by Grossman(1977). Also see Danthine(1978) and Bray(1981), which deal with the informational role of futures markets in the commodity speculation. [11]

3.2. The Informational Efficiency of Rational Expectations Equilibria.

Recent investigations into the informativeness of competitive price systems, initiated by Grossman, have successfully established the conditions under which equilibrium prices reveal the exogenous information possessed by traders. The question to be raised next is whether a REE is fully revealing in a class of more general economic models sine there might exist non-revealing REEs. This question is explored by, among others, Radner(1979), Jordan(1977), and Allen(1981) in a close relation to the existence of REEs. [12] A sofisticated technique, in particular differential topology tool, is heavily utilized to prove the generic existence of a revealing REE. While they show that a revealing REE generically exists in general equilibrium models of economies, the uniqueness question of a revealing REE is not adressed. For example, Radner assumes "a rule for choosing a single equilibrium q in the case of multiple equilibria" without any economic justifications.

We take up Radner's model as a representative of this sort of models. The model is that of a pure exchange economy in the absence of markets for contingency claims contracts. There are I traders and trader i chooses (c_i, z_i), where c_i=i's expenditure on current consumption, and z_i=i's portfolio with K different assets. So $c_i \in \mathbb{R}_+$ and $z_i \in \mathbb{R}_+^K$. The value of one unit of asset k next period will be v^k, which is not known to traders at the time of the current market. The vector $v = (v^k)$ of future asset values can take on one of finitely many values v_e,

$e \in E$. The set E represents the states of nature or economic environments. If the economic environment e obtains, then the future value of his portfolio will be the inner product $v_e z_i$. The i-th trader's utility function is given by a Von Neuman-Morgenstern Utility function $u_{0i}(c_i) + u_i(v_e z_i)$.

Before the market opens, each trader i has his private (exogenous) information s_i, drawn from a finite set S_i. The set S_i is interpreted as an "observation" set. S_i is called the signal received by trader i and the I-tuple $s = (s_1,..,s_I)$ the joint signal. Let S denote the finite set of all joint signals. A probability distribution is defined on $E \times S$. Different traders might have different beliefs about this probability distribution.

Let q denote the vector of asset prices, $q \in \mathbb{R}^K_+$ and the price of current consumption is normalized as 1. The i-th trader's budget equation is

$$c_i + qz_i \leq m_i + qw_i,$$

where m_i is an endowment in "cash" and w_i is an asset endowment except "cash".

Suppose that a price function $\phi(s)$ is ruling in the market when a joint signal s is received. Under the REH, every trader is assumed to know this price function ϕ. Given a price function ϕ, and given price vector q and a joint signal s, trader i chooses a (c_i, z_i) that maximizes his expected utility

$$E[u_{0i}(c_i) + u_i(v_c z_i) | s_i, s \in \phi^{-1}(q)]$$

subject to the budget equation. Let $\xi_i(q|s_i,\phi)$ denote i's demand

for assets. The total excess demand for assets is

$$Z_s(q,\phi) = \sum_{i\in I} [\xi_i(q|s_i,\phi) - w_i].$$

A rational expectations equilibrium is defined to be a price function ϕ^* such that

$$Z_s(\phi^*(s),\phi^*) = 0, \text{ for every } s\in S.$$

This is an ordinary definition of a REE.

Imagine that before entering the market the traders exchange all their private information, so that $s_i = s$, $\forall i\in I$. The trader i's demand $\xi_i(q|s)$ is computed from the maximization of

$$E[u_{0i}(c_i) + u_i(v_e z_i)|s].$$

Let $\hat{\phi}(s)$ denote an ordinary equilibrium price vector given s, which clears the excess demand corresponding to $\xi_i(q|s)$. $\hat{\phi}(s)$ is called a full communication equilibrium(FCE). $\hat{\phi}$ is revealing if $s \neq s'$ implies that $\hat{\phi}(s) \neq \hat{\phi}(s')$.

It immediately follows that if a FCE is revealing, the conditional probability distribution of e in E given $\hat{\phi}(s)$ is equal to the conditional distribution of e given s. Hence the rational expectations demand of trader i given s_i and $\hat{\phi}(s)$ would equal his ordinary demand given s, $\xi_i(q|s)$. Since $\hat{\phi}(s)$ clears market, the FCE $\hat{\phi}$ satisfies the definition of a REE above. Therefore, a revealing FCE is a REE.

Radner proceeds to show that, generically, a revealing REE exists. Let Π denote the set of probability distributions defined on $E\times S$. He shows that the set of points $\pi\in\Pi$ for which there is no corresponding revealing FCE is a set, the closure

of which has Lebesgue measure zero. It will then follow that a revealing REE exists except for a set with Lebesgue measure zero.

However, the generic existence of a revealing REE rests on the crucial assumption imposed on the information structure. If the set of economic environments and information signals $E \times S$ is not finite, then the generic existence of a revealing REE would not be true. A robust example of non-existence of a REE can be given in this situation. See for example Radner(1982) and Futia(1981).

As suggested by Radner, it is rather more important to explore "the case in which traders have models of market price determination that are imperfect or imprecise, but that are consistent with observations of the market." This is one of the issues on the stability property of a REE. Investigation in this direction is only at the preliminary stage. See, for example, Radner(1982, 6.4) and references cited there.

While the generic existence of a revealing REE is shown in approriate models, we have learned in 3.1 that a REE with the property that competitive prices alone reveal only part of the private information jointly available to all traders is economically more viable. Recently Futia(1981) has shown an Example in which the incentive to collect private information can persist even in a REE. He successfully constructed the examples of non-revealing REEs by utilizing a mathematical technique in Hilbert spaces. This example suggets that, even in a linear model with normal random variables, models in which

the set of environments and signals are infinite would give rise to a quite different result compared with the one in which it is finite. It would be no longer true in this class of models that the existence of revealing REEs is generic.

4. The Informational Efficiency of Monetary Policy In the Rational Expectations Equilibrium

The informational aspect of monetary policy in the REEs was exploited by Lucas(1972) to show the neutrality of money. This influencial paper is the seminal paper on rational expectations and information revealed by prices. The key idea is to take the explicit account of the informativeness of competitive prices in a REE. Due to the fact that the dimension of market signals is less than that of exogenous noises, each economic agent cannot completely distinguish a nominal shock from a real shock. This is so because the equilibrium price system reveals only part of exogenous information, in particular the ratio of money shocks to real shocks. In the Lucas model the qunatity of money can affect, by influencing the price level, the information revealed to agents and their behavior, and so this is only the way that monetary policy can have a real effect on the economy.

The Lucas Model

The structure of the Lucas model is constructed by introducing a production activity into the pure exchange economy presented in Section 2. In order to incorporate a real shock, exchnage is assumed to occur in two local markets which are physically separated. A simplifying assumption is that one unit of labor produces one unit of consumption goods and all agents have identical preferences. Since the model is a variant of the over-lapping generations model, each agent works in his first period and retires in the second. The old generation

is allocated across markets so as to equate total money demand between them. The young is allocated between two markets in proportions $\theta/2$ and $(1 - \theta/2)$, respectively. θ is a random variable and is i.i.d. with a probability density function $g(\theta)$, defined on the interval [0,2]. Once the assignment of persons to each market is made, no switching or communication between markets is possible and within each market, market clearing prices are ruling.

The government issues fiat money in the form of a transfer payments.[13] This transfer is made to the old at the beginning of each period and is assumed to be proportional to the pre-transfer holdings of agents. Let $\bar{m}_t$ denote the pre-transfer holding of money, per head of the old generation. The quantity of money $\bar{m}_t$ is assumed to be known to all agents. Then the post-transfer money balances are given by

$$\bar{m}_{t+1} = \bar{m}_t \cdot z,$$

where z is a random variable. $\bar{m}_{t+1}$ is not known until the next period unless it is revealed by current prices. This is so because z is not observable at period t. The random variable z is assumed to be i.i.d. with a pdf f(z) on $(0,\infty)$.

The state of the economy at period t is entirely described by the triple $(\bar{m}_t, z, \theta)$.

The old generation has a trivial decision problem at period t. They supply inelasically money in exchange for consumption. The young agent's utility is measured by a Neuman-Morgenstern utility function which has the form

$u(c_t,n_t) + V(c_{t+1})$. That is, his utility depends on his current consumption c_t, his future consumption c_{t+1} and on his supply of labor n_t. Each agent can observe the pre-transfer money supply $\bar{m}_t$ and a current local price at period t. The young agent's initial information structure is generated by $(\bar{m}_t,p_t)$. Therefore, under the REH his expected utility will be

$$u(c_t,n_t) + E[V(m_t z_{t+1}/p_{t+1})|\bar{m}_t,\ \phi(\bar{m}_t,z_t,\theta_t) = p_t].$$

The decision problem is to choose a non-negative triple (c_t,m_t,n_t) to maximize this expected utility subject to

$$p_t(c_t - n_t) + m_t \leq 0.$$

A price function ϕ is dependent of $\bar{m}$, z and θ since the state of the economy at period t is entirely described by the triple $(\bar{m}_t,z_t,\theta_t)$. The REH presupposes that each agent knows the price function .

To derive a property of the price function ϕ, consider the submarket that receives the fraction $\theta_t/2$ of young agents at priod t. Since consumption and labor are indistinguishable, there are effectively two commodities being traded, money and consumption. If the excess demand for one of them is zero, the excess demand for the other is zero by the Walras' law. Since old agents are allocated in such a way that the money supply in two markets is equal, the demand for consumption goods by the old generation is $N\bar{m}_t z_t/(2p_t)$, where N is the number of old agents in each period. Suppose that a typical young agent solves his decision problem and chooses an excess demand for consumption $c_t - n_t = \xi(p_t|\bar{m}_t,\phi(\bar{m}_t,z_t,\theta_t) = p_t)$. [14]

The total demand from young agents will be $N\theta_t\xi/2$ since $N\theta_t/2$ is the number of young agents in this market. Therefore the market-clearing condition is

$$\frac{\bar{m}_t z_t}{\theta_t} = p_t \xi(p_t | \bar{m}_t, \phi = p_t).$$

For each state $(\bar{m}, z, \theta)$, the price function ϕ must satisfy

$$\frac{\bar{m}_t z_t}{\theta_t} = \phi(\bar{m}_t, z_t, \theta_t)\xi(\phi(\bar{m}_t, z_t, \theta_t) | \bar{m}_t, \phi = p_t).$$

A REE in the model is defined by "a continuous, non-negative function $\phi(m,z,\theta)$, with $mz/(\theta\phi)$ bounded and bounded away from zero which satisfies the above market-clearing condition." [Lucas(1972)].

Two remarks are in order. Any equilibrium price system, i.e., ϕ, is a sufficient statistic for z_t/θ_t if $\bar{m}_t$ is known. It suggests that the price function ϕ might have the form $\bar{m}_t\psi(z_t/\theta_t)$. Lucas is able to show that this is the case [lucas, Theorem 1]. The money supply term $\bar{m}_t$ factors out because a fully anounced change in the quantity of money does not have a real effect in this model. If no non-monetary disturbance is present, the current value of z is fully revealed to each agent by the equilibrium price and so it is expected, with no surprise, that the neutrality of money holds. That is, current price will adjust proportional to changes in the money supply.[15]

On the other hand, the Lucas model implies that when real disturbances are present, agents have limited power to infer money disturbances from competitive prices and, as a result,

the monetary policy can affect real variables by influencing the informational content of prices. Since Lucas explored the case where the monetary policy is of the special form, i.e., positive proportinal changes in the quantity of money, the above aspect is not made clear. Nevertheless, this statement is expected to hold in more general models.

It is well known from the money literature that the Keynesian portfolio theory or a model utilized by Bruner and Meltzer states that financial signals exerts a significant influence on output by affecting the demand for new capital goods.[16] Therefore, when the Lucas model is extended so as to incorporate an informational role of financial signals, it can provide an additional transmission path from monetary shocks to changes in output. This approach is taken in the macroeconomic perspective by Barro(1980), Weiss(1980,1982), and King(1982). However, models utilized by them are postulated from the macroeconomic reasoning but abstract from a microeconomic foundation. Although this issue has not been settled yet on the ground of macroeconomic theory, it should be also explored by taking the explicit account of the rationality of economic agents to have clear insights.

5. The Optimality of the Rational Expectations Equilibrium

Now we come to the issue on economic efficiency of REEs in the absence of the complete set of contingency claims markets. A similar problem was taken up by Diamond(1967). A result obtained by Diamond is that an equilibrium in a one-good, two-period economy is Pareto optimal relative to the set of allocations that can be achieved through the existing market structure. This optimality is called a constrained Pareto optimum(CPO). Hart(1975) casts doubt on the generality of Diamond's result. Using a number of interesting examples, he suggests that the market equilibrium may not be a CPO in a two-goods economy that lasts more than two periods.

Recently Grossman(1977) and Grossman and Hart(1979) have explored this issue in the framework of a stock market economy. As mentioned above, Hart suggets that Diamond's conclusion cannot be generalized to an economy with many commodities, without changing the definition of the constrained Pareto optimality. One might expect that the notion of the CPO should be modified to take into account that allocations in incomplete markets are not fully coordinated across time and states of nature. In this context, Grossman has introduced the notion of incomplete coordination and then defined a social Nash optimum. Grossman shows that a REE is social Nash optimal(SNO) in a competitive stock market of an exchange economy. Further, Grossman and Hart extends this result to a competitive production exchange economy.

More recently Newbery and Stiglitz(1982) examined the

the optimal property of a REE. They show that a REE is not a CPO. A sufficient condition for a REE to be a CPO is the redundancy of risk markets. The necessary condition for a REE to be a CPO for all technologies are exactly the same as the the conditions for redundancy of risk markets, e.g., there is no risk, or producers are risk neutral, or consumers have an indirect utility function of a particular form. These conditions are very restrictive. This observation, therefore, leads to a strong presumption that a REE is in general not a CPO. Hence it is not surprising that a Pareto optimal improvement over existing market allocations can be achieved by a tax policy.

A constrained Pareto optimum and social Nash optimum are different concepts of economic efficiency. However, a SNO seems to be a weaker concept than a CPO. It is a conjecture that if a REE is a CPO, then it is a SNO but even if a REE is a SNO, it would not be a CPO. The fact that a SNO is a weaker concept pf optimality would not be a problem. The key point is that any REEs are a CPO in the incomplete market setting in exactly the same way as a full Pareto potimality is sustained as a competitive equilibrium in an economy with complete markets. Roughly speaking, a SNO characterizes a REE.

From the policy view point, an economic policy to improve ecoromic efficiency is desired in an economy with incomplete markets since equilibrium is not Pareto optimal. Newbery and Stiglitz gives a partial answer to this question.

A closely related issue is whether increasing the amount of information improves economic efficiency. In other words, a revealing REE is better than a non-revealing REE in the sense

of economic optimality. Since REEs in the incomplete market are subject to the theory of Second Best, it is ambiguous that revelation is better than non-revelation. This is one of currend research agendas.

Models of the rational expectations equilibrium mentioned above do not explain an active role of money. We need to explore the case in which money plays its own role as a means of stores of value and a medium of exchange. The famous Lucas model is a typical model of a monetary economy where money plays a substantial role. We consider the economic efficiency of the Lucas model in the remaining section since it caused a raging discussion in the field of monetary theory. Lucas claimed that the REE, when a monetary authority follows a k-percent policy rule, is Pareto optimal in some sense.

However, the Pareto optimality defined by Lucas has a weak nature. This is pointed out by Muench(1977). Muench attempted to show that the REE in the Lucas model is not Pareto optimal. To this end, he assumes a k-percent money supply rule in the model, i.e., money supply m is fixed to be 1 over time. A different aspect between the Lucas and Muench model is that the old generation is allocated into two markets each with N/2 individuals in the following manner. From each market indexed by θ into which this generation was allocated when young, one half members are sent into one market indexed by θ' , and one half into the other market. Except for this difference, the model used by Muench is entirely the same as the Lucas model.

Let $c(\theta)$ and $n(\theta)$ denote the young agent's consumption and labor supply when he is young and in θ, and let $C'(\theta,\theta')$ denote his consumption when old and in θ'. Then any feasible allocations $\{c(\theta), n(\theta), c'(\theta,\theta')\}$ must satisfy the feasiblity condition:

$$\Psi(\Theta') = n(\Theta') - c(\Theta') = \{c'(\theta,\theta') + (2-\theta)c'(2-\theta,\theta')\}/2\theta'.$$

An individual maximizes the problem:

$$\text{Max } u(c(\theta),n(\theta)) + \int V[\lambda(\theta)/p(\theta')]g(\theta')d\theta',$$

$$\text{sublect to } p(\theta)[n(\theta) - c(\theta)] = \lambda(\theta).$$

The rational expectations equilibrium is defined in the same way as in the Lucas model. The market clearing condition is $\lambda(\theta) = 1/\theta$. The REE allocation under the k-percent policy rule is called the L allocation. The functional equation which determines the L allocation is the first necessary condition:

$$\psi h(\psi) = \int (\theta'/\theta)\psi(\theta')V'[(\theta'/\theta)\psi(\theta')]g(\theta')d\theta',$$

where

$$\psi(\theta) = n(\theta) - c(\theta), \text{ and } h(\psi) = u_c(\psi).$$

To ensure a unique solution to the above problem, mathematical conditions must be imposed on utility functions u and V, of course.

To examine the Pareto optimality Lucas gave the definition.

> An allocation $\{\bar{c}(\theta), \bar{n}(\theta), \bar{c}'(\theta,\theta')\}$ is "Pareto" optimal if there is no allocation $\{c(\theta), n(\theta), c'(\theta,\theta')\}$ such that $u(c(\theta),n(\theta)) \geq u(\bar{c}(\theta),\bar{n}(\theta))$ and $c'(\theta,\theta') \geq \bar{c}'(\theta,\theta')$ for all (θ,θ'), with strict inequality over some subset of $(0,2)x(0,2)$ assigned positive probability by $g(\theta)$.

Muench gives a more suitable definition of the optimality.

An allocation is equal treatment Pareto optimal(ET-PO) if it maximizes

$$\int \{u[c(\theta),n(\theta)] + V[c'(\theta.\theta')]\}g(\theta)g(\theta')\theta d\theta d\theta'$$

subject to the feasibility condition given above. We can characterize the L allocation in terms of the ET-PO allocation. Muench ahows that the ET-PO allocation is not the L allocation. To this end, he departs from the original Lucas model. Muench introduces two markets, a futures market and a "spot" market, which are not allowed in the Lucas model. By a futures market, he meant a market for contingency claims that takes place before "history" begins. By a spot market meant a market which takes place each period after θ's are known and individuals trade present consumption for future contingencies. The ET-PO allocation is not a spot market equilibrium and the L allocation is a spot market equilibrium. The proposition mentioned above follows, then.

It is also shown that a REE allocation in the futures market yields the ET-PO allocation given appropriate conditions. It is expected that if the futures market is dropped and replaced by a social security scheme, the economy can achieve the ET-PO allocation without the futures market. Muench shows that this conjecture is true. It implies that fiscal policy (taxes and subsidies) is not neutral. Furtheremore, it can be demontsrated that an ET-PO allocation can be reached by appropriate monetary and fiscal policiea.

Thus the exploration of the Lucas model initiated by Muench seems to provide a theoretical basis for a microeconomic stabilization policy. It is noted that the message from Muench's study is interpreted as saying that if a microeconomic (monetary and fiscal) policy is able to replace a role of the futures mrket which optimally allocates endowments risk across time, the economy represented by the Lucas model can achieve the ET-PO allocation by such a policy. This observation is closely related to the economic implication of Newbery and Stiglitz's study.

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