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Feedback Stabilization of Linear Diffusion Systems

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Abstract. This paper treats the feedback stabilization of linear diffusion systems by using a finite-dimensional feedback dynamic controller. We construct a finite-dimensional observer using the output functions from sensors, and the control inputs to the system are given by the feedback of the observer output. Assuming, for some fixed finite number L, that the first L modes are controllable and observable, we prove that it is possible to construct a finite-dimensional feedback dynamic controller such that the diffusion system has an arbitrarily large damping constant.

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1. Introduction

In our previous paper [15], we discussed feedback stabilization of linear diffusion systems by designing actuator influence functions or sensor influence functions properly. In this paper, given arbitrary actuator and sensor influence functions, we construct a finite-dimensional feedback dynamic controller using an observer. By using the pole assignment theory for finite-dimensional linear systems [16], it is possible to stabilize the distributed systems so that it has an arbitrarily large damping constant.

Balas [3] discussed the same problem under the assumption that no observation "spillover" [2] is present. He also discussed the feedback stabilization problem for dissipative hyperbolic systems [4]. We do not neglect the observation spillover in this paper, and we obtain a sharper estimate for the influence of the control and observation spillovers on the stability of the system. It will be proved that the influence of the spillover on the stability of the system can be made arbitrarily small, if we increase the number of state variables of the dynamic controller.

2. Diffusion Systems

Let Ω be a bounded domain in a finite-dimensional Euclidean space, and let $L^2(\Omega)$ denote the Hilbert space of all square integrable real-valued functions with the inner product

$$(u_1, u_2) = \int_\Omega u_1(x)u_2(x)dx.$$

We consider diffusion processes in Ω described by the linear differential equation

$$\frac{du(t)}{dt} + Au(t) = Bf(t) = \sum_{k=1}^{r} b^k f^k(t), \quad t > 0, \tag{1}$$

where $u(t) \in L^2(\Omega)$, $b^k \in L^2(\Omega)$, $f^k(t)$ are scalar functions Hölder-continuous on $[0, \infty)$, and

$$B = (b^1, \cdots, b^r),$$

$$f(t) = \begin{bmatrix} f^1(t) \\ \vdots \\ f^r(t) \end{bmatrix}.$$

We assume that A is a <u>selfadjoint</u> operator with the domain D(A) which is dense in $L^2(\Omega)$, that the resolvent $(A - \lambda)^{-1}$ of A exists and is <u>compact</u> for some λ, and that A is bounded from below.

From the assumption we see that A is closed [9, p.16], that there is a constant γ such that [8, p.278]

$$(Au, u) \geqq \gamma(u, u), \quad u \in D(A), \tag{2}$$

and that the resolvent $(A - \lambda)^{-1}$ exists and is compact for any real λ satisfying $\lambda < \gamma$ [8, p.187].

From the Hilbert-Schmidt theory [11, p.159] for the compact selfadjoint operators, it is well-known that there exist the eigenvalues λ_i and the corresponding eigenfunctions

$\phi_{ij}(x)$ of the operator A satisfying the following conditions [6], [11, p.167]:

(i) $\gamma \leqq \lambda_1 < \lambda_2 < \cdots < \lambda_i < \cdots, \quad \lim \lambda_i = \infty.$

(ii) $A\phi_{ij} = \lambda_i \phi_{ij}, \quad j = 1, \cdots, m_i, \quad i = 1, 2, \cdots,$

where $m_i < \infty$ for each i.

(iii) The set $\{\phi_{ij}(\cdot)\}$ of the eigenfunctions forms a <u>complete orthonormal system</u> in $L^2(\Omega)$.

Since $u \in L^2(\Omega)$ has a unique expression

$$u(\cdot) = \sum_{i=1}^{\infty} \sum_{j=1}^{m_i} (u, \phi_{ij})\phi_{ij}(\cdot),$$

D(A) consists of all elements $u \in L^2(\Omega)$ such that

$$\sum_{i=1}^{\infty} \sum_{j=1}^{m_i} \lambda_i^2 (u, \phi_{ij})^2 < \infty. \tag{3}$$

The semigroup e^{-tA} generated by -A is analytic in $t > 0$, and is expressed as [11, p.309]

$$e^{-tA}u = \sum_{i=1}^{\infty} \sum_{j=1}^{m_i} e^{-\lambda_i t} (u, \phi_{ij})\phi_{ij}, \quad t \geqq 0, \tag{4}$$

where $u \in L^2(\Omega)$.

From (4) we see that

$$\| e^{-tA} \| \leqq e^{-\lambda_1 t}, \quad t \geqq 0. \tag{5}$$

If $\lambda_1 < 0$, the diffusion system

$$\frac{du(t)}{dt} + Au(t) = 0 \tag{6}$$

is clearly unstable. We consider this case, and we synthesize the input functions $f^k(t)$ by using a feedback dynamic controller so that (1) is stabilized.

We assume that there are p sensors, whose outputs are given by

$$y^k(t) = (c^k, u(t)), \quad k = 1, \cdots, p, \tag{7}$$

where $c^k(x)$ are sensor influence functions in $L^2(\Omega)$. Let us define the output vector function

$$y(t) = \begin{bmatrix} y^1(t) \\ \vdots \\ y^p(t) \end{bmatrix}.$$

Let $\sigma > 0$ be a given damping constant. We take an integer ℓ such that

$$\lambda_{\ell+1} > \sigma. \tag{8}$$

We take another integer n such that $n \geqq \ell$, and we define the orthogonal projection operators P_n and Q_n by

$$P_n u = \sum_{i=1}^{n} \sum_{j=1}^{m_i} (u, \phi_{ij})\phi_{ij},$$

$$Q_n u = (I - P_n)u = \sum_{i=n+1}^{\infty} \sum_{j=1}^{m_i} (u, \phi_{ij})\phi_{ij}.$$

Let $u(t)$ be the solution of (1) satisfying the initial condition

$$\lim_{t \to +0} u(t) = u_0 \in L^2(\Omega).$$

Since $u(t) \in D(A)$ for $t > 0$, from (1) we obtain

$$P_n \dot{u}(t) + AP_n u(t) = P_n Bf(t), \tag{9}$$

$$Q_n \dot{u}(t) + AQ_n u(t) = Q_n Bf(t). \tag{10}$$

The solution of (9) with the initial condition $P_n u(0) = P_n u_0$ can be expressed as

$$P_n u(t) = \sum_{i=1}^{n} \sum_{j=1}^{m_i} u_{ij}(t) \phi_{ij}(\cdot), \tag{11}$$

where $u_{ij}(t)$ is the unique solution of

$$\dot{u}_{ij}(t) + \lambda_i u_{ij}(t) = b_{ij} f(t), \tag{12}$$

$$j = 1, \cdots, m_i, \quad i = 1, \cdots, n,$$

satisfying the initial condition

$$u_{ij}(0) = (u_0, \phi_{ij}). \tag{13}$$

In (12), b_{ij} is a row vector defined by

$$b_{ij} = (b_{ij}{}^1, \cdots, b_{ij}{}^r),$$

where $b_{ij}{}^k = (b^k, \phi_{ij})$.

Now let us define the following vector and matrix

$$u_i(t) = \begin{bmatrix} u_{i1}(t) \\ \vdots \\ u_{im_i}(t) \end{bmatrix}, \quad \tilde{B}_i = \begin{bmatrix} b_{i1} \\ \vdots \\ b_{im_i} \end{bmatrix} = \begin{bmatrix} b_{i1}{}^1 & \cdots & b_{i1}{}^r \\ \vdots & & \vdots \\ b_{im_i}{}^1 & \cdots & b_{im_i}{}^r \end{bmatrix}. \tag{14}$$

Then (12) can be written as

$$\dot{u}_i(t) + \lambda_i u_i(t) = \tilde{B}_i f(t), \quad i = 1, \cdots, n. \tag{15}$$

Furthermore, let

$$L = m_1 + \cdots + m_\ell, \quad N = m_1 + \cdots + m_n.$$

Since $\ell \leqq n$, $L \leqq N$. Let us define an L-dimensional vector $x_1(t)$, $L \times r$ matrix B_1, and $L \times L$ diagonal matrix A_1 by

$$x_1(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_\ell(t) \end{bmatrix}, \quad B_1 = \begin{bmatrix} \tilde{B}_1 \\ \vdots \\ \tilde{B}_\ell \end{bmatrix}, \tag{16}$$

$$A_1 = \mathrm{diag}(-\lambda_1 I_{m_1}, \cdots, -\lambda_\ell I_{m_\ell}),$$

where I_m denotes an $m \times m$ unit matrix. Then from (15) we obtain

$$\dot{x}_1(t) = A_1 x_1(t) + B_1 f(t). \tag{17}$$

Similarly, let us define an (N-L)-dimensional vector $x_2(t)$, $(N-L) \times r$ matrix B_2, and $(N-L) \times (N-L)$ diagonal matrix A_2 by

$$\left.\begin{aligned} x_2(t) &= \begin{bmatrix} u_{\ell+1}(t) \\ \vdots \\ u_n(t) \end{bmatrix}, \quad B_2 = \begin{bmatrix} \tilde{B}_{\ell+1} \\ \vdots \\ \tilde{B}_n \end{bmatrix}, \end{aligned}\right\} \tag{18}$$

$$A_2 = \mathrm{diag}(-\lambda_{\ell+1} I_{m_{\ell+1}}, \cdots, -\lambda_n I_{m_n}).$$

Then we obtain

$$\dot{x}_2(t) = A_2 x_2(t) + B_2 f(t). \tag{19}$$

We see that (17) is controllable if and only if [7], [13]

$$\mathrm{rank}\ \tilde{B}_i = m_i, \quad i = 1, \cdots, \ell. \tag{20}$$

In order that the relation (20) holds, the number r of the control inputs should satisfy

$$r \geqq \max\{m_1, \cdots, m_\ell\}. \tag{21}$$

Since

$$u(t) = P_n u(t) + Q_n u(t),$$

the output functions (7) are expressed as

$$y^k(t) = \sum_{i=1}^{n} \sum_{j=1}^{m_i} c_{ij}{}^k u_{ij}(t) + (Q_n c^k, Q_n u(t)), \tag{22}$$

$$k = 1, \cdots, p,$$

where $c_{ij}{}^k = (c^k, \phi_{ij})$. By defining the matrices

$$\left.\begin{aligned} \tilde{C}_i &= \begin{bmatrix} c_{i1}{}^1 & \cdots & c_{im_i}{}^1 \\ \vdots & & \vdots \\ c_{i1}{}^p & \cdots & c_{im_i}{}^p \end{bmatrix}, \end{aligned}\right\} \tag{23}$$

$$C_1 = [\ \tilde{C}_1, \cdots, \tilde{C}_\ell\ ], \quad C_2 = [\ \tilde{C}_{\ell+1}, \cdots, \tilde{C}_n\ ],$$

the output vector function can be expressed as

$$y(t) = C_1x_1(t) + C_2x_2(t) + S_nQ_nu(t), \tag{24}$$

where $S_nQ_nu(t)$ is called the <u>observation spillover</u> [2], and the operator S_n mapping $Q_nL^2(\Omega)$ into R^p is defined by

$$S_nu = \begin{bmatrix} (Q_nc^1, u) \\ \vdots \\ (Q_nc^p, u) \end{bmatrix}, \quad u \in Q_nL^2(\Omega). \tag{25}$$

The system (A_1, C_1) is observable if and only if [7], [14]

$$\text{rank } \tilde{C}_i = m_i, \quad i = 1, \cdots, \ell. \tag{26}$$

In order that the rank condition (26) holds, the number p of sensors should be such that

$$p \geqq \max\{m_1, \cdots, m_\ell\}. \tag{27}$$

3. Feedback Control Using Observers

First, we construct two kinds of finite-dimensional observer defined by

$$\dot{z}_1(t) = (A_1-G_1C_1)z_1(t) + G_1[y(t)-C_2z_2(t)] + B_1f(t), \tag{28}$$

$$\dot{z}_2 = A_2z_2 + B_2f(t), \tag{29}$$

where $z_1(t)$ is an L-dimensional vector and $z_2(t)$ is an (N-L)-dimensional vector which estimate $x_1(t)$ and $x_2(t)$, respectively, and G_1 is an $L \times p$ matrix to be determined.

It is clear from (19) and (29) that

$$x_2(t) - z_2(t) = e^{A_2 t}(x_{20} - z_{20}), \tag{30}$$

where $x_{20} = x_2(0)$, and $z_{20} = z_2(0)$. Let us define a 2L-dimensional vector $q_1(t)$ by

$$q_1(t) = \begin{bmatrix} x_1(t) \\ z_1(t) \end{bmatrix}. \tag{31}$$

Let the control input vector function $f(t)$ be given by

$$f(t) = F_1 z_1(t), \tag{32}$$

where F_1 is an $r \times L$ matrix to be determined. Substituting (32) into (10), (17), and (28), and using (24) and (30) gives

$$\frac{d}{dt}\begin{bmatrix} q_1(t) \\ Q_n u(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} q_1(t) \\ Q_n u(t) \end{bmatrix} + \begin{bmatrix} \psi(t) \\ 0 \end{bmatrix}, \tag{33}$$

where

$$\left.\begin{aligned} A_{11} &= \begin{bmatrix} A_1 & B_1F_1 \\ G_1C_1 & A_1 - G_1C_1 + B_1F_1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 \\ G_1S_n \end{bmatrix}, \end{aligned}\right\} \tag{34}$$

$$A_{21} = [0, \quad Q_n B F_1], \qquad\qquad A_{22} = -AQ_n, \quad \Big\}$$

$$\psi(t) = \begin{bmatrix} 0 \\ G_1 C_2 e^{A_2 t}(x_{20} - z_{20}) \end{bmatrix}. \tag{35}$$

We see that

$$\begin{bmatrix} I_L & 0 \\ -I_L & I_L \end{bmatrix} \begin{bmatrix} A_1 & B_1 F_1 \\ G_1 C_1 & A_1 - G_1 C_1 + B_1 F_1 \end{bmatrix} \begin{bmatrix} I_L & 0 \\ -I_L & I_L \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} A_1 + B_1 F_1 & B_1 F_1 \\ 0 & A_1 - G_1 C_1 \end{bmatrix}. \tag{36}$$

Suppose the rank conditions (20) and (26) hold. Then the linear system (A_1, B_1, C_1) is controllable and observable. Consequently, there exist matrices F_1 and G_1 such that all the eigenvalues of the matrices $A_1 + B_1 F_1$ and $A_1 - G_1 C_1$ take any preassigned values $\{-\nu_1, -\nu_2, \cdots, -\nu_{2L}\}$[10], [16]. Here, the real numbers $\nu_i > 0$ are such that

$$\left.\begin{aligned} \sigma < \lambda_{\ell+1} < \nu_1 < \nu_2 < \cdots < \nu_{2L}, \\ \nu_1 < \lambda_{n+1}. \end{aligned}\right\} \tag{37}$$

Let us construct the matrices F_1 and G_1 as stated above. In view of (34) and (36), we see that the matrix A_{11} is similar to the diagonal matrix

$$\mathrm{diag}(-\nu_1, -\nu_2, \cdots, -\nu_{2L}),$$

and the matrix $e^{A_{11}t}$ is also similar to the diagonal matrix

$$\mathrm{diag}(e^{-\nu_1 t}, e^{-\nu_2 t}, \cdots, e^{-\nu_{2L} t}).$$

In other words, there is a nonsingular matrix T_1 such that [1], [5]

$$T_1 e^{A_{11}t} T_1^{-1} = \mathrm{diag}(e^{-\nu_1 t}, \cdots, e^{-\nu_{2L} t}). \tag{38}$$

From (38) we obtain

$$\| e^{A_{11}t} \| \leqq M_1 e^{-\nu_1 t}, \quad t \geqq 0, \tag{39}$$

where M_1 is the so-called condition number of a nonsingular matrix T_1 defined by

$$M_1 = \| T_1 \| \| T_1^{-1} \| \geqq 1. \tag{40}$$

Since

$$e^{A_{22}t} Q_n u_0 = \sum_{i=n+1}^{\infty} \sum_{j=1}^{m_i} e^{-\lambda_i t} (Q_n u_0, \phi_{ij}) \phi_{ij}, \tag{41}$$

It is clear that

$$\| e^{A_{22}t} \| \leqq e^{-\lambda_{n+1} t}, \quad t \geqq 0. \tag{42}$$

Let us define the operators

$$\tilde{A} = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}, \tag{43}$$

where $\tilde{A}$ is unbounded, whereas $\tilde{B}$ is bounded. Then

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \tilde{A} + \tilde{B}. \tag{44}$$

Let us introduce infinite-dimensional vectors

$$w(t) = \begin{bmatrix} q_1(t) \\ Q_n u(t) \end{bmatrix}, \quad \tilde{\psi}(t) = \begin{bmatrix} \psi(t) \\ 0 \end{bmatrix}, \tag{45}$$

with the norm

$$\| w(t) \| = [\| q_1(t) \|^2 + \| Q_n u(t) \|^2]^{1/2}.$$

Then (33) can be written as

$$\dot{w}(t) = (\tilde{A} + \tilde{B}) w(t) + \tilde{\psi}(t). \tag{46}$$

It is clear that

$$e^{\tilde{A}t} = \begin{bmatrix} e^{A_{11}t} & 0 \\ 0 & e^{A_{22}t} \end{bmatrix},$$

and that

$$\| e^{\tilde{A}t} \| \leqq \max(\| e^{A_{11}t} \|, \| e^{A_{22}t} \|). \tag{47}$$

Since $0 < \nu_1 < \lambda_{n+1}$, and $M_1 \geqq 1$, it follows from (39) and (42) that

$$\| e^{\tilde{A}t} \| \leqq M_1 e^{-\nu_1 t}, \quad t \geqq 0. \tag{48}$$

From (43) we see that

$$\|\tilde{B}\| \leqq \max(\| A_{12} \|, \| A_{21} \|) \leqq \max(\| Q_n B \| \, \| F_1 \|, \| G_1 \| \, \| S_n \|), \tag{49}$$

where

$$\begin{aligned} \| Q_n B \| &= (\| Q_n b^1 \|^2 + \cdots + \| Q_n b^r \|^2)^{1/2}, \\ \| S_n \| &\leqq (\| Q_n c^1 \|^2 + \cdots + \| Q_n c^p \|^2)^{1/2}. \end{aligned} \tag{50}$$

The second relation of (50) is derived from (25).

Now, applying the perturbation theory of semigroups [8, p. 495], [12, p. 80], we obtain

$$\| e^{(\tilde{A}+\tilde{B})t} \| \leqq M_1 e^{-(\nu_1 - M_1 \| \tilde{B} \|)t}, \quad t \geqq 0. \tag{51}$$

Let

$$\bar{\nu}_1 = \nu_1 - M_1 \| \tilde{B} \|.$$

Since $b^i \in L^2(\Omega)$ $(i=1, \cdots, r)$, $c^i \in L^2(\Omega)$ $(i=1, \cdots, p)$, and M_1 is independent of n, in view of (49) and (50), for any small number $\varepsilon > 0$ there is an integer $n(\geqq \ell)$ such that

$$M_1 \| \tilde{B} \| \leqq \varepsilon. \tag{52}$$

Since $\lambda_{\ell+1} < \nu_1$ from (37), proper choice of n gives the relation

$$\sigma < \lambda_{\ell+1} < \bar{\nu}_1 < \nu_1. \tag{53}$$

The solution of (46) is clearly given as

$$w(t) = e^{(\tilde{A}+\tilde{B})t} w_0 + \int_0^t e^{(\tilde{A}+\tilde{B})(t-s)} \tilde{\psi}(s) ds. \tag{54}$$

By using (51), $\| w(t) \|$ can be estimated as

$$\| w(t) \| \leqq M_1 e^{-\bar{\nu}_1 t} [\| w_0 \| + \int_0^t e^{\bar{\nu}_1 s} \| \tilde{\psi}(s) \| ds]. \tag{55}$$

From (18) we see that

$$\| e^{A_2 t} \| = e^{-\lambda_{\ell+1} t}, \quad t \geqq 0. \tag{56}$$

Using (35) and (56), we obtain

$$\| \tilde{\psi}(t) \| \leqq \| G_1 \| \| C_2 \| e^{-\lambda_{\ell+1} t} \| x_{20} - z_{20} \|. \tag{57}$$

Substituting (57) into (55) yields

$$\| w(t) \| \leqq M_2 e^{-\lambda_{\ell+1} t}, \quad t \geqq 0, \tag{58}$$

where

$$M_2 = M_1 [\| w_0 \| + \| G_1 \| \| C_2 \| (\bar{\nu}_1 - \lambda_{\ell+1})^{-1} \| x_{20} - z_{20} \|]. \tag{59}$$

Define a 2(N-L)-dimensional vector $q_2(t)$ by

$$q_2(t) = \begin{bmatrix} x_2(t) \\ z_2(t) \end{bmatrix}. \tag{60}$$

From (19) and (29) we obtain

$$\dot{q}_2(t) = \hat{A}q_2(t) + \hat{B}z_1(t), \tag{61}$$

where

$$\hat{A} = \begin{bmatrix} A_2 & 0 \\ 0 & A_2 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B_2F_1 \\ B_2F_1 \end{bmatrix}. \tag{62}$$

Integrating (61) gives

$$q_2(t) = e^{\hat{A}t}q_{20} + \int_0^t e^{\hat{A}(t-s)}\hat{B}z_1(s)ds, \tag{63}$$

where $q_{20} = q_2(0)$. Using the estimates

$$\begin{aligned} &\| e^{\hat{A}t} \| \leqq \| e^{A_2t} \| = e^{-\lambda_{\ell+1}t} \leqq e^{-\sigma t}, \quad t \geqq 0, \\ &\| \hat{B} \| \leqq \sqrt{2} \| F_1 \| \| B_2 \|, \\ &\| z_1(t) \| \leqq \| w(t) \| \leqq M_2 e^{-\lambda_{\ell+1}t}, \quad t \geqq 0, \end{aligned} \tag{64}$$

we obtain the following relation

$$\| q_2(t) \| \leqq e^{-\sigma t}[\| q_{20} \| + \sqrt{2}M_2 \| F_1 \| \| B_2 \| (\lambda_{\ell+1} - \sigma)^{-1}]. \quad (65)$$

Putting (58) and (65) together, we finally obtain

$$\| w(t) \| \leqq M_1[\| w_0 \| + \sqrt{2} \| G_1 \| \| C_2 \| (\bar{\nu}_1 - \lambda_{\ell+1})^{-1} \| q_{20} \|] e^{-\sigma t}, \quad (66)$$

$$\| q_2(t) \| \leqq M_1[\sqrt{2} \| F_1 \| \| B_2 \| (\lambda_{\ell+1} - \sigma)^{-1} \| w_0 \| + \{1$$

$$+ 2 \| F_1 \| \| G_1 \| \| B_2 \| \| C_2 \| (\lambda_{\ell+1} - \sigma)^{-1} (\bar{\nu}_1 - \lambda_{\ell+1})^{-1}\} \| q_{20} \|] e^{-\sigma t}, \quad (67)$$

for $t \geqq 0$. Define an infinite-dimensional vector $\tilde{w}(t)$ by

$$\tilde{w}(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ Q_n u(t) \end{bmatrix} \in R^{2N} \times Q_n L^2(\Omega).$$

It is obvious that $\tilde{w}(t)$ represents the state of the diffusion system as well as the state of the dynamic controller. From (66) and (67) we see that

$$\| \tilde{w}(t) \| \leqq K e^{-\sigma t} \| \tilde{w}(0) \|, \quad t \geqq 0, \quad (68)$$

where K is a constant dependent on ℓ, n, etc..

Thus we can summarize what we have discussed so far as follows:

<u>Theorem</u>. Given an arbitrary damping constant $\sigma > 0$, suppose that the rank conditions (20) and (26) hold, where ℓ is an integer satisfying (8). Then a finite-dimensional feedback dynamic con-

troller, described by (28), (29), and (32), can be constructed such that the state $\tilde{w}(t)$ of the overall system satisfies (68), where K is a constant dependent of ℓ, n, and so on.

Remark 1. The bounded operator $\tilde{B}$ defined by (43) results from the control and observation spillovers [2]. If

$$b^k(\cdot) \in P_n L^2(\Omega), \quad k = 1, \cdots, r,$$

$$c^k(\cdot) \in P_n L^2(\Omega), \quad k = 1, \cdots, p,$$

for some integer n, then $\tilde{B} = 0$.

Remark 2. If $\ell = n$, (52) does not hold. Because in this case the constant M_1 depends on $\ell = n$ and $M_1 \to \infty$ $(\ell \to \infty)$, in general. Thus, boundedness of $M_1\|\tilde{B}\|$ with respect to n is not clear. The key point of this paper lies in the introduction of two different integers ℓ and n.

Remark 3. In this paper, an identity observer has been used for estimating the state of the first L modes of the diffusion system. It is also possible to construct a feedback dynamic controller by use of a reduced order observer [10].

Remark 4. Curtain [17] discusses, under some conditions, the case where operator B in (1) and the observation operator C defined by $y(t) = Cu(t)$ are not bounded.

Remark 5. Mitkowski [18] considered the stabilization of linear distributed systems by using an infinite-dimensional observer. The idea in this paper is partially due to Mitkowski.

4. An Example of Partial Differential Equation

In this section, we show an example of partial differential equation as well as boundary condition which can be described in the abstract form as in (1).

Let us consider the partial differential equation

$$\frac{\partial u(t, x)}{\partial t} - (\Delta - c(x))u(t, x) = \sum_{k=1}^{r} b^k(x) f^k(t), \tag{69}$$

where $x \in \Omega$, Δ denotes the Laplacian, and $c(x)$ is a bounded measurable function. The boundary condition is assumed to be either of the Dirichlet type

$$u(t, x) = 0, \quad t > 0, \quad x \in \Gamma, \tag{70}$$

or of the third kind

$$\frac{du(t, x)}{dn} + \sigma(x)u(t, x) = 0, \quad t > 0, \quad x \in \Gamma, \tag{71}$$

where Γ is a sufficiently smooth boundary of Ω, d/dn is the derivative in the direction of the inner normal, and $\sigma(x)$ is a sufficiently smooth function on Γ. In view of (69), let us define the operator A by

$$Au = (-\Delta + c(x))u(\cdot). \tag{72}$$

It is proved in [11] that the operator A defined on the domain

$$D(A) = \{u \in H^2(\Omega) : \frac{du}{dn} + \sigma u = 0 \quad (\text{ or } \quad u = 0 \ ) \quad \text{on } \Gamma\} \tag{73}$$

is selfadjoint, where $H^2(\Omega)$ is the Sobolev space of order 2, that $D(A)$ is dense in $L^2(\Omega)$, that the resolvent $(A - \lambda)^{-1}$ exists and is compact for any real λ satisfying $\lambda < \inf_{x\varepsilon\Omega} c(x)$, and that A is bounded from below. Therefore, the diffusion system (69) with the boundary condition (70) or (71) can be expressed as in (1).

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