数理解析研究所講究録
487巻 1983年 79-81

FINITENESS OF SYMMETRIES ON 3-MANIFOLDS

Sadayoshi Kojima

This is a summary of my talk on finiteness of symmetries on 3-manifolds at Research Institute for Mathematical Science.

A symmetry of a manifold M is a periodic diffemorphism with non-empty fixed point set. It then determines a cyclic group action on M and hence a cyclic subgroup of Diff M. We shall identify two symmetries if they induce conjugate subgroups in Diff M. My exclusive concern is finiteness properties of isometric symmetries on a geometric manifold. However this object is not restrictive for the general study of symmetries in the light of recent Thurston's theorem [3].

One of the finiteness properties we can conclude from my study and Thurston's theorem is

Theorem A. *Let M be a closed orientable prime 3-manifold. Then there are only finitely many conjugacy classes of cyclic subgroups of given order in Diff M generated by an orientation preserving symmetry.*

This means that M admits only finitely many symmetries of given order. Theorem A almost answers Problem 3.39 (A) in [1] posed by Tollefson. This can be generalized for compact non-prime manifolds without $S^1 \times S^2$ summands. As a corollary to the generalization, we can negetively answer Problem 3.41 in [1] posed by Montesinos.

Corollary B. _No closed 3-manifold admits an infinite number of noneqivariant involutions with_ S^3 _as the orbit space_.

The other finiteness property we conclude concerns with the possible periods of symmetries on a given manifold. On non-spherical surfaces, it is known to be bounded. We got the analogus result for 3-manifolds. That is

Theorem C. _Let_ M _be a closed orientable irreducible non-spherical 3-manifold. Then the order of cyclic subgroups of_ Diff M _generated by an orientation preserving symmetry is bounded_.

This also almost answers Problem 3.39 (C) in [1] posed by Thurston. There is a similar generalization of the theorem for non-irreducible manifolds.

These theorems are still valid for compact manifolds with toral boundary with slight change. This generalizations are useful for the study of symmetries of links. A symmetry of a link is by definition a symmetry of the link exterior which extends a symmetry of S^3. A link is said to have period n if it admits a symmetry of order n. Two symmetries are identified if they are conjugate by a diffeomorphism of S^3 leaving the link invariant. Though the equivalence turns out fine, the boundedness theorem of the possible periods of symmetries still does work, and we get a corollary which answers a question posed by Sakuma in [4].

Corollary D. Any non-trivial link has only finitely many periods.

The complete proof of these theorems (except Thurston's theorem as usual!) is available as a preprint form [2].

References

[1] Kirby, R.: Problems in low-dimensional manifold theory, Proc. Symp. Pure Math. vol 32 (1978), 273 - 312.

[2] Kojima, S.: Finiteness of symmetries on 3-manifolds, Preprint.

[3] Thurston, W.: Three manifolds with symmetries, preliminary report (1982).

[4] 3 and 4-dimensional manifolds, RIMS Kokyoroku 467 (Japanese).