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Tomographic Reconstruction Problem in Hilbert Spaces

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Section 1.

In this note we consider the problem of reconstructing a function from its projections. This problem has arisen in many fields such as medicine, astronomy and electron microscopy. Let B be a closed ball in R^n and we denote $L^2(B)$ a Hilbert space of all square integrable functions on B. Let $f \in L^2(B)$ and let $\{(R_{\omega_i} f)(\cdot)\}$ be a given finite set of hyperplane integrals where $\{\omega_1, \cdots, \omega_k\}$ is a finite set of unit vectors in R^n and hyperplane integral $R_{\omega_i} f$ of f is defined as follows:

$$R_{\omega_i} f(p) = \int_{<x,\omega_i>=p} f(x)\,dx$$

Under these notations the two main questions for this problem are stated as follows: [1]

Q1. Estimate f by $\{(R_{\omega_i} f)(\cdot)\}$ and evaluate $\| f - \tilde{f} \|$, where $\tilde{f}$ is an estimator of f,

Q2. for f and k are given, determine $\{\omega_i\}$ which optimizes the estimation.

Section 2.

If we have a complete set of hyperplane integrals, then the problem is nothing but an inversion theorem of Radon transform. In our case we consider an approximate reconstruction from finitely many projections. S.Helgason[2] and D.C.Solmon [3,4]showed several elegant results for Q1. Conversely we show some results for Q2 which give optimal projections. Let μ be the measure of $L^2(B)$ and let S be a covariance operator of μ with respect to f. Then the eigen vectors of S form an optimal coordinates system for approximating f. Furthermore this coordinates system has the entropy minimizing property.

References

[1] I.SUZUKI, Computed Tomography and Radon transform. Fourth Seminor on Applied Functional Analysis 1981,67-72.

[2] S.Helgason, The Radon Transform. Progress in Mathematics Birkhauser, Cambrige 1980.

[3] D.C.Solmon, The X-ray Transform, J. Math. Anal. Appl. 31,1976,61-83.

[4] __________, Practical and Mathematical Aspect of The Problem of Reconstructing Obects From Radiographs, Bulletin of the Amer. Math. Soc. 83,6,1977,1227-1268.