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REFINABLE MAPS AND SHAPE

By Hisao Kato

In [9], J. J. Kelley defined very important notion "property [K] and he proved that if X is a continuum which has property [K], then the hyperspace C(X) of subcontinua of X is contractible. In [6], R. W. Wardle proved that every confluent map preserves property [K]. It is well-known that every refinable map is weakly confluent (see [1]), but simple examples show that weakly confluent maps do not preserve property [K].

In [12, (16.38) Question], S. B. Nadler asked the following question: what kinds of mappings preserve property [K]? We show that every refinable map preserves property [K]. In [1], J. Ford and J. W. Rogers proved that every refinable map onto a Peano continuum (locally connected) is monotone. In [10], S. B. Nadler proved that if \( f: X \to Y \) is a near-homeomorphism between compacta and Y has property [K], then f is confluent.

Note that every near-homeomorphism is a refinable map but the converse is not true. We show that if \( r: X \to Y \) is a refinable map between compacta and Y has property [K], then r is confluent. The condition that Y has property [K] cannot be omitted. We give a example in which refinable maps are not confluent.

Also, we show that if \( r: X \to Y \) is a refinable map between continua, then X is irreducible iff Y is irreducible. Moreover, in shape theory, we have the following: If \( r: X \to Y \) is a refinable
map between compacta and \( Y \) is calm, then \( r \) is a shape equivalence. As a corollary, if \( r: X \to Y \) is a refinable map between compacta and either \( X \) or \( Y \) is \( S^n \)-like \((n \geq 1)\), then \( r \) is a shape equivalence, where \( S^n \) denotes the \( n \)-sphere (cf. [3]). Several properties concerning refinable maps have been studied in \([1, 2, 3, 4, 5, 6, 7, 8, \text{ etc.}]\).

The word compactum means a compact metric space. A connected compactum is called a continuum. If \( x \) and \( y \) are points of a metric space, \( d(x, y) \) denotes the distance from \( x \) to \( y \). For any subsets \( A, B \) of a metric space, let \( d(A, B) = \inf \{d(a,b) | a \in A, b \in B\} \). Also, let \( d_H(A, B) = \max \{\sup_{a \in A} d(a,B), \sup_{b \in B} d(b,A)\} \). \( d_H \) is called the Hausdorff metric (see [9], [10]). A compactum \( X \) is said to have property \([K]\) (see [9]) provided that given \( \epsilon > 0 \) there exists \( \delta > 0 \) such that if \( a, b \in X \), \( d(a,b) < \delta \), and \( A \) is a subcontinuum of \( X \) with \( a \in A \), then there exists a subcontinuum \( B \) of \( X \) such that \( b \in B \) and \( d_H(A, B) < \epsilon \). Note that every locally connected compactum has a property \([K]\), but the converse is not true. A map \( f: X \to Y \) between compacta is confluent (weakly confluent) if for every subcontinuum \( Q \) of \( Y \) each (at least one, respectively) component of the inverse image \( f^{-1}(Q) \) is mapped by \( f \) onto \( Q \). A map \( r: X \to Y \) between compacta is refinable \([1]\) if for every \( \epsilon > 0 \) there exists an onto map \( f: X \to Y \) such that \( \operatorname{diam} f^{-1}(y) < \epsilon \) for each \( y \in Y \) and \( d(r, f) = \sup \{d(r(x), f(x)) | x \in X\} < \epsilon \). By definitions, each refinable map is surjective, each near-homeomorphism is refinable and if there is a refinable map from a compactum \( X \) to a compactum \( Y \), then \( X \) is \( Y \)-like (see [5] for the definition that \( X \) is \( Y \)-like). But any converse assertions of them are not true.
Theorem. Let \( r : X \to Y \) be a refinable map between compacta. If \( X \) has property \([K]\), then \( Y \) has the same property.

Corollary. If \( r : X \to Y \) is a refinable map between continua and \( X \) has property \([K]\), then the hyperspaces \( 2^Y \) and \( C(\gamma) \) are contractible.

Theorem. Let \( r : X \to Y \) be a refinable map between compacta. If \( Y \) has property \([K]\), then \( r \) is confluent.

Remark. In the statement of above theorem, we cannot omit the condition that \( Y \) has property \([K]\). In the plane \( \mathbb{R}^2 \), put

\[
A = \{(2,y) \mid -1 \leq y \leq 2\}, \\
B = \text{Cl}\{(x,\sin \frac{2\pi}{x}) \mid -1 \leq x < 0\}, \\
C = \text{Cl}\{(x,\sin \frac{2\pi}{x}) \mid 0 < x \leq 1\}, \\
D = \text{Cl}\{(x,\sin \frac{2\pi}{x-2}) \mid 1 \leq x < 2\}, \text{ and} \\
E = \{(0,y) \mid -1 \leq y \leq 2\}.
\]

Also, let \( X = A \cup B \cup C \cup D \) and \( Y = B \cup E \). Define a map \( r : X \to Y \) by

\[
 r(p) = \begin{cases} 
(0,\sin \frac{2\pi}{x}) & \text{if } p = (x,\sin \frac{2\pi}{x}) \in C, \\
(0,\sin \frac{2\pi}{x-2}) & \text{if } p = (x,\sin \frac{2\pi}{x-2}) \in D, \\
(0,y) & \text{if } p \in A,
\end{cases}
\]

Then it is easily seen that \( r \) is a refinable map, but not confluent.

Corollary. If \( r : X \to Y \) is a refinable map between compacta and \( X \) has property \([K]\), then \( r \) is confluent.
It is well-known that the condition that the hyperspaces $2^X$ and $C(X)$ of a continuum $X$ is contractible does not imply that $X$ has property [K]. Hence, the following question is raised.

Question. Let $r: X \to Y$ be a refinable map between continua. If the hyperspaces $2^X$ and $C(X)$ are contractible, are the hyperspaces $2^Y$ and $C(Y)$ contractible?

Recall that a continuum $X$ is irreducible if there exist two points $p, q \in X$ such that no proper subcontinuum of $X$ contains $p$ and $q$. A continuum is hereditarily decomposable (hereditarily indecomposable) if for any non-degenerate subcontinuum $A$ of $X$, there exists (there does not exist) a decomposition of $A$ into two proper subcontinua $A_1$ and $A_2$ of $A$ such that $A = A_1 \cup A_2$.

A continuum $T$ is a trioid if there are three subcontinua $A$, $B$, and $C$ of $T$ such that $T = A \cup B \cup C$, $A \cap B \cap C = A \cap B = B \cap C = C \cap A$ and this common part is a proper subcontinuum of each of them. A continuum is atriodic if $X$ fails to contain a trioid ($\mathcal{U}$).

Theorem. Let $r: X \to Y$ be a refinable map between continua. Then $X$ is irreducible iff $Y$ is irreducible.

To prove the above theorem, we need the following characterization of irreducible continua.

Theorem (R. H. Sorgenfrey [7]). A necessary and sufficient condition that $X$ is irreducible is that if $X$ is the essential sum of three proper subcontinua, then some pair fails to intersect.
Proposition. Let \( r: X \rightarrow Y \) be a refinable map between compacta. If either \( X \) or \( Y \) is a Cantor set, then \( r \) is a near-homeomorphism, i.e., \( X \) and \( Y \) are Cantor sets.

Proposition. Let \( r: X \rightarrow Y \) be a refinable map between continua. Then

1. if \( X \) is hereditarily decomposable, then \( Y \) is also,
2. \( X \) is hereditarily indecomposable iff \( Y \) is also, and
3. \( X \) is atriodic iff \( Y \) is also.

Corollary. Let \( r: X \rightarrow Y \) be a refinable map between continua. If either \( X \) or \( Y \) is the pseudo-arc, then \( r \) is a near-homeomorphism, i.e., \( X \) and \( Y \) are pseudo-arcs.

A compactum \( X \) is calm if whenever \( X \subset M \subset ANR \), there is a neighborhood \( V \) of \( X \) in \( M \) such that for any neighborhood \( U \) of \( X \) in \( M \) there is a neighborhood \( W \) of \( X \) in \( M \), \( W \subset U \) such that if \( f, g: Y \rightarrow W \) are maps with \( f \lessdot g \) in \( V \), then \( f \lessdot g \) in \( U \) for all \( Y \subset ANR \).

Theorem. If \( r: X \rightarrow Y \) is a refinable map between compacta and \( Y \) is calm, then \( r \) is a shape equivalence, i.e., \( sh(X) = sh(Y) \).

Corollary. If \( r: X \rightarrow Y \) is a refinable map between compacta and \( Y \) is an FANR, then \( r \) is a shape equivalence (see [3]).

Corollary. If \( r: X \rightarrow Y \) is a refinable map between compacta and \( Y \) is an \( AANR_N \), then \( r \) is a shape equivalence.

Remark. In the statements of above results, we cannot replace "calm" by "movable". Also, we cannot replace "\( AANR_N \)" by "\( AANR_C \)" (see [4]).
As a corollary, we have

Corollary. If $r: X \to Y$ is a refinable map between compacta and if either $X$ or $Y$ is $S^n$-like $(n \geq 1)$, then $r$ is a shape equivalence, where $S^n$ denotes the $n$-sphere.

Question. Does every refinable map preserve calmness (FANR, AANR)?
References

