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Dihedral-free groups.

by

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Recently, G.Glauberman showed that all S^4-free simple groups, that is, all simple groups which don't involve the symmetric group of degree 4, are known.

So, this classification shows that 3-elements in 2-local subgroups are very important and all simple groups else contain a dihedral subgroup D_6. So we naturally ask to characterize the following groups:

" A group G does not involve any dihedral subgroups of order 2p for all primes p greater than 3. "

From now on, I call such groups to be D-free. So my purpose is to characterize all D-free groups. An inspection of all the known simple groups containing 26 sporadic simple groups shows that there are only three such simple groups, they are, $PSL_2(7) \cong GL_3(2)$, $PSL_3(3)$, and $U_3(3)$.

And we easily show the structure of D-free groups whose composition factors are all known.

Lemma 1. Let H be a known D-free group, then H has the following structure;

$$O^{2'}(H/O_{\{2,3\}}(H)) \cong 1 \text{ or } E_1\times\cdots\times E_n ,$$

where each E_i is isomorphic to one of the known three simple groups and $O_{\{2,3\}}(H)$ denotes the unique maximal normal $\{2,3\}$-subgroup of H.

So my aim is to prove the following:

Conjecture. Let G be a finite D-free group, then G is of known type.

A result that I have already got is;

Theorem. Let G be a finite simple D-free group. Assume that every proper subgroup of G is of known type. Then we have the following:

1) every 2-local subgroup is 2-constrained,

2) $e_2(3)\leq 2$ (2-local 3-rank is at most 2).

Furthermore, is $e_2(3)\leq 1$, then G is isomorphic to one of the known simple groups.

Here, $e_2(3)=\max\{m_3(N_G(H)) : 1\neq H \text{ 2-subgroups of } G\}$.