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On certain non-continuous functions and shape

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In the shape category of topological spaces a shape morphism is constructed by a system of maps (= continuous functions); it is, in general, not generated by a single map. Hence we have the following questions:

Question 1. Is it possible that a kind of non-continuous function induces a shape morphism?

Question 2. Can a shape equivalence be generated by a certain non-continuous function?

Definition 1. Let X and Y be topological spaces. A function $f : X \to Y$ is a connectivity function if for any connected $C \subseteq X$, the graph $G(f|C)$ of $f|C$ is connected.

Definition 2. A function $f : X \to Y$ is almost continuous if for any open set $N \subseteq X \times Y$ containing $G(f)$ there is a continuous function $g : X \to Y$ such that $G(g) \subseteq N$.

These notions have been considered to generalize Brouwer's fixed point theorem (cf. Stallings[10]).

Each of the following is intermediate to answer the questions.
Proposition 1. Let $f : X + Y$ be an almost continuous function between compact metric spaces. Then there are ANR-sequences $X = \{X_i, p_{ij}\}$ and $Y = \{Y_i, q_{ij}\}$ with limits $X$ and $Y$, respectively, and a system $f : X + Y$ of almost continuous functions $f_i : X_i \to Y_i$ such that $f_ip_{ij} = q_{ij}f_j$ for $i \leq j$.

Proof. There are ANR-sequences $X$ and $Y$ with limits $X$ and $Y$, respectively, and each projection $p_i, q_i$ surjective, since both $X$ and $Y$ are compact metric. Define a function $f_i : X_i \to Y_i$ for each $i$, by the formula $f_ip_i = q_if$. The almost continuity of $f$ implies that of $f_i$.

Proposition 2. A bijective connectivity function with connectivity inverse function does not induce a shape equivalence.

Proof. By the example of Stallings [10,p.262].
Let $X$ be the circle represented as the real numbers mod 1.
Define a function $f : X \to X \times X$ by the formula
$$f(x \text{ mod } 1) = 1/x \text{ mod } 1, \text{ where } 0 < x \leq 1.$$ Let $Y$ be the graph of $f$ and $f^* : X \to Y$ such that
$$f^*(x) = (x,f(x)).$$ Then $f^*$ is a bijection, and both $f^*$ and $f^{*-1}$ are connectivity functions, but $\text{Sh}(X) \neq \text{Sh}(Y)$, because their 1-dimensional Čech cohomology groups are different.

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REFERENCES ON NON-CONTINUOUS FUNCTIONS


