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The Godbillon-Vey class of codimension one foliations without holonomy

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In this note we prove the following result.

THEOREM. Let F be a codimension one C^2-foliation on a compact smooth manifold M and assume that F is without holonomy, namely the holonomy group of each leaf is trivial. Then the Godbillon-Vey characteristic class of F defined in $H^3(M; \mathbb{R})$ ([3]) vanishes.

For the proof of the above result, the argument of Herman used in [4] to prove the triviality of the Godbillon-Vey invariant of foliations by planes of T^3 and also the work of Novikov [7] and Imanishi [5] on codimension one foliations without holonomy play very important roles.

1. Codimension one foliations without holonomy.

Let M be a compact connected smooth manifold and let F be a codimension one C^2-foliation without holonomy on M. We fix a base point x_0, a flow $\Phi : M \times \mathbb{R} \longrightarrow M$ whose orbits are transverse to leaves of F and we denote $\varphi(t)$ for $\Phi(x_0, t)$ $(t \in \mathbb{R})$. Following Novikov [7] (also see Imanishi [5]), we define a homomorphism

$$\chi : \pi_1(M, x_0) \longrightarrow \mathrm{Diff}^2_+(\mathbb{R})$$

as follows, where $\mathrm{Diff}^2_+(\mathbb{R})$ is the group of orientation preserving diffeomorphisms of class C^2 of $\mathbb{R}$. Let ω be an element of $\pi_1(M, x_0)$ represented by a closed curve $p : (I, \dot{I}) \rightarrow (M, x_0)$

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and let t be a point of $\mathbb{R}$. Then $\chi(\omega)(t)$ is defined to be a point t_1 of $\mathbb{R}$ such that there is a leaf curve ℓ : $(I, 0, 1) \longrightarrow (L, \varphi(t_1), \varphi(t))$ (L is the leaf passing through $\varphi(t)$) satisfying the condition: two curves p_+ and ℓ_- are homotopic, where p_+ is the product of two curves p and $\varphi([0, t])$ (if $t \geqq 0$) or $\varphi([t, 0])$ (if $t < 0$), while ℓ_- is the product of two curves $\varphi([0, t_1])$ (or $\varphi([t_1, 0])$) and ℓ.

χ is a well defined homomorphism (we define the product of two elements f and g of $\mathrm{Diff}_+^2(\mathbb{R})$ to be $f \circ g$) and it is known that Image (χ) is abelian (see [5] [7]). Now using the homomorphism χ, we can construct a locally trivial foliated $\mathbb{R}$-bundle (or the suspension foliation) E over M as follows. Let $\widetilde{M}$ be the universal covering space of M. Then $\pi_1(M, x_0)$ acts on $\widetilde{M} \times \mathbb{R}$ by the deck transformation on the first factor and through the homomorphism χ on the second. This action is free and preserves the trivial foliation on $\widetilde{M} \times \mathbb{R}$ defined by $\{t = \text{constant}\}$. Therefore the quotient manifold $E = \widetilde{M} \times \mathbb{R} / \pi_1(M, x_0)$ has the structure of a locally trivial foliated $\mathbb{R}$-bundle over M.

Now our first important step is the following.

PROPOSITION 1. Let E be the locally trivial foliated $\mathbb{R}$-bundle over M defined by the homomorphism χ. Then there is a cross-section $\sigma : M \longrightarrow E$ such that Image(σ) is transverse to the codimension one foliation on E and the induced foliation on M is the same as the original one F.

Proof. We define a mapping $\psi : \widetilde{M} \longrightarrow \mathbb{R}$ as follows. Let $\tilde{q}$ be a point of $\widetilde{M}$ represented by a path $q : (I, 0) \longrightarrow (M, x_0)$. Then $\psi(\tilde{q})$ is defined to be a point of $\mathbb{R}$ such that there is a leaf curve $\ell : (I, 0, 1) \longrightarrow (M, \varphi \circ \psi(\tilde{q}), q(1))$, so that two curves q and ℓ_- are homotopic where ℓ_- is the product of two curves $\varphi([0, \psi(\tilde{q})])$ (or $\varphi([\psi(\tilde{q}), 0])$) and ℓ. Now we define an imbedding $\tilde{\sigma} : \tilde{M} \to \tilde{M} \times \mathbb{R}$ by $\tilde{\sigma}(\tilde{q}) = (\tilde{q}, \psi(\tilde{q}))$. Then it can be checked that $\tilde{\sigma}$ is equivariant with respect to the $\pi_1(M, x_0)$-actions. Moreover $\tilde{\sigma}$ is transverse to the trivial foliation on $\tilde{M} \times \mathbb{R}$ defined by $\{t = \text{constant}\}$ and the induced codimension one foliation on $\tilde{M}$ coincides with the lift to $\tilde{M}$ of the original foliation F. Therefore the induced mapping $\sigma : M \to E$ satisfies the required conditions.

q.e.d.

REMARK 2. In the construction above, suppose that the orbit Image(φ) is periodic, namely for some k the equality $\varphi(t + k) = \varphi(t)$ holds for every $t \in \mathbb{R}$. Then for any element ω of $\pi_1(M, x_0)$, $\chi(\omega)$ is a periodic diffeomorphism of $\mathbb{R}$; $\chi(\omega)(t+k) = \chi(\omega)(t)$. Thus χ induces a homomorphism χ' : $\pi_1(M, x_0) \to \mathrm{Diff}^2_+(S^1)$ where we identify $\mathbb{R}$ mod $k\mathbb{Z}$ with S^1. Imanishi [5] has proved, among other things, that Image(χ') is topologically conjugate to rotations. Now the same proof as that of Proposition 1 gives the following.

PROPOSITION 1'. Let E' be the foliated S^1-bundle over M defined by the homomorphism χ'. Then there is a cross-section $\sigma' : M \to E'$ such that Image (σ') is transverse to the codimension one foliation on E' and the induced foliation on M is the same as the original one F.

2. The Godbillon-Vey class of foliated S^1 and $\mathbb{R}$-bundles.

Let E be a foliated S^1-bundle of class C^2 over a smooth manifold M defined by a homomorphism $\pi_1(M) \to \mathrm{Diff}^2_+(S^1)$. For such object, the Godbillon-Vey class (integrated over the fibres) is defined as an element of $H^2(\mathrm{Diff}^2_+(S^1); \mathbb{R})$ (the 2-dimensional cohomology group with trivial coefficients $\mathbb{R}$ of $\mathrm{Diff}^2_+(S^1)$ considered as an abstract group). According to Thurston (cf. [1] [4]), this element is represented by the following cocycle $\alpha \in C^2(\mathrm{Diff}^2_+(S^1); \mathbb{R})$.

DEFINITION 3. Let u, v be elements of $\mathrm{Diff}^2_+(S^1)$. Then

$$\alpha(u, v) = \int_{S^1} \log Dv(t)\, D \log D(u)(v(t))dt.$$

Now let E be a locally trivial foliated $\mathbb{R}$-bundle over a smooth manifold M defined by a homomorphism $\pi_1(M) \longrightarrow \mathrm{Diff}^2_+(\mathbb{R})$. Then similarly as above, the Godbillon-Vey class for such objects is defined as an element of $H^3(\mathrm{Diff}^2_+(\mathbb{R}); \mathbb{R})$ as follows.

Let f, g, h be elements of $\mathrm{Diff}^2_+(\mathbb{R})$ and we set

$$A = \log Df^{-1}(t)$$
$$B = \log Dg^{-1}(f^{-1}(t))$$
$$C = \log Dh^{-1}(g^{-1}f^{-1}(t)).$$

Let $\Delta^3 = \{(x_1, x_2, x_3) \in \mathbb{R}^3;\ x_1, x_2, x_3 \geq 0,\ x_1 + x_2 + x_3 \leq 1\}$ be the 3-simplex and let $s : \Delta^3 \longrightarrow \mathbb{R}$ be a function defined by

$$s(x_1, x_2, x_3) = \begin{cases} (x_1+x_2+x_3)f\left(\dfrac{x_2+x_3}{x_1+x_2+x_3}\, g\left(\dfrac{x_3}{x_2+x_3}\, h(0)\right)\right), & x_2+x_3 \neq 0 \\ x_1 f(0), & x_2+x_3 = 0. \end{cases}$$

s is C^∞ on the interior of Δ^3, $\mathring{\Delta}^3$, and continuous on Δ^3. Let $S : \Delta^3 \longrightarrow \Delta^3 \times \mathbb{R}$ be defined by $S(x_1, x_2, x_3) = (x_1, x_2, x_3, s(x_1,x_2,x_3))$. Now we define a cochain $\beta \in C^3(\mathrm{Diff}^2_+(\mathbb{R}); \mathbb{R})$ by the formula

DEFINITION 4.

$\beta(f,g,h)$

$$= \int_{\mathring{\Delta}^3} s^*\{Adx_1+(A+B)dx_2+(A+B+C)dx_3\}\{A'dtdx_1+(A'+B')dtdx_2+(A'+B'+C')dtdx_3\}.$$

Since the derivatives $\frac{\partial s}{\partial x_1}$, $\frac{\partial s}{\partial x_2}$, $\frac{\partial s}{\partial x_3}$ are bounded over $\mathring{\Delta}^3$, the integral exists. We can show

PROPOSITION 5. The cochain β is a cocycle.

Thus β defines an element $[\beta] \in H^3(\mathrm{Diff}^2_+(\mathbb{R}); \mathbb{R})$.

A proof of Proposition 5 together with related topics will be given in [6]. This is because, for a proof of our THEOREM, the form of the cocycle β is not essential. We need only the fact that the Godbillon-Vey class of a locally trivial foliated $\mathbb{R}$-bundle can be calculated by group cohomology argument. More precisely, let $\rho : \pi_1(T^3) = \mathbb{Z}^3 \longrightarrow \mathrm{Diff}^2_+(\mathbb{R})$ be a homomorphism defined by three mutually commuting diffeomorphisms f, g, h of $\mathbb{R}$ and let E be the locally trivial foliated $\mathbb{R}$-bundle over T^3 defined by ρ. Then the Godbillon-Vey class of this foliation on E is an element of $H^3(E; \mathbb{R}) \cong H^3(T^3; \mathbb{R}) \cong \mathbb{R}$. Let us denote GV(f, g, h) for the corresponding real number. Under these situation, we have

PROPOSITION 6. Let f, g, h be mutually commuting elements of $\mathrm{Diff}^2_+(\mathbb{R})$. Then $z = (f, g, h) - (f, h, g) + (g, h, f) - (g, f, h) + (h, f, g) - (h, g, f)$ is a cycle (of the group $\mathrm{Diff}^2_+(\mathbb{R})$) and the equality

$$GV(f, g, h) = \beta(z)$$

holds.

A proof of this Proposition will also be given in [6].

3. Foliated S^1 and $\mathbb{R}$-bundles over tori.

In [4], Herman has proved the following

THEOREM 7. Let E be a foliated S^1-bundle of class C^2 over T^2. Then the Godbillon-Vey invariant of the codimension one foliation on E is zero.

In this section, we prove the following results which can be considered as generalizations of Theorem 7.

THEOREM 8. Let E be a foliated S^1-bundle of class C^2 over a torus T^k $(k \geq 2)$. Then the Godbillon-Vey class of the codimension one foliation on E vanishes.

THEOREM 9. Let E be a locally trivial foliated $\mathbb{R}$-bundle over a torus T^k $(k \geq 3)$. Then the Godbillon-Vey class of the codimension one foliation on E vanishes.

Before proving the above Theorems, let us recall the argument of Herman [4] briefly. Let E be a foliated S^1-bundle over T^2 defined by commuting diffeomorphisms $u, v \in \mathrm{Diff}^2_+(S^1)$. Then $c = (u, v) - (v, u)$ is a cycle of the group $\mathrm{Diff}^2_+(S^1)$ and by Thurston (cf. [1] [4]), the Godbillon-Vey invariant of E, denoted by $Gv(u, v)$, is given by

$$Gv(u, v) = \alpha(c).$$

Herman has proved $\alpha(c) = 0$ by an elegant argument using known properties of elements of $\mathrm{Diff}^2_+(S^1)$. Now we prove Theorems 8 and 9.

Proof of Theorem 9. Since the cohomology group $H^3(T^k; \mathbb{R})$ $(k \geq 3)$ is generated by 3-dimensional cohomologies of various 3-dimensional subtori of T^k, we have only to prove the case $k = 3$. Thus let $f, g, h \in \mathrm{Diff}^2_+(\mathbb{R})$ be mutually commuting diffeomorphisms and let E be the locally trivial foliated

$\mathbb{R}$-bundle over T^3 defined by them. We have to prove $GV(f, g, h) = 0$. We consider two cases.

Case 1. All of f, g, h have fixed points.

In this case it can be proved that f, g, h have a common fixed point. In fact this follows from the following general statement.

PROPOSITION 10. Let $f_1, \ldots, f_r$ be mutually commuting homeomorphisms of $\mathbb{R}$ and assume that all of f_i have fixed points. Then there is a common fixed point of $f_1, \ldots, f_r$.

Proof. If f is an orientation reversing homeomorphism of $\mathbb{R}$, then f has a unique fixed point p and for any homeomorphism g of $\mathbb{R}$ such that $f \circ g = g \circ f$, clearly $g(p) = p$ holds. Therefore if at least one of $f_1, \ldots, f_r$ reverses the orientation, then the assertion is clear. Hence we assume that all of $f_1, \ldots, f_r$ preserve the orientation. Now first assume that at least one of $f_1, \ldots, f_r$, say f_i, has a maximum (or minimum) fixed point p. Then since any f_j $(j = 1, \ldots, r)$ leaves the fixed point set of f_i, $F(f_i)$, invariant, we have $f_j(p) = p$. So p is a common fixed point. Next assume the contrary and let (a, b) be a maximal open interval contained in $\mathbb{R} - F(f_1)$, thus $a, b \in F(f_1)$. Let (a_1, b_1) be the maximal open interval containing (a, b) such that (a_1, b_1) is contained in $\mathbb{R} - F(f_i)$ for some i. We claim that a_1 and b_1 are common fixed points of $f_1, \ldots, f_r$. For from the definition, either $(a_1, b_1) \subset \mathbb{R} - F(f_j)$ or f_j has a fixed point on (a_1, b_1). But in either case we should have $f_j(a_1) = a_1$ and $f_j(b_1) = b_1$. This completes the proof of Proposition 10.

REMARK 11. In Proposition 10, if we assume that $f_1, \ldots, f_r$ are orientation preserving diffeomorphisms of class C^2, then we can obtain a stronger statement that if (a, b) is a maximal open interval contained in $\mathbb{R} - F(f_1)$, then a and b are common fixed points of $f_1, \ldots, f_r$ (cf. [4] Lemma 1).

Now we go back to the proof of Theorem 9, Case 1.

We have just proved that f, g, h have a common fixed point p. Then this fixed point defines a cross-section $\sigma : T^3 \longrightarrow E$ such that Image(σ) is a compact leaf of the foliation on E. Since the restriction of the Godbillon-Vey class to any leaf is trivial and since Image(σ) generates the 3-dimensional homology group of E, we conclude that $GV(f, g, h) = 0$.

Case 2. At least one of f, g, h has no fixed point.

First we claim that

$$GV(f, g, h) = GV(g, h, f) = GV(h, f, g).$$

This follows from the definition of GV. It also follows from Proposition 6. Therefore to prove our assertion $GV(f, g, h) = 0$, we may assume that h has no fixed points. Now let us define a $\mathbb{Z}$-action on $\mathbb{R}$ by $n(t) = h^n(t)$ $(n \in \mathbb{Z},\ t \in \mathbb{R})$. Then since h has no fixed points, this action is free and the quotient manifold can be identified with S^1 by an orientation preserving diffeomorphism $k : \mathbb{R}/\{h^n\} \cong S^1$. Let $\tilde{k} : \mathbb{R} \longrightarrow \mathbb{R}$ be the lift of k such that $\tilde{k}(0) = 0$. It is a diffeomorphism of class C^2. Now we set $f_1 = \tilde{k}^{-1} f \tilde{k}$, $g_1 = \tilde{k}^{-1} g \tilde{k}$, $h_1 = \tilde{k}^{-1} h \tilde{k}$. Then f_1, g_1, h_1 are mutually commuting diffeomorphisms of class C^2 of $\mathbb{R}$. Let $\mathfrak{z} = (f, g, h) - (f, h, g) + (g, h, f) - (g, f, h) + (h, f, g) - (h, g, f)$ and $\mathfrak{z}_1 = (f_1, g_1, h_1) - (f_1, h_1, g_1) + (g_1, h_1, f_1) - (g_1, f_1, h_1) + (h_1, f_1, g_1) - (h_1, g_1, f_1)$. Then the cycle $\mathfrak{z}_1$ is conjugate to $\mathfrak{z}$: $\mathfrak{z}_1 = \tilde{k}^{-1} \mathfrak{z} \tilde{k}$. Since inner automorphisms of a group induce the identity on the homology groups ([2]), we have

$$\beta(\mathfrak{g}_1) = \beta(\mathfrak{g}).$$

Therefore from Proposition 6, we obtain

$$GV(f, g, h) = GV(f_1, g_1, h_1).$$

Now from the construction, h_1 is the translation of $\mathbb{R}$ by 1 (denoted by T) or by -1 according as $h(0) > 0$ or $h(0) < 0$ respectively. By the definition of GV, clearly we have

$$GV(f_1, g_1, h_1) = - GV(f_1, g_1, h_1^{-1}).$$

Therefore we may assume that $h_1 = T$. Since f_1 and g_1 commute with $h_1 = T$, f_1 and g_1 are lifts of some diffeomorphisms f_1' and g_1' of S^1. Now we claim

PROPOSITION 12. Let u, v be mutually commuting elements of $\mathrm{Diff}_+^2(S^1)$ and let $\tilde{u}$, $\tilde{v}$ be their arbitrary lifts to $\mathbb{R}$. Then we have

$$GV(\tilde{u}, \tilde{v}, T) = Gv(u, v).$$

Proof. We consider $\mathbb{R}^2 \times \mathbb{R} = \{(x_1, x_2, t);\ x_i,\ t \in \mathbb{R}\}$, $\mathbb{R}^3 \times \mathbb{R} = \{(x_1, x_2, x_3, t);\ x_i, t \in \mathbb{R}\}$ and let

$$\lambda(x_1,x_2,t) = (x_1+1,x_2,\tilde{u}(t)),\quad \lambda_1(x_1,x_2,x_3,t) = (x_1+1,x_2,x_3,\tilde{u}(t))$$

$$\mu(x_1,x_2,t) = (x_1,x_2+1,\tilde{v}(t)),\quad \mu_1(x_1,x_2,x_3,t) = (x_1,x_2+1,x_3,\tilde{v}(t))$$

$$\nu(x_1,x_2,t) = (x_1,x_2,t+1),\quad \nu_1(x_1,x_2,x_3,t) = (x_1,x_2,x_3+1,t+1).$$

Then λ, μ, ν and λ_1, μ_1, ν_1 generate free $\mathbb{Z}^3$-actions on $\mathbb{R}^2 \times \mathbb{R}$ and $\mathbb{R}^3 \times \mathbb{R}$ respectively. These actions preserve the trivial foliations defined by $\{t = \text{constant}\}$. The quotient manifolds E and E_1 carry the structures of foliated S^1-bundle over T^2 defined by u and v and locally trivial foliated

$\mathbb{R}$-bundle over T^3 defined by $\tilde{u}$, $\tilde{v}$, T respectively. Now define a mapping $\pi : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^2 \times \mathbb{R}$ by $\pi(x_1, x_2, x_3, t) = (x_1, x_2, t)$. Then π is equivariant with respect to the $\mathbb{Z}^3$-actions. Therefore it induces a mapping $\pi' : E_1 \to E$. Moreover it is easy to see that the pull back of the foliation on E by the submersion π' coincides with the given foliation on E_1. Therefore from the naturality of the Godbillon-Vey class, we obtain

$$(\pi')^*(gv(E)) = gv(E_1) ,$$

where $gv(E)$ (resp. $gv(E_1)$) is the Godbillon-Vey class of the foliation on E (resp. E_1). Now since $(\pi')^*$ gives an isomorphism $H^3(E; \mathbb{R}) \cong H^3(E_1'; \mathbb{R}) \cong \mathbb{R}$, we obtain

$$GV(\tilde{u}, \tilde{v}, T_1) = Gv(u, v) .$$

This completes the proof of Proposition 11.

Now by the above Proposition and the argument before it, we have

$$GV(f, g, h) = Gv(f_1', g_1') .$$

But Herman's result (Theorem 7) implies

$$Gv(f_1', g_1') = 0.$$

Hence $GV(f, g, h) = 0$. This completes the proof of Case 2 and hence Theorem 9. q.e.d.

Next we prove Theorem 8.

Proof of Theorem 8. Since the case $k = 2$ is just Theorem 7, we assume that $k \geqq 3$ and let E be a foliated S^1-bundle of class C^2 over T^k defined by mutually commuting diffeomorphisms $u_1, \ldots, u_k \in \mathrm{Diff}_+^2(S^1)$. Since E is a trivial bundle as a differentiable S^1-bundle, there is a cross-section $\sigma : T^k \to E$.

σ defines an isomorphism $E \cong T^k \times S^1$. Now the Godbillon-Vey class of the foliation on E, $gv(E)$, lies in $H^3(E; \mathbb{R}) \cong H^3(T^k; \mathbb{R}) \oplus H^2(T^k; \mathbb{R}) \otimes H^1(S^1; \mathbb{R})$. However Herman's result (Theorem 7) implies that the second component of $gv(E)$ is zero. Now let $\tilde{E} = T^k \times \mathbb{R}$ be the covering space of $E = T^k \times S^1$ corresponding to the subgroup $\pi_1(T^k) \subset \pi_1(E)$. Then the projection $\pi : \tilde{E} \to E$ induces a codimension one foliation on $\tilde{E}$. In fact $\tilde{E}$ has the structure of locally trivial foliated $\mathbb{R}$-bundle over T^k defined by mutually commuting diffeomorphisms $\tilde{u}_1, \ldots, \tilde{u}_k \in \mathrm{Diff}^2_+(\mathbb{R})$, where $\tilde{u}_i$ is a suitable lift of u_i to $\mathbb{R}$ defined by the cross-section σ. Hence $gv(\tilde{E}) = 0$ by Theorem 9. Therefore we obtain $\pi^*(gv(E)) = gv(\tilde{E}) = 0$. Now since $gv(E)$ lies in $H^3(T^k; \mathbb{R}) \subset H^3(E; \mathbb{R})$ as remarked before, we conclude $gv(E) = 0$. q.e.d.

5. Proof of THEOREM.

Let M be a compact smooth manifold, F a codimension one foliation of class C^2 over M and assume that F is without holonomy. Then by Proposition 1, there is a locally trivial foliated $\mathbb{R}$-bundle E over M defined by a homomorphism $\chi : \pi_1(M) \to \mathrm{Diff}^2_+(\mathbb{R})$ and an imbedding of M in E transverse to the codimension one foliation on E such that the induced foliation on M coincides with the original one F. Moreover $\mathrm{Image}(\chi)$ is abelian. Therefore by Theorem 9, we conclude that $gv(E) = 0$. Then by the naturality of the Godbillon-Vey class, we obtain $gv(F) = 0$. This completes the proof of THEOREM. We could also use Proposition 1' and Theorem 8 instead of Proposition 1 and Theorem 9. q.e.d.

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