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Group Duality and the Kubo-Martin-Schwinger Condition

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For ω an α-invariant state of a C*-algebra $\mathfrak{A}$ endowed with an action $g \in G \to \alpha_g \in \mathrm{Aut}\,\mathfrak{A}$ of a locally compact abelian group G, we study the relationship between the two following facts:

(i) ω is KMS w.r.t. some continuous 1-parameter subgroup of G

(ii) there exists a linear map T_0, closable in an appropriate topology, such that

$T_0 F_{AB} = G_{AB}$, $A, B \in \mathfrak{A}$, where $F_{AB}(g) = \omega(B\alpha_g(A))$,

$G_{AB}(g) = \omega(\alpha_g(A)B)$, $g \in G$.

1) Typical result: Assume ω extremal τ-invariant for an action τ of an amenable group H on $\mathfrak{A}$ which is asymptotically abelian and commutes with α; and imbed the set $\mathfrak{F} = \{F_{AB};\ A, B \in \mathfrak{A}\}$ in either $L^\infty(G)$ or $B(G)$ endowed with their weak*-topologies. If there is a closable linear map T_0 s.t. $T_0 F_{AB} = G_{AB}$ for all $A, B \in \mathfrak{A}$, (i) holds.

Ingredients are:

2) Duality results; e.g: Let $\mathcal{D}$ be a weak*-dense translation invariant subalgebra of $L^\infty(G)$ (or $B(G)$). If T is a weak*-closed homomorphism of $\mathcal{D}$ into $L^\infty(G)$ (or $B(G)$) commuting with the translations, there is a continuous 1-parameter group $\{g(t),\ t \in R\}$ of G and $s_0 \in G$ s.t. for each $f \in \mathcal{D}$ and $s \in G$, $t \to f(s + g(t))$ and $t \to Tf(s + s_0 + g(t))$ are the boundary values of a function in $H^\infty(\{|\mathrm{Im} z| \leq 1\})$.

3) Density results, e.g. : Let ω be an α-invariant state of $\mathfrak{A}$ generating the covariant representation $\{\pi, H\}$. Then
(i) $\mathcal{F}$ is weak $*$-total in $B(G)$ iff $\mathrm{Sp}U = \hat{G}$
(ii) Assuming ω extremal τ-invariant for an action τ of an amenable group H on $\mathfrak{A}$ which is asymptotically abelian and commutes with α, $\mathcal{F}$ is weak$*$-total in $L^\infty(G)$.

Results of the type 3) allow to apply results of the type i) to the closure T of the operator T_0 in results like 1) to obtain the conclusion.

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