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<td>著者</td>
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<td>引用</td>
<td>数理解析研究所講究録 1980年8月号395号 18-28</td>
</tr>
<tr>
<td>発行日</td>
<td>1980-08</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/105012">http://hdl.handle.net/2433/105012</a></td>
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<tr>
<td>タイプ</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>テキストバージョン</td>
<td>publisher</td>
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A SEMIGROUP OF ISOMORPHISM CLASSES OF SOME QUADRATIC EXTENSIONS OF RINGS

Takasi NAGAHARA (Okayama University)

Throughout this paper, \( B \) will mean a (non-commutative) ring with identity element 1 which has an automorphism \( \rho \). By \( B[X;\rho] \), we denote the ring of all polynomials \( \sum_{i} x^{i} b_{i} (b_{i} \in B) \) with an indeterminate \( X \) whose multiplication is given by \( bX = X\rho(b) \). Moreover, by \( B[X;\rho]_{2} \), we denote the subset of \( B[X;\rho] \) of all polynomials \( f = X^{2} - Xa - b \) with \( fB[X;\rho] = B[X;\rho]f \). If \( X^{2} - Xa - b \in B[X;\rho]_{2} \) then \( \rho(b) = b \). By \( B[X;\rho]_{(2)} \), we denote the subset \( \{ X^{2} - Xa - b \in B[X;\rho]_{2} \mid \rho(a) = a \} \). Now, for \( f, g \in B[X;\rho]_{2} \), if the factor rings \( B[X;\rho]/fB[X;\rho] \) and \( B[X;\rho]/gB[X;\rho] \) are \( B \)-ring isomorphic then we write \( f \sim g \). Clearly the relation \( \sim \) is an equivalence relation in \( B[X;\rho]_{2} \). By \( B[X;\rho]_{2}^{\sim} \) (resp. \( B[X;\rho]_{(2)}^{\sim} \)), we denote the set of equivalence classes of \( B[X;\rho]_{2} \) (resp. \( B[X;\rho]_{(2)} \)) with respect to the relation \( \sim \). Moreover, for \( f \in B[X;\rho]_{2} \), if the factor ring \( B[X;\rho]/fB[X;\rho] \) is separable (resp. Galois) over \( B \) then \( f \) will be called to be separable (resp. Galois). As is well known, any Galois polynomial in \( B[X;\rho]_{2} \) is separable. By [6, Th.1], any separable polynomial of \( B[X;\rho]_{2} \) is contained in \( B[X;\rho]_{(2)} \). For \( f = X^{2} - Xa - b \in B[X;\rho]_{2} \), we denote \( a^{2} + 4b \) by \( \delta(f) \), which will be called the discriminant of \( f \). We shall use here the convention: \( B(\rho^{n}) = \{ u \in B \mid au = u\rho^{n}(a) \text{ for all } a \in B \} \) (where \( n \) is any integer). If \( X^{2} - Xa - b \in B[X;\rho]_{2} \) then \( a \in B(\rho), b \in B(\rho^{2}) \),
\( \rho(b) = b \) (and conversely). Clearly \( a^2 + 4b \in B(\rho^2) \). An element \( a \) of \( B(\rho^n) \) is said to be \( \pi \)-regular if there exists an element \( c \) in \( B \) and an integer \( t \geq 0 \) such that \( a^t = a^{t+1}c \).

Now, in [1], K. Kitamura studied free quadratic extensions of commutative rings and its isomorphism classes. In his study, the set of polynomials of degree 2 plays an important rôle. Indeed, [1] is a study on \( B[X;\rho]_2 \) and \( B[X;\rho]_{2} \) where \( B \) is commutative and \( \rho = 1 \).

In [2], K. Kishimoto studied the sets \( B[X;\rho]_{(2)} \) and \( B[X;\rho]_{(2)}^\sim \) in case \( B[X;\rho]_{(2)} \) contains a Galois polynomial \( x^2 - b \) (and hence \( 2b \) is invertible in \( B \)).

In [5], the present author studied the sets \( B[X;\rho]_{(2)} \) and \( B[X;\rho]_{(2)}^\sim \) in case \( B[X;\rho]_{(2)} \) contains a Galois polynomial \( x^2 - xa - b \) (and hence the discriminant \( a^2 + 4b \) is invertible in \( B \)). The study contains a generalization of [2]. Moreover, in [1], [2] and [5], it was shown that \( B[X;\rho]_{(2)}^\sim \) forms an abelian semigroup with identity element under some composition, and the structure of this semigroup was studied to characterize the separable polynomials in \( B[X;\rho]_{(2)} \).

In this paper, we shall study the separable polynomials in \( B[X;\rho]_{(2)} \) and the structure of \( B[X;\rho]_{(2)}^\sim \) in case \( B[X;\rho]_{(2)} \) contains a separable polynomial whose discriminant is \( \pi \)-regular, and we shall show that \( B[X;\rho]_{(2)}^\sim \) forms also an abelian semigroup with identity element under some composition such that for \( C \in B[X;\rho]_{(2)}^\sim \) and \( f \in C, C \) is invertible in this semigroup if and only if \( f \) is separable. Moreover, this semigroup will be studied in various ways.
In the rest of this paper, $Z$ will mean the center of $B$. Moreover, $U(B)$ denotes the set of invertible elements in $B$, and for any subset $S$ of $B$, $U(S)$ denotes the intersection of $S$ and $U(B)$. Clearly, $U(Z)$ coincides with the set of invertible elements in $Z$. Further, for any subset $S$ of $B$, we use the following conventions: $S^0 = \{ s \in S \mid \rho(s) = s \}$; $\rho^n|S$ = the restriction of $\rho^n$ to $S$ (where $n$ is any integer).

By [5, (2, xvii)] and [6, Th. 1], we see that if $B[X;\rho]_2$ contains a separable polynomial then $\rho^2|Z$ is identity. As is easily seen, if an element $a$ of $B(\rho^n)^0$ is $\pi$-regular then there exists an integer $n \geq 0$ and an idempotent $e$ of $Z^0$ such that $a^n B = e B$. This idempotent will be denoted by $e(a)$.

First, we shall prove the following

Lemma 1. Let $2$ be nilpotent, and assume that $B[X;\rho]_2$ contains a separable polynomial $X^2 - b$. Then, $b \in U(B)$, and there exists an element $z \in Z$ such that $z + \rho(z) = 1$.
Moreover, $B(\rho) = \{ 0 \}$, $B(\rho^2) = bZ$, $B(\rho^2)^0 = bZ^0$, and $B[X;\rho]_2 = \{ x^2 - v \mid v \in B(\rho^2)(0) \}$.

Proof. The first assertion is a direct consequence of [5, Lemma 2.3] and [6, Th.1]. Now, since $2$ is nilpotent, there exists an integer $n > 0$ such that $2^n = 0$. Let $u \in B(\rho)$. Then, we have $u = u(z + \rho(z))^n = u(z + \rho(z))(z + \rho(z))^{n-1} = 2zu(z + \rho(z))^{n-1} = 2^n z^n u = 0$. The rest assertion will be easily seen.

Next, we shall prove the following
Lemma 2. Let $\varepsilon$ be an idempotent in $\mathbb{Z}^\rho$ such that $\varepsilon 2^n = 2^n$ for some integer $n > 0$. Let $f$ be a polynomial in $B[X;\rho]_2$, such that $\varepsilon f$ is Galois in $\varepsilon B[X;\rho]$ and $(1 - \varepsilon)f$ is separable in $(1 - \varepsilon)B[X;\rho]$. Then $\delta(f)$ is $\pi$-regular, $e(\delta(f))B \supset \varepsilon B$, and $(1 - e(\delta(f)))B[X;\rho]_2 = \{(1 - e(\delta(f)))(x^2 - v) \mid v \in B(\rho^2)\}$.

Proof. By [6, Th.2], we have $\varepsilon B = e\delta(f)B$. Moreover, $f$ is separable, and so, $f \in B[X;\rho]_{(2)}$. We write here $f = x^2 - Xa - b$. Then, by [5, Lemma 2.2 (2, xix)], we have $a = \delta(f)s_0 = \delta(f)^{n+1}s_0^{n+1}a$ for some $s$ in $B$. Since $\varepsilon 4^n = 4^n$, it follows that $(1 - \varepsilon)\delta(f)^n_B = (1 - \varepsilon)(ac + 4^n b^n)_B = (1 - \varepsilon)acB = (1 - \varepsilon)\delta(f)^{n+1}_B$, and whence, $\delta(f)^n_B = e\delta(f)^n_B + (1 - \varepsilon)\delta(f)^{n+1}_B = e\delta(f)^{n+1}_B + (1 - \varepsilon)\delta(f)^{n+1}_B = \delta(f)^{n+1}_B$. Thus $\delta(f)$ is $\pi$-regular, and $e(\delta(f))B = \delta(f)^n_B \supset e\delta(f)^n_B = e\delta(f)B = \varepsilon B$. Moreover, noting $e(\delta(f))a = a$, the other assertion will be easily seen from the result of Lemma 1.

Corollary 3. Let 2 be $\pi$-regular. If $f \in B[X;\rho]_2$ is separable then $\delta(f)$ is $\pi$-regular.

Proof. Let $f = x^2 - Xa - b$ be a separable polynomial in $B[X;\rho]_2$. Since any invertible element of $B$ is $\pi$-regular in $B$, we may assume that $\delta(f)$ is not invertible in $B$. If $e(2) = 1$ then 2 is invertible in $B$, and so, $\delta(f)$ is invertible in $B$ by [6, Th.3]. Hence $e(2) \neq 1$. First, we assume that $e(2) = 0$. Then $2^n = 0$ for some integer $n > 0$.

By [5, Lemma 2.2 (2, xix)], we have $a = \delta(f)^n ta = a^2 r$ for some $t, r \in B$. Hence $a$ is $\pi$-regular, and $e(a)$ is in $\mathbb{Z}^\rho$. 

Since $e(a)a$ is invertible in $e(a)B$, so is $e(a)\delta(f)$ in $e(a)B$. Hence, it follows from [6, Th.2] that $e(a)f$ is Galois in $e(a)B[X;p]$. Moreover, $1 - e(a) \neq 0$, and $(1 - e(a))f$ is separable in $(1 - e(a))B[X;p]$. Therefore, $\delta(f)$ is $\pi$-regular by Lemma 2. Next, we assume that $e(2) \neq 0$. Then $e(2) \in Z^0$, $e(2)B = 2^n B$, and $e(2)2^n = 2^n$ for some integer $n > 0$. Noting that $e(2)2$ is invertible in $e(2)B$, $e(2)f$ is Galois in $e(2)B[X;p]$ by [6, Th.2]. Moreover, $(1 - e(2))f$ is separable in $(1 - e(2))B[X;p]$. Hence by Lemma 2, $\delta(f)$ is $\pi$-regular.

Now, we shall prove the following theorem which is one of our main results.

**Theorem 4.** Assume that $B[X;p]_2$ contains a separable polynomial $f$ whose discriminant is $\pi$-regular. Set $\varepsilon = e(\delta(f))$ and $\omega = 1 - \varepsilon$. Then, $\omega 2$ is nilpotent, and $\omega B[X;p]_2 = \{\omega X^2 - v \mid v \in B(p^2)^0\}$. Moreover, $g = X^2 - Xu - v \in B[X;p]_2$, the following conditions are equivalent:

(a) $g$ is separable.

(b) $\delta(g)$ is $\pi$-regular, $e(\delta(g)) = \varepsilon$, and $\omega B = \omega vB$.

(c) $\varepsilon B = \varepsilon \delta(g)B$, and $\omega B = \omega vB$.

**Proof.** Let $f = X^2 - Xa - b$. If $\varepsilon = 1$ then $\delta(f)$ is invertible in $B$, and whence, the assertion holds obviously. Now, we assume that $\varepsilon = 0$. Then, by [5, Lemma 2.2 (2, xix)], 2 is nilpotent and $a = 0$. Hence by Lemma 1, we have $B[X;p]_2 = \{X^2 - v \mid v \in B(p^2)^0\}$. Hence by [5, Lemma 2.3], it will be easily seen that (a), (b) and (c) are equivalent.
Next, we shall consider the case $\epsilon \neq 1, 0$. Since $\epsilon B = \delta(f)^n B$ for some integer $n > 0$, it follows that $\rho(\epsilon) = \epsilon$, and $4^n = \delta(f)^n r = \epsilon \delta(f)^n r = \epsilon 4^n$ for some $r$ in $B$ ([5, Lemma 2.2]). Moreover, since $\epsilon \delta(f)$ is inversible in $\epsilon B$, $\epsilon f$ is Galois in $\epsilon B[X; \rho]$. Obviously, $\omega f$ is separable in $\omega B[X; \rho]$. Hence by Lemma 2, we have $\omega B[X; \rho]^2 = \{ \omega(X^2 - v) \mid v \in B(\rho^2) \}$. Now, let $g = X^2 - Xu - v \in B[X; \rho]^2$. Assume (a). Then, since $\epsilon g$ is separable in $\epsilon B[X; \rho]$, it follows from [6, Th.2] that $\epsilon g$ is Galois in $\epsilon B[X; \rho]$.

Moreover, $\omega g$ is separable in $\omega B[X; \rho]$. Hence by Lemma 2, $\delta(g)$ is $\pi$-regular, and $e(\delta(g))B \supset \epsilon B = e(\delta(f))B$. By a similar way, we have $e(\delta(g))B \subset e(\delta(f))B$. This implies $e(\delta(g)) = \epsilon$. Since $\omega g = \omega(X^2 - v)$ is separable in $\omega B[X; \rho]$, $\omega v$ is inversible in $\omega B$ by [5, Lemma 2.3], that is, $\omega B = \omega v B$. Thus we obtain (b). Assume (b). Then $\epsilon B = e(\delta(g))B = \delta(g)^m B$ for some integer $m > 0$. This shows that $\epsilon B = \epsilon \delta(g) B$.

Finally, assume (c). Since $\epsilon B = \epsilon \delta(g) B$, $\epsilon \delta(g)$ is inversible in $\epsilon B$. Hence $\epsilon g$ is Galois in $\epsilon B[X; \rho]$ by [6, Th.2], and so, $\epsilon g$ is separable in $\epsilon B[X; \rho]$. Moreover, $\omega v$ is inversible in $\omega B$. Since $\omega f$ is separable in $\omega B[X; \rho]$, there exists an element $z$ in $\omega Z$ with $z + \rho(z) = \omega$. Hence $\omega g = \omega(X^2 - v)$ is separable in $\omega B[X; \rho]$ by [5, Lemma 2.3]. Therefore $g = \epsilon g + \omega g$ is separable, completing the proof.

In the rest of this note, we shall deal with the set $B[X; \rho]^{(2)}$ (of $B$-ring isomorphism classes of the ring extensions $B[X; \rho]/gB[X; \rho]$ ($g \in B[X; \rho]^{(2)}$) of $B$).

Now, if $C \in B[X; \rho]^{(2)}$ and $g \in C$ then we write $C = \langle g \rangle$. 


Moreover, for \( g = x^2 - xu - v \), \( g_1 = x^2 - xu_1 - v_1 \) and \( s \in B \), we write

\[
\begin{align*}
g \times s &= x^2 - xus - vs^2 \\
g \times g_1 &= x^2 - xu_1 - (uv_1 + vu_1^2 + 4vv_1) \\
g \ast s &= x^2 - vs^2 \\
g \ast g_1 &= x^2 - vv_1.
\end{align*}
\]

If \( B[X; \rho]_2 \) contains a separable polynomial then \( \rho^2 \mid Z = 1 \), and in this case, for any element \( \alpha \) (resp. any subset \( S \)) of \( Z \), we denote \( \alpha \rho(\alpha) \) (resp. \( \{ \alpha \rho(\alpha) \mid \alpha \in S \} \)) by \( N_\rho(\alpha) \) (resp. \( N_\rho(S) \)).

Now, by virtue of Lemma 1, [5, Lemma 2.10] and [3, Lemma 1.8], we obtain the following

**Lemma 5.** Let \( 2 \) be nilpotent, and assume that \( B[X; \rho]_2 \) contains a separable polynomial \( f = x^2 - b \). Let \( g_1 = x^2 - v_1 \) and \( g_2 = x^2 - v_2 \in B[X; \rho]_2 \) \( (= \{ x^2 - v \mid v \in B(\rho^2) \}) \). Then, \( g_1 \sim g_2 \) if and only if \( v_1 = v_2 N_\rho(\alpha) \) for some \( \alpha \in U(Z) \).

From the preceding lemma and [5, Lemma 2.3], we obtain

**Corollary 6.** Let \( 2 \) be nilpotent, and assume that \( B[X; \rho]_2 \) contains a separable polynomial \( f = x^2 - b \). Let \( g_1 \sim g_2 \) in \( B[X; \rho]_2 \), and \( h_1 \sim h_2 \) in \( Z[X; \rho | Z]_2 \). Then for any \( g \in B[X; \rho]_2 \) and \( h \in Z[X; \rho | Z]_2 \), there holds the following

\[
\begin{align*}
(i) & \quad g_1 \ast g \ast b^{-1} \sim g_2 \ast g \ast b^{-1} \text{ in } Z[X; \rho | Z]_2. \\
(ii) & \quad h_1 \ast h \sim h_2 \ast h \text{ in } Z[X; \rho | Z]_2.
\end{align*}
\]
(iii) \( h_1 \ast g \sim h_2 \ast g \) in \( B[X; \rho](2) \).
(iv) \( g_1 \ast g \ast f \ast b^{-1} \sim g_2 \ast g \ast f \ast b^{-1} \) in \( B[X; \rho](2) \).
(v) \( g \ast f \ast f \ast b^{-1} = g \), and \( h \ast f \ast f \ast b^{-1} = h \).
(vi) \( g \) is separable in \( B[X; \rho](2) \) if and only if \( g \ast g' \ast f \ast b^{-1} \sim f \) which is equivalent to that \( g \ast g' \ast f \ast b^{-1} \sim f \) for some \( g' \in B[X; \rho](2) \).
(vii) \( h \) is separable in \( Z[X; \rho\mid Z](2) \) if and only if \( h \ast h' \sim f \ast f \ast b^{-1} \) which is equivalent to that \( h \ast h' \sim f \ast f \ast b^{-1} \) for some \( h' \in Z[X; \rho\mid Z](2) \).

By making use of Cor. 6, we can prove the next

Lemma 7. Let 2 be nilpotent, and assume that \( B[X; \rho](2) \) contains a separable polynomial \( f = X^2 - b \). Then, the set \( B[X; \rho](2) \) (resp. \( Z[X; \rho\mid Z](2) \)) forms an abelian semigroup under the composition \( \langle g_1 \rangle \langle g_2 \rangle = \langle g_1 \ast g_2 \ast f \ast b^{-1} \rangle \) (resp. \( \langle h_1 \rangle \langle h_2 \rangle = \langle h_1 \ast h_2 \rangle \)) with identity element \( \langle f \rangle \) (resp. \( \langle f \ast f \ast b^{-1} \rangle \)), and the subset \( \{ \langle g \rangle \in B[X; \rho](2) \mid g \) is separable\} \) (resp. \( \{ \langle h \rangle \in Z[X; \rho\mid Z](2) \mid h \) is separable\} \) coincides with the set of all inversible elements in the semigroup \( B[X; \rho](2) \) (resp. \( Z[X; \rho\mid Z](2) \)) which is a group of exponent 2. Moreover, \( B[X; \rho](2) \cong Z[X; \rho\mid Z](2) \), which is isomorphic to the multiplicative semigroup \( Z^0 / N_{\rho}(U(Z)) \).

Now, let \( \varepsilon \) be an idempotent in \( Z^0 \). Then \( \varepsilon B = (\varepsilon B)^\rho \), \( \varepsilon B(\rho) = (\varepsilon B)(\rho\mid\varepsilon B) \), and \( \varepsilon B(\rho)^\rho = (\varepsilon B)(\rho\mid\varepsilon B)^\rho \). Hence we have a bijective map: \( \varepsilon B[X; \rho](2) \cong (\varepsilon B)[X; \rho\mid\varepsilon B](2) \) given by
\[ \varepsilon(X^2 - Xu - v) + X^2 - X\varepsilon u - \varepsilon v. \] Hence we shall identify \( \varepsilon B[X; \rho]_2(2) \) with \( (\varepsilon B)[X; \rho | \varepsilon B](2) \), and by \( \varepsilon B[X; \rho]_2 \), we denote \( (\varepsilon B)[X; \rho | \varepsilon B]_2 \). We set here \( \omega = 1 - \varepsilon \). Then, as is easily seen, the map:

\[ B[X; \rho]_2 \rightarrow \varepsilon B[X; \rho]_2(2) \times \omega B[X; \rho]_2 \] (direct product)
given by \( g \rightarrow (\varepsilon g, \omega g) \) is bijective. This induces a bijective map:

\[ B[X; \rho]_2(2) \rightarrow \varepsilon B[X; \rho]_2(2) \times \omega B[X; \rho]_2(2) \]
where \( \langle g \rangle \rightarrow (\langle \varepsilon g \rangle, \langle \omega g \rangle) \). Clearly, \( g \) is separable in \( B[X; \rho] \) if and only if \( \varepsilon g \) and \( \omega g \) are separable in \( \varepsilon B[X; \rho] \) and \( \omega B[X; \rho] \) respectively. If \( B[X; \rho]_2 \) contains a separable polynomial \( f = X^2 - Xa - b \) whose discriminant is \( \pi \)-regular and \( \varepsilon = e(\delta(f)) \) \( (\omega = 1 - \varepsilon) \) then \( B[X; \rho]_2 \) contains a Galois polynomial \( \varepsilon f \), \( \omega 2 \) is nilpotent and \( \omega B[X; \rho]_2 \) contains a separable polynomial \( \omega f = \omega (X^2 - b) \) (Th.4, [5, Lemma 2.2], [6, Th.2]).

Now, our main results are the following theorems which can be proved by making use of the preceding remarks, Lemma 7, [5, Th.2.17], Cor. 3, [5, Lemma 2.10], [3, Lemma 1.8], [6, Th.2], [4, Th.1.2], and etc.

Theorem 8. Assume that \( B[X; \rho]_2 \) contains a separable polynomial \( f = X^2 - Xa - b \) whose discriminant is \( \pi \)-regular. Set \( \varepsilon = e(\delta(f)) \) and \( \omega = 1 - \varepsilon \). Then the set \( B[X; \rho]_2(2) \) (resp. \( Z[X; \rho | 2]_2(2) \)) forms an abelian semigroup under the composition
\[ \langle g_1, g_2 \rangle = \langle \varepsilon g_1 \times \varepsilon g_2 \times \varepsilon f \times (\varepsilon \delta(f))^{-1} + \omega g_1 \times \omega g_2 \times \omega f \times (\omega b)^{-1} \rangle \]

(resp. \( \langle h_1, h_2 \rangle = \langle \varepsilon h_1 \times \varepsilon h_2 + \omega h_1 \times \omega h_2 \rangle \))

with identity element

\[ \langle f \rangle = \langle \varepsilon f \times \varepsilon f \times (\varepsilon \delta(f))^{-1} + \omega f \times \omega f \times (\omega b)^{-1} \rangle \]

, and the subset

\[
\{ \langle g \rangle \in B[X; \rho]_{(2)} \mid g \text{ is separable} \} \\
(\text{resp. } \{ \langle h \rangle \in Z[X; \rho] \times Z_{(2)} \mid h \text{ is separable} \})
\]

coincides with the set of all invertible elements of \( B[X; \rho]_{(2)} \) (resp. \( Z[X; \rho] \times Z_{(2)} \)) which is a group of exponent 2. Moreover

\[
B[X; \rho]_{(2)} \cong Z[X; \rho] \times Z_{(2)} = \varepsilon Z[X; \rho] \times \omega Z[X; \rho] \times \omega Z_{(2)} = \varepsilon Z[X; \rho] \times \omega Z_{(2)} \times \omega Z_{(2)}
\]

Theorem 9. Let \( 2 \) be \( \pi \)-regular and assume that \( B[X; \rho]_{(2)} \) contains a separable polynomial \( f \). Then, there exists an idempotent \( \varepsilon (\omega = 1 - \varepsilon) \) of \( Z^p \) such that

\[
B[X; \rho]_{(2)} = \varepsilon Z[X]_{(2)} \times \omega Z_{(2)} / N_{(U(\omega Z))}
\]

where if \( e(2) = e(\delta(f)) \) then \( \varepsilon = 0 \).

Corollary 10. Let \( 2 = 0 \) and assume that \( B[X; \rho]_{(2)} \) contains a separable polynomial. Let \( U(B[X; \rho]_{(2)}) \) be a group of invertible elements of \( B[X; \rho]_{(2)} \). Then, there exists an idempotent \( \varepsilon (\omega = 1 - \varepsilon) \) of \( Z^p \) such that

\[
U(B[X; \rho]_{(2)}) = \varepsilon Z / \varepsilon \{ z^2 - z \mid z \in Z \} \times U(\omega Z) / N_{(U(\omega Z))}
\]

where \( \omega Z \) is an additive subgroup of \( Z \), and if \( B[X; \rho]_{(2)} \) contains a Galois polynomial then \( \omega = 0 \).
References


