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Kyoto University

京都大学
NEW APPLICATIONS OF THE PRINCIPAL PARTITION OF GRAPHS
TO ELECTRICAL NETWORK ANALYSIS

Takao Ozawa

Department of Electrical Engineering, Kyoto University

I. INTRODUCTION

The principal partition of a graph was defined by Kishi and Kajitani.\(^1\) It was introduced in order to prove the validity of an algorithm for finding a pair of trees* in a graph which have as few edges as possible in common. Its first application to electrical network analysis was found a few months after its birth by Kishi and Kajitani themselves and a group of Nippon Electric Company.\(^2\) The application is concerned with the minimum number of equations for the mixed analysis of electrical networks. This minimum number is called the topological degree of freedom of a network. The concept of the principal partition was found in the decomposition of a matrix by Iri.\(^3\)(\(^4\)) He related it to the decomposition by Dulmage and Mendelsohn. It was also extended to that of a matroid in a more detailed form by Tomizawa\(^5\) and independently by Narayanan.\(^6\)

In 1972 the author was trying to derive a set of state equations for an electrical network by a graphical method. At that time he found some examples to which the method did not work:

* A tree means a spanning tree.
Actually there can exist no state equations for the examples.\(^{(7)}\)

The reason for the non-existence is the lack of common trees of the current graph and the voltage graph. The existence of a

common tree turns out to be a necessary condition for the exist-

ence of a unique solution of network equations.\(^{(8)}\)\(^{(9)}\)

In connection with the existence of a common tree of 2-graphs the author introduced the principal partition of 2-graphs(current and voltage graphs). A more detailed discussion of the principal

partition will be found elsewhere in this issue.\(^{(10)}\)

The new applications of the principal partition discussed here are those which can be related to the solvability problem mentioned above. The results obtained concerning the principal

partition of 2-graphs are applicable to the network problems defined here. The problem themselves are of Menger type. The

partition of graphs defined in connection with Menger's theorem in matroids\(^{(11)}\) is shown to have a one-to-one correspondence

with the principal partition of 2-graphs.

II. DIAGNOSIS AND SEQUENTIAL ANALYSIS OF ELECTRICAL NETWORKS

The linear active network to be considered here is represented

by 2-graphs, that is, the current graph \(G_i\) and the voltage graph \(G_v\). The current[voltage]* graph represents the relations among the currents[voltages] in the network. Kirchhoff's current law (KCL)[Kirchhoff's voltage law(KVL)] is applied to \(G_i[G_v]\) to get network equations. \(G_i[G_v]\) is derived from the network as fol-

lows. First \(G\) is the graph obtained from the network by replac-

* A dual sentence is obtained by replacing the words just before

\[\] with those in \[

\].
ing elements with edges. Then $G_i[G_v]$ is obtained from $G$ by contracting/deleting the edges corresponding to dependent voltage/current sources and norators, and deleting/contracting the edges corresponding to voltage/current sensors and nullators. An edge in $G_i$ and an edge in $G_v$ which correspond to an element in the network, are considered to be the same edge. A voltage/current sensor and a dependent current/voltage source always form a pair, and an edge corresponding to only one of them is left in $G_i$ or $G_v$. The edge represents the pair. Thus $G_i$ and $G_v$ have a common edge set, which is denoted by $E$.

It is assumed that there is no special relation among the element values. Therefore the results obtained here are of a topological nature. Moreover it is assumed that $G_i$ and $G_v$ have at least one common tree and the network has a unique solution.

For any graph $G$, its edge set $E$ and a subset $E_s$ of $E$, we denote by $G \cdot E_s[G \cdot E_s]$ the graph obtained from $G$ by deleting/contracting the edges of $E - E_s$. The rank of $G$ is denoted by $r(G)$, and the nullity, by $n(G)$. For any set $A$, $|A|$ denotes the cardinality of $A$. $\oplus$ denotes the union of edge-disjoint sets.

**Diagnosability of linear active networks** \(^{(12)}\) It may happen in an electrical network that the resistances, capacitances and/or inductances, etc. (called element values) change in a lapse of time or by some other reasons, or there are stray elements whose element values are unknown. The diagnosis of a network is to detect such faults by determining currents and/or voltages of desired elements from the measured currents and voltages of certain other elements. $E$ is partitioned into sets $E_m, E_k$ and
$E_u$: $E_m$ is further partitioned into $E_b$, $E_j$ and $E_e$.

$E_b$: set of edges whose currents and voltages are both measurable,

$E_j$: set of edges whose currents only are measurable,

$E_e$: set of edges whose voltages only are measurable,

$E_m = E_b \oplus E_j \oplus E_e$,

$E_k$: set of edges whose element values are known,

$E_u$: set of edges whose element values are unknown,

$F_i$: set of edges whose currents are required to determine,

$F_v$: set of edges whose voltages are required to determine.

If the current[voltage] of an edge is measured and its element value is known, then its voltage[current] can be determined. Therefore such an edge is included in $E_b$, even if only its current[voltage] is actually measurable. As for an independent source, its voltage[current] cannot be determined from its current[voltage] only. Therefore, if its current[voltage] only can be measured or known, it is included in $E_j[E_e]$, and if neither current nor voltage is measurable or known, it is included in $E_u$.

The network is said to be diagnosable if the required currents of the edges in $F_i$ and the voltages of the edges in $F_v$ can be all determined from the measured currents and/or voltages of the edges in $E_m$. If the current and voltage of an edge can [cannot] be determined from the measurements, it is called a determinate[indeterminate] edge.

Sequential Network Analysis$^{(13)}$ A sequential method of network analysis was defined by Moad.$^{(14)}$ The unknown currents and voltages in the network are sequentially related to properly
chosen independent variable by use of KCL or KVL. At the end of this sequential process, a set of simultaneous equations called constraint equation, are obtained. These equations are then solved to determine independent variables. The detail of the sequential process is given by Algorithm 1 below. It is a modified version of Mood's method. For the simplicity of description, the treatment of independent sources is omitted.

\[E_j\]: set of edges corresponding to independent current sources, 
\[E_e\]: set of edges corresponding to independent voltage sources,

\[E_p \equiv \overline{E_j - E_e}

\[G_{ip} \equiv G_i \cdot (E_e \cdot E_p) \times E_p, \quad G_{vp} \equiv G_v \cdot (E_e \cdot E_p) \times E_p.

The edges associated with the independent variables are called independent edges.

**Algorithm 1 (Sequential process)**

**Step 0.** \[E_\beta = \emptyset \] (\(\emptyset\): null set)

**Step 1.** Choose, as the initial set of independent edges, a set of edges, denoted by \(E_{bi}\), which contains neither cutset in \(G_{ip}\) nor tieset in \(G_{vp}\).

**Step 2.** If \(E_\beta = E_p\), stop.

**Step 3.** If an edge, \(e\), in \(E_p - E_\beta\) forms a cutset in \(G_{ip}\) with some edges in \(E_\beta\), then add \(e\) to \(E_\beta\), and go to Step 2.

**Step 4.** If an edge, \(e\), in \(E_p - E_\beta\) forms a tieset in \(G_{vp}\) with some edges in \(E_\beta\), then add \(e\) to \(E_\beta\), and go to Step 2.

**Step 5.** Choose, as an additional independent edge, an edge, \(e\), in \(E_p - E_\beta\). Add \(e\) to \(E_\beta\), and go to Step 3.

If an edge is added to \(E_\beta\), it is said to be covered. An edge in \(E_p - E_\beta\) which forms a cutset[tieset] in \(G_{ip}\)[\(G_{vp}\)] with some of
of the edges in $E_B$ is called a current[voltage] dependent edge. Edge $e$ in Step 3 is a current dependent edge. It may also be voltage dependent. If so, $e$ is called a constraint edge. Edge $e$ in Step 4 is a voltage dependent edge. If an edge is either current dependent or voltage dependent, but not both, then it is called a single-dependent edge.

The current[voltage] of a current[voltage] dependent edge can be given, by applying KCL[KVL] to the cutset[tieset], in terms of the currents[voltages] of edges in $E_B$. The current and the voltage of an edge are related by Ohm's law. Now at the beginning of the sequential process, $E_B = E_{b_1}$. Then if a current or voltage dependent edge is added to $E_B$, its current and voltage can be given in terms of the independent variables. This can be repeated as Algorithm 1 proceeds, and all the currents and the voltages in the network can be given in terms of the independent variables. The current and the voltage of a constraint edge can be related independently to the independent variables. Then Ohm's law for the edge gives a constraint equation which must be satisfied by the independent variables. Although omitted in Algorithm 1, the independent sources can be handled in a similar way to the independent edges. The source currents and/or voltages appear in the constraint equations, which are solved to determine the independent variables.

$E_b$: set of all the independent edges

$E_k$: set of all the single-dependent edges

$E_u$: set of all the constraint edges.
III. PRINCIPAL PARTITION OF 2-GRAPHS

The following 2-graphs, $G_{ik}$ and $G_{vk}$, are formed from $G_i$ and $G_v$ respectively.

\[ G_{ik} := G_i \cdot (E_u \oplus E_k) \times E_k = G_{ip} \cdot (E_u \oplus E_k) \times E_k \]  \hspace{1cm} (1)

\[ G_{vk} := G_v \times (E_u \oplus E_k) \cdot E_k = G_{vq} \times (E_u \oplus E_k) \cdot E_k \]  \hspace{1cm} (2)

The principal partition of $G_{ik}$ and $G_{vk}$ results in a partition of $E_k$ into three sets, $E_1$, $E_2$ and $E_0$: $E_1$ and $E_2$ are the minimum sets which give

\[ \delta_b = \min_{E_s \subseteq E_k} \{ r(G_{vk} \cdot E_s) - r(G_{ik} \cdot E_s) \} \]  \hspace{1cm} (3)

\[ \delta_u = \min_{E_s \subseteq E_k} \{ r(G_{ik} \cdot E_s) - r(G_{vk} \cdot E_s) \} \]  \hspace{1cm} (4)

respectively. $\delta_b$ and $\delta_u$ are called deficiencies.

$E_0 \subseteq E_k \setminus (E_1 \cup E_2)$.

We can define a pair of trees, $T_{ik}$ and $T_{vk}$ as follows.

$T_{ik}, T_{vk}$: $T_{ik}$ is a tree of $G_{ik}$ and $T_{vk}$ is a tree of $G_{vk}$ such that $T_{ik}$ and $T_{vk}$ have as many edges as possible in common. ($T_{ik}$ and $T_{vk}$ are called maximally-common trees.)

$\overline{T_{ik}}, \overline{T_{vk}}$: cotree of $T_{ik}, T_{vk}$ in $G_{ik}, G_{vk}$.

Then $E_1$ and $E_2$ can be characterized as follows.

**Proposition 1.** $E_1$ is the minimum edge set which satisfies

(i) $(T_{ik} \subseteq T_{vk}) \subseteq E_1$, (ii) $T_{ik} \setminus E_1$ is a forest of $G_{ik} \times E_1$, (iii) $T_{vk} \setminus E_1$ is a forest of $G_{vk} \cdot E_1$.

**Proposition 2.** $E_2$ is the minimum edge set which satisfies

(i) $(T_{vk} \subseteq T_{ik}) \subseteq E_2$, (ii) $T_{vk} \setminus E_2$ is a forest of $G_{vk} \times E_2$, (iii) $T_{ik} \setminus E_2$ is a forest of $G_{ik} \cdot E_2$.

Actually a pair of trees $T_{ik}$ and $T_{vk}$ and the partition of $E_k$ into $E_1$, $E_2$ and $E_0$ can be obtained at the same time by an
algorithm. \( E_0 \) can be further partitioned into subsets \( E_{01}, E_{02}, \ldots, E_{0n} \), and a partial ordering can be given to them. If the second subscripts of the subsets are given in accordance with the partial ordering, then \( E_1 \oplus E_{01} \oplus E_{02} \oplus \ldots \oplus E_m = E_1^* \) for proper \( m \) gives \( \delta_b \), and \( E_2 \oplus E_{n-1} \oplus \ldots \oplus E_{m+1} = E_2^* \) gives \( \delta_u \), that is,

\[
\begin{align*}
  r(G_{vk} \cdot E_1^*) - r(G_{ik} \cdot xE_1) &= \delta_b \\
  r(G_{ik} \cdot E_2^*) - r(G_{vk} \cdot xE_2) &= \delta_u. 
\end{align*}
\]  

(5)  

(6)

Now,

\[
\kappa_b = \min_{E_b \subseteq E} \{ r(G_{vp} \cdot E_t) - r(G_{ip} \cdot xE_t) \},
\]

(7)

\[
\kappa_u = \min_{E_u \subseteq E} \{ r(G_{ip} \cdot E_t) - r(G_{vp} \cdot xE_t) \},
\]

(8)

are called the electrical connectivities or the nonseparabilities. The relation between \( \kappa_b \) and \( \delta_b \) can be obtained by noting that \( E_t = E_s \oplus E_b \) and that

\[
\kappa_b = \min_{E_s \subseteq E_k} \{ r(G_{vp} \cdot E_b) + r(G_{vp} \cdot xE_t \cdot xE_s) - r(G_{ip} \cdot xE_b) - r(G_{ip} \cdot xE_t \cdot E_s) \} = r(G_{vp} \cdot E_b) - r(G_{ip} \cdot xE_b) + \min_{E_s \subseteq E_k} \{ r(G_{vk} \cdot E_s) - r(G_{ik} \cdot xE_s) \}.
\]

(9)

Then we have

\[
\kappa_b = \rho_b - \delta_b
\]

(10)

where

\[
\rho_b \equiv r(G_{vp} \cdot E_b) - r(G_{ip} \cdot xE_b).
\]

(11)

Dually we have

\[
\kappa_u = \rho_u - \delta_u
\]

(12)

where

\[
\rho_u \equiv r(G_{ip} \cdot E_u) - r(G_{vp} \cdot xE_b).
\]

(13)

If we denote the minimum edge sets giving \( \kappa_b \) and \( \kappa_u \) by \( E_0 \) and
\( E_w \) respectively, then we have
\[
E_c = E_b \oplus E_1, \\
E_w = E_u \oplus E_2.
\] (14) (15)

\( E_c^\ast \equiv E_b \oplus E_1^\ast \) and \( E_w^\ast \equiv E_u \oplus E_2^\ast \) are edge sets giving \( \kappa_b \) and \( \kappa_u \) respectively.

Let
\[
G_{i0} \equiv G_{ik} \cdot (E_0 \oplus E_1) \cdot E_0 \\
G_{v0} \equiv G_{vk} \cdot (E_0 \oplus E_1) \cdot xE_0.
\]

Then \( G_{i0} \) and \( G_{v0} \) have a common tree, which is denoted by \( T_0 \).
\[
T_0 = \bigoplus_{m=1}^n T_{0m}
\] (16)

where \( T_{0m} \) is a common tree of \( G_{i0m} \) and \( G_{v0m} \):
\[
G_{i0m} \equiv G_{i0} \cdot (E_0 \oplus E_2 \oplus \cdots \oplus E_{0m}) \cdot E_{0m} \\
G_{v0m} \equiv G_{v0} \cdot (E_0 \oplus E_2 \oplus \cdots \oplus E_{0m}) \cdot xE_{0m}.
\]
The decomposition of \( G_{i0} \) and \( G_{v0} \) into \( G_{i0m} \) and \( G_{v0m} (m=1,2,\ldots,n) \) is called the fine decomposition. An elementary common-tree transformation is possible within \( G_{i0m} \) and \( G_{v0m} \) only, that is, no common tree can be obtained from \( T_0 \) by exchanging an edge in \( E_{0m} \) with that in \( E_{0i} (i \neq m) \).

IV. SOLUTION TO THE DIAGNOSIS PROBLEM

From the definition of \( E_1 \) and \( E_2 \),
\[
\delta_b = -r(G_{vk} \cdot E_1) + r(G_{ik} \cdot xE_1) = r(G_{ik} \cdot xE_1) + n(G_{vk} \cdot E_1) - |E_1| \\
\delta_u = -r(G_{ik} \cdot E_2) + r(G_{vk} \cdot xE_2) = |E_2| - r(G_{ik} \cdot E_2) - n(G_{vk} \cdot xE_2).
\] (17) (18)

Now \( r(G_{ik} \cdot xE_1) + n(G_{vk} \cdot E_1) \) is the number of equations obtained by use of KCL and KVL for the unknown variables associated with \( E_1 \). \(|E_1|\) is the number of unknown variables. Thus \( \delta_b \) is the
number of excess equations to determine the unknown variables associated with the edges of $E_1$. Dually $\delta_{u}$ is the number of equations which are short of to determine the unknown variables associated with the edges of $E_2$. Even if $\delta_{b} > 0$, the equations for $E_1$ must be consistent, since we have assumed that the original network has a unique solution. Only a part of the equations for $E_1$ need be solved. On the other hand, if $\delta_{u} > 0$, then the unknown variables associated with the edges of $E_2$ can not be determined.

The current[voltage] of an edge in $E_u$ or $E_{e}[E_u$ or $E_j]$ must be determined, if possible, by use of KCL[KVL] only, since Ohm's law cannot be used for it. It must be a bridge[self-loop] in

$$G_i \cdot (E_u \oplus E_{E_2}) \cdot G_v \cdot (E_u \oplus E_{E_2}).$$

$E_{id}$ : set of bridges in $G_i \cdot (E_u \oplus E_{E_2})$

$E_{vd}$ : set of self-loops in $G_v \cdot (E_u \oplus E_{E_2})$

For an edge in $E_{id}[E_{vd}]$ there is a cutset in $G_i$[tieset in $G_v$] which consists of the edge and those in $E_{b} \oplus E_{j} \oplus E_{0} \oplus E_{1} \oplus E_{b} \oplus E_{e} \oplus E_{0}$ $E_1$, and the current[voltage] of the edge can be determined by KCL[KVL] applied to the cutset[tieset]. Thus we have:

Theorem 1. The network is diagnosable if and only if

$$F_{i} \subseteq E_0 \oplus E_{E_{id}} \quad (19)$$

and

$$F_{v} \subseteq E_0 \oplus E_{E_{vd}}. \quad (20)$$

Example 1. An Example is given in Fig.1. From $G_i$ and $G_j$ shown in Fig.1(a), $G_{ip}$ and $G_{vp}$ in Fig.1(b) are derived. $G_{ik}$ and $G_{vk}$ are shown in Fig.1(c). From the principal partition of $G_{ik}$ and $G_{vk}$, we get $E_1 = \{4\}$, $E_2 = \{11\}$ and $E_0 = \{5, 8, 9, 10\}$. $E_0$ is further
Fig. 1. Example 1. (a) $G_i$ and $G_v$. $E_i = \{1, 2, 3\}$, $E_j = \{7\}$, $E_e = \{6\}$, $E_b = \{4, 5, 8, 9, 10, 11\}$ and $E_u = \{12, 13, 14\}$. (b) $G_{ip}$ and $G_{vp}$. 
Fig. 1. Example 1 (continued) (c) $G_{ik}$ and $G_{vk}$.
(d) $G_{i0}$ and $G_{v0}$. (e) Partial ordering of the edges in $E_0$. (f) $G_i \cdot (E_u \oplus E_2)$ and $G_v \cdot (E_u \oplus E_j \oplus E_2)$
partitioned into $E_{01} = \{9\}$, $E_{02} = \{5\}$, $E_{03} = \{8\}$ and $E_{04} = \{10\}$, and a partial ordering can be given to them as shown in Fig. 1(e). From Fig. 1(f) we get $E_{id} = \{6, 12\}$ and $E_{vd} = \{7, 14\}$. Thus if we are given $F_i$ and $F_v$, we can determine the diagnosability of the network. The partial ordering of Fig. 1(e) also shows the order to determine the currents and voltages in the network from the measured currents and voltages: The voltage of edge 9 is first determined from those of edges 2 and 3, and then its current is obtained by use of Ohm's law. Next the current of edge 5 or 8 can be determined by use of KCL, and then the voltage, and so on.

V. TOPOLOGICAL PROPERTIES OF THE SEQUENTIAL ANALYSIS

For a single-dependent edge obtained in the sequential process, either KCL or KVL equation is used to relate its current or voltage to the independent variables. Thus the number of unknown variables associated with the edges in $E_k$ is equal to the number of equations for them. (Ohm's law is used to relate the current and the voltage of an edge, and thus only one of the current and the voltage is considered to be the unknown variable associated with the edge.) Thus we have the following theorem from eqs. (17) and (18).

Theorem 2. For the principal partition of $G_{ik}$ and $G_{vk}$ derived from the sequential process,

$$\delta_b = 0, \delta_u = 0, E_1 = \emptyset, E_2 = \emptyset \text{ and } E_0 = E_k.$$  \hspace{1cm} (21)

Now let

$$e_{s1}, e_{s2}, \ldots, e_{sn} (n = |E_k|): \text{ single-dependent edges covered in this}$$


order in the sequential process,

\[ E_{sl} = \{e_{sl}\}, \quad E_{sm} = E_{sm-1} \ominus \{e_{sm}\} \quad (m=2,3,\ldots,n) \quad (E_{sn} = E_{k}). \]

Then each of \( E_{sm} \) \((m=1,2,\ldots,n)\) gives \( \delta_b = 0 \), and thus \( \{e_{sm}\} \)
\((m=1,2,\ldots,n)\) must be the subsets of \( E_0 \) defined with respect to
the principal partition of \( G_{ik} \) and \( G_{vk} \) (which are now equal to
\( G_{i0} \) and \( G_{v0} \) respectively). The order of \( e_{sm} \) \((m=1,2,\ldots,n)\) is
in accordance with the partial ordering of the subsets of \( E_0 \)
(The order of \( e_{sm} \) \((m=1,2,\ldots,n)\) may not be unique, since more
than one edge may become current or voltage dependent to \( E_{\beta} \),
and then an edge can be arbitrarily chosen next to cover.).

**Theorem 2 (continued)** Each of the subsets \( E_{01}', E_{02}', \ldots, E_{0n} \)
of \( E_0 \) consists of exactly one edge, and \( n = |E_k| \). They can
be set as \( E_{0m} = \{e_{sm}\} \) \((m=1,2,\ldots,n)\). Each of \( E_{sm} \) \((m=1,2,\ldots,n)\) gives
\( \delta_b = 0 \). A necessary and sufficient condition for this to hold
is that there exists exactly one common tree of \( G_{ik} \) and \( G_{vk} \).

Next \( E_b[E_u] \) contains neither cutset in \( G_{ip}[G_{vp}] \) nor tieset
in \( G_{vp}[G_{ip}] \). Thus we have the following theorem.

**Theorem 3.**

\[ \rho_b = |E_b|, \quad \rho_u = |E_u|, \quad \rho_b = \rho_u. \quad (22) \]

\[ \kappa_b = \kappa_u = |E_b|, \quad (23) \]

and each of \( E_{tm} = E_{sm} \ominus E_{sb} \) \((m=1,2,\ldots,n)\) gives \( \kappa_b \).

**Example 2.** An example is given in Fig. 2. The edges of \( G_i \) and
\( G_v \) in Fig. 2(a) are numbered according to the sequential process.
\( E_j = \{j\} \) and \( E_e = \{e\} \). We get \( G_{ip} \) and \( G_{vp} \) as shown in Fig. 2
(b), and then \( G_{ik} \) and \( G_{vk} \), in Fig. 2(c). The partial ordering
of edges is given in Fig. 2(d). It can be easily seen that

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Fig. 2. Example 2. (a) $G_i$ and $G_v$. $E_j=\{j\}$, $E_e=\{e\}$.
(b) $G_{ip}$ and $G_{vp}$.
(c) $G_{ik}$ and $G_{vk}$.
(d) Partial ordering of the edges in $E_0=E_k$.
there exists exactly one common tree of $G_{ik}$ and $G_{vk}$. It is indicated by the thick lines in Fig.2(c). It consists of the current dependent edges, and its cotree, of voltage dependent edges. The edges of the common tree and those of $E_u \oplus E_e [E_b \oplus E_e]$ form a tree of $G_i [G_v]$. These trees of $G_i$ and $G_v$ are called a linkage pair of trees.

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