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Homomorphisms of differentiable dynamical systems

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1) In this note we consider the following problems.

Let (M, φ_t) and (N, ψ_t) be differentiable dynamical systems (D.D.S.). Assume that there exists a homomorphism, i.e. differentiable mapping $\pi : M \to N$ such that $\pi \cdot \varphi_t = \psi_t \cdot \pi$ for all $t \in R$. Under these assumptions, what relations can exist between the structures of (M, φ_t) and (N, ψ_t)?

Then we obtain the following results. For the proofs, see [1].

2) Theorem 1. Let (M, φ_t) and (N, ψ_t) be D.D.S.'s and π be a homomorphism of (M, φ_t) to (N, ψ_t).

If M is compact and the system (N, ψ_t) is minimal, then π is a surjective mapping of maximal rank, and as a consequent of it, M is the total space of a locally trivial fibre space over N, the system (φ_t) preserves the fibres, and the naturally induced system on the base space is isomorphic to (N, ψ_t).

Theorem 2. Let $\pi : T^m \to N$ be a homomorphism of a quasi-periodic motion (T^m, τ_t) to D.D.S. (N, ψ_t), and $r = \text{rank of } \pi$. Then $\pi(T^m)$, image of π is an r-dimensional invariant submanifold of N, which is homeomorphic to an r-dimensional torus T^r, and the restricted system of (N, ψ_t) to $\pi(T^m) \subset N$, $(\pi(T^m), \psi_t|_{\pi(T^m)})$ is C^0-isomorphic to some quasi-periodic motion $(T^r, \tilde{\tau}_t)$, i.e. there exists a homeomorphism h of T^r to $\pi(T^m)$ such that

$$h \cdot \tilde{\tau}_t = \psi_t|_{\pi(T^m)} \cdot h \quad \text{for all } t.$$

Here (T^m, τ_t) is called a quasi-periodic motion, when
$T^n = \{(x^1, x^2, \cdots, x^n) \; ; \; x^i \in R \pmod 1, \; i=1,2,\cdots,n\}$, and
$\tau_t : (x^1, \cdots, x^n) \mapsto (x^1 + \omega^1 t, \cdots, x^n + \omega^n t)$, mod 1, where $\omega^1, \cdots, \omega^n$ are rationally independent.

References

1 . T. Niwa : Homomorphisms of differentiable dynamical systems, to appear.

2 . T. Niwa : Classical flows with discrete spectra, J. Math. Kyoto Univ. 9-1 (1969) p.p. 55-68.

3 . V.I. Arnold- A.Avez : Problemes ergodiques de la mecanique classique, Gauthier-Villars, Paris (1966).

4 . S. Smale : Differentiable Dynamical Sydtems, Bull. A. M. S. 73 (1967) p.p. 747-817.