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Homomorphisms of differentiable dynamical systems
By Toshio Niwa

1) In this note we consider the following problems.
Let \((M, \mathcal{U}_M)\) and \((N, \mathcal{V}_N)\) be differentiable dynamical systems
(D.D.S.). Assume that there exists a homomorphism, i.e. differentiable
mapping \(\Phi : M \to N\) such that \(\Phi \cdot \mathcal{U}_M = \mathcal{V}_N \cdot \Phi\) for all \(t \in \mathbb{R}\). Under
these assumptions, what relations can exist between the structures of
\((M, \mathcal{U}_M)\) and \((N, \mathcal{V}_N)\)?
Then we obtain the following results. For the proofs, see [1].

2) Theorem 1. Let \((M, \mathcal{U}_M)\) and \((N, \mathcal{V}_N)\) be D.D.S.'s and \(\Phi\) be
a homomorphism of \((M, \mathcal{U}_M)\) to \((N, \mathcal{V}_N)\).
If \(M\) is compact and the system \((N, \mathcal{V}_N)\) is minimal, then \(\Phi\) is
a surjective mapping of maximal rank, and as a consequent of it, \(M\)
is the total space of a locally trivial fibre space over \(N\), the system
\((\mathcal{U}_M)\) preserves the fibres, and the naturally induced system on the
base space is isomorphic to \((N, \mathcal{V}_N)\).

Theorem 2. Let \(\Phi : T^m \to N\) be a homomorphism of a quasi-
periodic motion \((T^m, \tau_t)\) to D.D.S. \((N, \mathcal{V}_N)\), and \(r = \text{rank of } \Phi\).
Then \(\Phi(T^m)\), image of \(\Phi\) is an \(r\)-dimensional invariant submanifold
of \(N\), which is homeomorphic to an \(r\)-dimensional torus \(T^r\), and the
restricted system of \((N, \mathcal{V}_N)\) to \(\Phi(T^m) \subseteq N\), \((\Phi(T^m), \mathcal{V}_N | \Phi(T^m))\)
is \(C^r\)-isomorphic to some quasi-periodic motion \((T^r, \tilde{\tau}_t)\), i.e. there exists
a homeomorphism \(h\) of \(T^r\) to \(\Phi(T^m)\) such that

\[ h \cdot \tilde{\tau}_t = \mathcal{V}_N | \Phi(T^m) \cdot h \quad \text{for all } t.\]

Here \((T^m, \tau_t)\) is called a quasi-periodic motion, when
\(T^m = \left\{ (x^1, x^2, \ldots, x^n) : x^i \in \mathbb{R} \text{ (mod 1)} \right\} \), \(i=1, 2, \ldots, n\), and
\(\tau_t: (x^1, \ldots, x^n) \mapsto (x^1 + \omega_1 t, \ldots, x^n + \omega_n t), \mod 1\), where \(\omega, \ldots, \omega^n\)
are rationally independent.
References


