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<td>Author(s)</td>
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<tr>
<td>Citation</td>
<td>数理解析研究所講究録 (1975), 258: 104-105</td>
</tr>
<tr>
<td>Issue Date</td>
<td>1975-12</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/105782">http://hdl.handle.net/2433/105782</a></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
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On the stability of incompressible viscous fluid motions past objects

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Let $E$ be the exterior domain in $3$-space. Let us consider the steady flow in $E$ governed by

$$
\begin{align*}
-\nu \Delta w + (w \cdot \nabla) w + \nabla p, &= 0, \\
\nabla \cdot w &= 0
\end{align*}
$$

(1)

(2) $w(x) \to w^\infty$ $(|x| \to \infty)$

(3) $w(x) = b(x)$ $(x \in \partial E)$

where the viscousity coefficient $\nu$ is a positive constant, $w^\infty$ is some fixed constant vector, $b$ is some prescribed function on $E$.

R. Finn showed that if $w^\infty - b$ is "small" enough, then there exists a smooth solution $w$ with

$$
\sup_{x \in E} |x| |w(x) - w^\infty| < \infty
$$

$$
\nabla w \in L^3(E)
$$
Given the disturbance \( u_0 \in L^2(E) \) to \( w \). Then the perturbed flow \( v \) is governed by

\[
\begin{align*}
\frac{\partial v}{\partial t} + \nabla \Delta v + (v \cdot \nabla) v - \nabla p &= 0, \\
\nabla \cdot v &= 0
\end{align*}
\]

\((4)\)

\[
\begin{align*}
\lim_{|x| \to \infty} v(x,t) &= w^a, \\
v(x,t) &= b(x) \quad (x \in \partial E, t > 0) \\
\lim_{t \to t_0} v(x,t) &= w(x) + u_0(x)
\end{align*}
\]

\((5)\)

Now our result is:

Assume that

(i) \( \sup_{x \in E} |x| | w(x) - w^a | < \frac{1}{2} \)

(ii) \( \nabla w \in L^3(E) \)

(iii) \( \nabla \cdot u = 0 \).

Then every weak solution \( v \) of \((4), (5)\) becomes analytic (in \( t \) and \( x \)) after some definite time \( T_0 \), and then converges to steady flow \( w \) uniformly in \( x \) on \( E \) like

\[
| v(x,t) - w(x) | \leq M | t |^{-\frac{1}{2}} \quad (t \to \infty)
\]

(\( M \); constant)