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<th>On the Analyticity in Time of Solutions of Initial Boundary Value Problems for Semi-Linear Parabolic Differential Equations with Monotone Nonlinearity (非線形問題の解析)</th>
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On the analyticity in time of solutions of initial boundary value problems for semi-linear parabolic differential equations with monotone nonlinearity

By Sunao Ouchi

In this note we consider the following initial boundary value problems:

\[ \frac{\partial u}{\partial t} = \Delta u + f(u) \]

(I.B.V.P.) \[ u(t,x)|_{\partial \Omega} = 0 \]

\[ u(0,x) = a(x) \]

where \( \Omega \) is a bounded domain in \( \mathbb{R}^n \) with smooth boundary \( \partial \Omega \).

The purpose of this note is to report that solutions of (I.B.V.P.) are extensible holomorphically in time \( t \) to a sector \( \sum \theta = \{ t \in \mathbb{C} : |\arg t|^\theta \} \) in the complex domain which does not depend on initial values, if the nonlinear term \( f(u) \) is a monotone decreasing polynomial.

Let us now introduce definitions to state results.

**Definition 1.** A polynomial with real coefficients \( f(u) \) is said to be monotone or to satisfy condition \( (M) \), if \( f(0) = 0 \) and \( f'(u) \leq 0 \) for \( -\infty < u < +\infty \).

**Examples.** \( f(u) = -u^{2p+1}, -u-u^3, -u^3-u^5 \).

**Definition 2.** A polynomial with real coefficients \( f(u) \) is said to be monotone on \( \mathbb{R}^+ = [0, +\infty) \) or satisfy condition \( (M^+) \), if \( f(0) = 0 \) and \( f'(u) \leq 0 \)

for \( 0 \leq u < +\infty \).

**Examples.** \( f(u) = -u^p, -u-u^4, -u^2-u^6 \).

**Theorem 1.** Suppose that the nonlinear term \( f(u) \) in (I.B.V.P.) satisfies condition \( (M) \) and the initial value \( a = a(x) \) is realvalued and boundedly continuous in \( \Omega \). Then there is a sector \( \sum \theta = \{ t : |\arg t|^\theta \} \) in the complex domain which is independent of \( a(x) \) such that the solution \( u(t,x) \) of (I.B.V.P.) is analytically extensible in \( t \) to the sector.

**Theorem 2.** Suppose that the nonlinear term \( f(u) \) in (I.B.V.P.) satisfies condition \( (M^+) \) and the initial value \( a = a(x) \) is a nonnegative and boundedly continuous function. Then there is a sector \( \sum \theta = \{ t : |\arg t|^\theta \} \) in the complex domain which does not depend on \( a(x) \) such that the nonnegative solution \( u(t,x) \) of (I.B.V.P.) is analytically extensible in \( t \) to the sector.
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