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Unitary representations of p-adic division algebras

By Hiroaki Hijikata
Tetsuo Mizuno

Let ; p = a prime, k = a $\mathfrak{p}$-adic field or $F_q((t))$ such that $\mathfrak{p}$ divides p or q is a power of p, E = a central simple algebra over k, $E^\times$ = the multiplicative group of E, $E^{(1)} = \{x \in E^\times \mid$ reduced norm of $x = 1\}$ and $n^2 = [E : k]$. Then, by the natural topology from k, $E^\times$ is a locally compact (totally disconnected) topological group, and its center is exactly $k^\times$.

<u>Problem (I) : Find all the unitary representations of $E^\times$, $E^{(1)}$ and $E^\times/k^\times$</u>.

Note that $E^\times$ (resp. $E^{(1)}$; resp. $E^\times/k^\times$) is the group of k-rational points of a k-form of GL_n (resp. SL_n ; resp. PGL_n). The problem (I) is solved so far only when n = 2, p odd and $E = M_2(k)$ [by Kirillov]. Suppose E to be a division algebra, then E has a unique maximal compact subring $R = R(E)$, and for any subring L of E, $R(L) = R(E) \cap L$ is a unique maximal compact subring of L. Then since we have $E^{(1)} \subset R(E)$ (hence $E^{(1)} = R^\times \cap E^{(1)}$), and since $R^\times/R(k)^\times$ is of finite index in $E^\times/k^\times$, the solution of (I) is not very far from that of the following.

<u>Problem (I') : Find all the unitary representations of $R(E)^\times/R(k)^\times$</u>.

Again under the assumption that n = 2, (i.e. E is a quaternion) and p odd, (I') is solved by Gelfond-Graev. Now we have solved (I') and (I) under the following

<u>Assumption : (1) E is a division algebra, (2) n is an odd prime different from p, (3) k contains a primitive n-th root of unity</u>.

Our result is surprisingly simple, for instance simpler than the

Gelfond-Graev case of $n = 2$. Indeed every representation of $R^\times/R(k)^\times$ can be obtained as a monomial representation. We start with a maximal subfield L, take a character χ of $R(L)^\times/R(k)^\times$, extend χ to a linear character $\tilde{\chi}$ of a certain open subgroup A which we choose as big as possible, then induce $\tilde{\chi}$ from A to whole $R^\times/R(k)^\times$. Then we get all the representations.

Severe as our assumptions are, our result provides the first example of a solution of (I) for groups with (absolute) rank greater than 2.