

Teizyou mondai wo toku tetuzuki ni kansuru  
ni-san no tyuui

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§1. Hasigaki.

Navier-Stokes houteisiki no teizyoukai wo keisansuru noni, teizyou houteisiki wo sono mama toku no ga yoi ka, hiteizyou houteisiki wo toite  $t \rightarrow \infty$  no kyokugen wo motomeru no ga yoi ka ni tuite, Kawaguti Si ga sugu mae ni ronzirareta. Koko dewa, ikutuka no kantan na houteisiki wo rei ni toqte, kono mondai ni tuite no hosokuteki na tyuui wo noberu.

§2. Kantan na rei.

Mazu, mitisuu  $u$  ni kansuru tugi no daisuu-houteisiki

$$(2.1) \quad u^2 - a = 0, \quad a > 0$$

wo suutiteki ni toku koto wo kangaeyou. Kono houteisiki no kai wa, iu made mo naku  $u = \sqrt{a}$  oyobi  $u = -\sqrt{a}$  de aru. Kore wo suutikeisan de motomeru niwa hutuu tugi no youni suru. Sunawati, kaqte na atai  $u^{(0)}$  kara hazimete, tugi no siki

$$(2.2) \quad u^{(n+1)} = \frac{1}{2} \left( \frac{a}{u^{(n)}} + u^{(n)} \right)$$

$$n = 0, 1, 2, \dots$$

ni sitagaqte  $u^{(n)}$  wo tugitugi ni keisansite ikeba, sono kyokugenti ga motomeru mono de aru.

Kono keisan tetuzuki wa, yoku miruto, tugi no bibunhouteisiki no syokiti mondai

$$(2.3) \quad \begin{cases} \frac{du}{dt} = \frac{a}{u} - u, & * \\ u(0) = u_0 \end{cases}$$

wo moqtomo kantan na sabunhouteisiki de kinzisite toite iru noni hokanaranai koto ga wakaruru. Kono koto wo miru tame ni, hensuu  $t$  no sabunkankaku wo  $\Delta t$ , kansuuti  $u(n\Delta t)$  no kinziti wo  $u^{(n)}$  ( $n = 0, 1, 2, \dots$ ) to kakou. (2.3) no kinzi to site tugi no sabunhouteisiki wo

kangaeru:

$$(2.5) \quad \begin{cases} \frac{u^{(n+1)} - u^{(n)}}{\Delta t} = \frac{a}{u^{(n)}} - u^{(n)}, \\ n = 0, 1, 2, \dots, \\ u^{(0)} = u_0. \end{cases}$$

\* Syokiti mondai (2.3) no kai wa

$$(2.4) \quad u(t) = \pm \sqrt{a} \left\{ 1 + \left( \frac{u_0^2}{a} - 1 \right) e^{-2t} \right\}^{\frac{1}{2}}$$

de ataerareru.

Hazime no hou no siki wo kakinaoseba

$$(2.6) \quad \begin{aligned} u^{(n+1)} &= u^{(n)} + \Delta t \left( \frac{a}{u^{(n)}} - u^{(n)} \right) \\ &= \Delta t \left\{ \frac{a}{u^{(n)}} + \left( \frac{1}{\Delta t} - 1 \right) u^{(n)} \right\}. \end{aligned}$$

Sokode, tokuni  $\Delta t = 1/2$  to oite mireba, kore wa (2.2) to kanzen ni iqtisuru.

Sunawati, "teizyou mondai" (2.1) wo iwayuru Newton-hou de toku koto wa, zituwa "hiteizyou mondai" (2.3) wo toite,  $t \rightarrow \infty$  no kyokugen wo motomeru no to maqtaku onazi koto nano de aru.

Soredewa,  $a$  no heihoukon wo keisansuru tame no hiteizyou mondai wa moqto hoka nimo nai no darou ka? Sore wa ikura demo aru. Tatoeba, tugi no syokiti mondai wa sono hitotu de aru:

$$(2.7) \quad \begin{cases} \frac{du}{dt} = a - u^2, \\ u(0) = u_0. \end{cases}$$

Kono mondai no kai wa

$$(2.8) \quad u(t) = \sqrt{a} \frac{1 + \rho e^{-2\sqrt{a}t}}{1 - \rho e^{-2\sqrt{a}t}}, \quad \rho = \frac{u_0 - \sqrt{a}}{u_0 + \sqrt{a}}$$

de ataerareru kara,  $a > 1$  no baai niwa, syokiti mondai (2.3) wo toku yorimo hayaku  $u(\infty)$  no atai ga erareru kanousei ga aru.

§ 3. Douti de nai hiteizyou mondai.

§ 2 de nobeta rei wa, sei no kazu  $a$  no heihoukon wo motomeru to iu teizyou mondai kara hazimaqta.

Tugi ni, onazi mondai wo sukosi betu no kakudo kara nagamete miyou.

Mazu, hiteizyou mondai (2.3) no kai no,  $t \rightarrow \infty$  de no kyokugen wo motomeru to iu mondai kara syuqpatusuru. Kono kyokugen ga sonzaisuru naraba, sore wa houteisiki

$$(3.1) \quad \frac{a}{u} - u = 0$$

no kon de aru. Sokode, hazime no mondai wo surikaete,

(3.1) to douti na houteisiki

$$(3.2) \quad a - u^2 = 0$$

kara mitibikareru hiteizyou mondai (2.7) wo kangae, sono kai no kyokugen wo motomeru koto ni sitemo yoi de arou.

Sikasi, koko de tyuuisubeki koto ga aru.

Mazu, syokiti mondai (2.3) ni oite wa, sei no syokiti  $u_0 > 0$  kara wa sei no heihoukon ga, mata, hu no syokiti  $u_0 < 0$  kara wa hu no heihoukon ga, sorezore  $t \rightarrow \infty$  no kyokugen to site erareru. Tokoroga, syokiti mondai (2.7) dewa, syokiti  $u_0 > -\sqrt{a}$  kara wa  $\sqrt{a}$  ga erareru ga,  $u_0 < -\sqrt{a}$  de aru youna syokiti ni taisite wa,  $u(t)$  wa  $t \rightarrow \infty$  de haqsansite simau. Tumari, (2.3) ni yoru naraba erareru hazu no kyokugenti  $-\sqrt{a}$  ga, (2.7)

wo toku baai niwa motomeru koto ga dekinai. Kono zizyou wa,  $du/dt$  no houkou-ba wo  $(t, u)$ -men ni egaitte mireba sugu wakaru de arou.

Kore to maqtaku gyaku ni, kondo wa houteisiki (2.7) ni sitagau "gensyou" no teizyoukai wo motomeru to iu mondai ga hazime ni ataerareta to suru. Kore wo motomeru tame ni, (2.3) wo toku keisan tetuzuki wo motiita to suruto, moto no mondai (2.7) dewa "buturiteki" imi wo motanakaqta (huantei na) kai  $-\sqrt{a}$  ga, mitatokoro imi no aru kai de aru ka no youni motomaqte simau koto ni naru.

#### § 4. Teizyoukai wo keisansuru tetuzuki.

Sate tugi ni, kanari ranbou dewa aru ga, mae ni ageta rei to, Navier-Stokes houteisiki no teizyoukai wo keisansuru mondai to no kanren ni tuite nobeyou.

Navier-Stokes houteisiki no teizyoukai wa, koremade iroiro no baai ni tuite keisansarete iru. Sabunhou ni yoru suutikeisan ni hanasi wo kagireba, ookiku wakete tugi no 2-toori no yarikata ga aru to iqte yoi de arou. Sore wa,

(i) akumademo teizyou houteisiki dake ni tyuumokusi, kyoukaiti mondai to site, tikuzikinzi no houhou de sono kai wo motomeru to iu yarikata, oyobi

(ii) hiteizyou Navier-Stokes houteisiki ni tati-modori, kore wo tekitou na syokizyouken (oyobi kyoukai-zyouken) no moto de toki,  $t \rightarrow \infty$  no kyokugen wo motomeru to iu yarikata, de aru.

Sokode, korera hutatu no yarikata no uti, dotira ga yori nozomasii mono de aru ka wo kangaete miyou.

Mazu, (i) ni okeru tikuzikinzi no keisan wa, teizyou houteisiki wo toku keisan dewa aru keredomo, zituwa nanika aru hiteizyou houteisiki no syokiti mondai wo toku keisan ni naqte iru to kangaerareru koto ni tyuuisiyou — motiron, sono houteisiki no gutaiteki na katati wo siki de kakiarawasu koto wa iqpan niwa muzukasii de arou ga. Tugi ni, (ii) wa (i) de kangaerareru iroiro no tikuzikinzihou no naka no tokubetu no hitotu de aru. Sunawati, (i) wa (ii) ni kurabete, harukani takusan no keisanhou wo hukunde iru.

Sitagaqte, (i) no yarikata de kuhuu wo koraseba, motomeyou to suru keqka ni (ii) yorimo itizirusiku hayaku taqsuru koto ga dekiru de arou. Sono imi dewa, (ii) yorimo (i) no hou ga sugureta yarikata de aru koto wa akiraka de aru.

Sikasi iqpou, (ii) no yarikata wa (i) niwa nai tuyomi wo moqte iru koto mo tasika de aru. Honrai,

Navier-Stokes houteisiki no teizyoukai to iu no wa, buturiteki ni iqte, hiteizyou Navier-Stokes houteisiki no kai no  $t \rightarrow \infty$  de no kyokugen to kaisyaku subeki mono dakara de aru. Tumari, (i) de kangaeru iroiro no "hiteizyou" houteisiki no uti, hitezyou Navier-Stokes houteisiki to iwaba "karakuri ga onazi" mono de nakereba ansinsite tukau koto ga dekinai.

Mae ni nobeta kantan na rei to kurabete miyou. (2.3) no kai no kyokugen wo motomeru to iu toki ni, syuusoku ga hayai kara to iu dake no riyuu de (2.7) wo tokou to suruto, houteisiki no karakuri no tigai ni yoqte kai  $-\sqrt{a}$  wo mituke sokonau. Mata, (2.7) ga toku beki houteisiki de aru toki ni (2.3) wo toita to suruto, (2.7) ni tuite wa buturiteki ni imi no nai kai  $-\sqrt{a}$  wo, atakamo imi no aru (antei na) kai to toritigaeru. Korera no koto wa, honrai nara (ii) wo torubeki nano ni (i) wo eranda tame ni okoriuru kiken no hitotu de aru to ieyou.

Dewa, koremade ni keisansarete iru Navier-Stokes houteisiki no teizyoukai wa, ue ni nobeta youna imi de huzyuubun na mono aruiwa maborosi no youna mono nano darou ka? Korera no suutikai wa, sukunakutomo wareware no buturiteki tyoqkan ni uqtaeru kagiri dewa kiwamete moqtomorasii mono bakari de aru youni omowareru. Sikasi, Navier-Stokes houteisiki no karakuri ga mada

zyuubun niwa wakaqte inai genzai, erarete iru keqka no seitousa ni tuite tadasiku ronziro koto wa motiron dekinai.

Sokode, kono ato dewa, honsituteki ni senkei dewa aru ga, houbutugata de aru to iu imi de Navier-Stokes houteisiki ni nite iru netuhouteisiki ni tuite, sono teizyokai wo keisansuru tikuzikinzi no tetuzuki to, hiteizyokai wo keisansuru tetuzuki to no aida no kankei wo sirabete miru koto ni suru.

§ 5. Netuhouteisiki kara no ruisui.

1-zigen netuhouteisiki no kyokaiti mondai

$$(5.1) \quad \begin{cases} \frac{d^2u}{dx^2} + f(x, u) = 0, & 0 < x < 1, \\ u(0) = a, \quad u(1) = b, \end{cases}$$

wo kangaeyou. Koko de,  $f(x, u)$  wa ataerareta kansuu de aru to suru.

Kono mondai wo sabunkinzi de toku houhou to site, tugi no mono ga aru. Hensuu  $\mathcal{X}$  no sabunkankaku wo  $\Delta x (= 1/J)$ ,  $u(j\Delta x)$  ni taisuru kinziti wo  $u_j$  ( $j = 0, 1, \dots, J$ ) to kaku. Kaqte na syuqpatuti  $u_j^{(0)}$  wo kateisite, tugi no tetuzuki

$$(5.2) \quad u_j^{(n+\frac{1}{2})} = \frac{1}{2} \left\{ u_{j+1}^{(n)} + u_{j-1}^{(n)} + (\Delta x)^2 f_j^{(n)} \right\},$$

$$(5.3) \quad u_j^{(n+1)} = (1-\omega) u_j^{(n)} + \omega u_j^{(n+\frac{1}{2})},$$

ni sitagaqte  $u_j^{(n)}$  ( $j = 1, 2, \dots, J - 1; n = 0, 1, \dots$ )  
wo keisansite iku. Tadasi,  $\omega$  wa  $0 < \omega < 1$  wo mitasu  
teisuu no paramêtâ, mata  $f_j^{(n)} = f(j\Delta x, u_j^{(n)})$  de aru.

Tokorode, kono tikuzikinzi no tetuzuki wa,  
zituwa netuhouteisiki no hiteizyou mondai wo tyuuzitu ni  
toku tetuzuki de aru. Kono koto wo miru tame ni, syokiti-  
kyoukaiti mondai

$$(5.4) \quad \left\{ \begin{array}{l} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(x, u), \quad 0 < x < 1, \quad t > 0, \\ u(0, t) = a, \quad u(1, t) = b, \\ u(x, 0) = u^{(0)}(x), \end{array} \right.$$

ni taisi,  $u(j\Delta x, n\Delta t)$  no kinzi wo  $u_j^{(n)}$  to kaite, moqtomo  
kantan na sabunkinzisiki wo tukureba,

$$(5.5) \quad \frac{u_j^{(n+1)} - u_j^{(n)}}{\Delta t} = \frac{u_{j+1}^{(n)} - 2u_j^{(n)} + u_{j-1}^{(n)}}{(\Delta x)^2} + f_j^{(n)},$$

aruiwa

$$(5.6) \quad u_j^{(n+1)} = u_j^{(n)} + \frac{\Delta t}{(\Delta x)^2} \left\{ u_{j+1}^{(n)} - 2u_j^{(n)} + u_{j-1}^{(n)} + (\Delta x)^2 f_j^{(n)} \right\}$$

to naru. Sokode

$$(5.7) \quad \frac{\Delta t}{(\Delta x)^2} = \frac{\omega}{2}$$

to okeba, (5.6) wa (5.2) to (5.3) wo matometa siki to  
maqtaqu onazi ni naru.

Soredewa, onazi kyoukaiti mondai (5.1) ni  
taisuru moqto nouritu no yoi tugi no houhou (iwayuru

kasoku Liebmann-hou) wa dou de arou ka? Sunawati,

$$(5.8) \quad u_j^{(n+\frac{1}{2})} = \frac{1}{2} \left\{ u_{j+1}^{(n)} + u_{j-1}^{(n+1)} + (\Delta x)^2 f_j^{(n)} \right\}^*$$

$$(5.9) \quad u_j^{(n+1)} = (1-\omega) u_j^{(n)} + \omega u_j^{(n+\frac{1}{2})}$$

Tokoroga, zituwa kono keisan tetuzuki mo mata, hiteizyou netuhouteisiki no kai wo tyuuzitu ni tusekisuru tetuzuki ni naqte iru. Ziqsai, (5.8) to (5.9) wo matomereba

$$(5.10) \quad \left\{ u_j^{(n+1)} - u_j^{(n)} \right\} - \frac{\omega}{2} \left\{ u_{j+1}^{(n+1)} - u_{j-1}^{(n)} \right\} \\ = \frac{\omega}{2} \left\{ u_{j+1}^{(n)} - 2u_j^{(n)} + u_{j-1}^{(n)} + (\Delta x)^2 f_j^{(n)} \right\}$$

to kakeru. Sitagaqte, kore wa, tugi no kankei

$$(5.11) \quad \frac{\Delta t}{(\Delta x)^2} = \frac{\omega/2}{1-\omega/2}$$

kara kimaru sabunkankaku  $\Delta t$  to  $\Delta x$  to wo motiite hiteizyou houteisiki (5.4) wo sabunka sita mono to minasu koto ga dekiru no de aru.

Kono youni site, tokubetu na 2-kai houteisiki no kyoukaiti mondai wo toku hutuu no keisan tetuzuki wa, netuhouteisiki no hiteizyou mondai wo toku tetuzuki to

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\* Mae no houhou (5.2) to no tigai wa, (5.8) no migigawa no dai-2 kou ga  $u_{j-1}^{(n)}$  de nakute  $u_{j-1}^{(n+1)}$  to naqte iru koto de aru. Kore ga syuusoku wo itizirusiku hayameru genqin ni naqte iru.

kanzen ni onazi de aru koto ga wakaqta. Navier-Stokes houteisiki no teizyoukai no keisan ni motiirarete iru tetuzuki no ooku no mono mo, honsituteki niwa ue ni nobeta mono to onazi de aru to kangaerareru. Teizyou Navier-Stokes houteisiki no suutikai to site, buturiteki ni okasii youna keqka ga koremade ni erarete inai no wa osoraku kono zizyou ni yoru no de arou. Sikasi, Navier-Stokes houteisiki to netuhouteisiki to no honsituteki na tigai ga arawareru no wa Reynolds-suu ga ookii baai, sunawati hisenkeisei no eikyou ga ookiku dete kuru baai de aru. Kono baai niwa Navier-Stokes houteisiki no kai sonomono no seisitu ga mada yoku wakaqte orazu, mata, singyou no okeru suutikai mo mada erarete inai. Kono koto wo kangaeruto, teizyoukai no suutikeisan niwa, narubeku hayaku sosite seimitu ni kotae wo dasu tame no houhou no kaihatu to douzi ni, sono houhou no karakuri ni taisuru zyuubun na tyuui ga hituyou de arou.