

Uzu-undou to uzuito-kinzi

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Uzusou o hukumu nagare o siraberu no ni, uzusou o takusan no uzuito ga naranda mono to minasite, sorema no undou o tusekisuru houhou ga aru. Uzusou aruiwa uzukuda o hukumu nagare ni taisite, kono kinzihou o motiite okonatta ikutuka no suutizikken ni tuite noberu.

§ 1 Uzuito-kinzi.

Reynolds-suu no ookii nagare dewa, ryuutai no nensei no eikyou wa kiwamete usui sou —— kyoukaisou —— no naka dake ni arawareru. Kyoukaisou no soto no nagare wa uzunasi de aru. Ippou, kyoukaisou no naka wa uzuari de atte, nagare no sokudo wa kuukanteki ni kyuugeki ni henkasite iru. Kono youna nagare ni tuite Navier-Stokes houteisiki o kaisekiteki ni toku koto wa, tateo kinzi o okonatta to sitemo, ippan niwa kiwamete muzukasii mondai de aru.

Dewa, Navier-Stokes houteisiki o kaisekiteki ni atukau koto wa akirameru to site, suutiteki ni toku koto ni sitara dou de arou ka? Sono tame no houhou to site hutuu suguni kangaerareru no wa, bibunhouteisiki de aru Navier-Stokes

houteisiki o sabunhouteisiki de kinzisite toku koto de arou. Kou sureba, sukunakutomo genriteki niwa, dono youni hukuzatu na nagare demo atukau koto ga dekisou ni omowareru. Sikasi, Reynolds-suu no ookii baai wa kono houhou mo zissai no yaku niwa tatanai. Kyoukaisou no youna, ba no henka no hagesii ryouiki o seimitu ni toraeru koto wa, tatoe oogata-keisanki o motiita to sitemo youi na koto dewa nai.

Sokode, kyoukaisou ga kiwamete usui to iu zizitu o musiro gyakuni riyousite, kyoukaisou o atusa ga zero de aru uzusou to minasi, sarani kono uzusou o, takusan no uzuito ga 1-retu ni naranda mono to minasu kinzi ga kangaerareru. Kore o "uzuito-kinzi" to yobu koto ni siyou.

Uzuito-kinzi dewa, uzunasi-nagare no naka ni okareta uzuito no hitotu-hitotu ga zibunzisin no tukuru ba o nozoita nagare no naka de okonau undou o tusekisuru. Suugakuteki niwa, renritu-zyoubibunhouteisiki o toku koto ni naru kara, moto no henbibunhouteisiki o tyokusetu atukau yori wa keisan no tema ga harukani sukunakute sumu.

Uzuito-kinzi wa, koremade tatoeba, hantaimuki ni nagareru hutatu no ryuutai no kyoukaimen no huanteisei (Rosenhead, 1931), hikouki no tubasa kara nagare no naka ni deta uzusou no makiagari (Westwater, 1936), Kármán uzuretu ga dekiru katei (Abernathy & Kronauer, 1962), nado o siraberu no ni motiirarete iru.

Koko dewa tugi no 2-syurui no nagare o uzuito-kinzi de sirabeta kekka ni tuite noberu:

(1) Seisisita ryuutai no naka de heiban ga naname ni ugokidasu toki no, ita no usirohasi kara deru uzusou no seityou to

makiagari.

(2) Daenkei no danmen o moti, sono naka de tuyosa ga itiyou na uzukuda no undou.

(1) wa, nagare no naka ni sudeni sonzaisuru uzusou dake de naku, buttai no hyoumen kara wakidete iku uzusou o dono youni atukau ka o simesu hitotô no rei de aru. (2) wa genmitukai ga sirarete iru baai de atte, sore to kuraberu koto ni yori, uzuito-kinzi ga dono teido no seido to zituyousei o motu ka o teiryouteki ni siru koto o mokuteki to sita mono de aru.

§ 2 Heiban no usirohasi kara deru uzusou no hattatu.

Seisisita ryuutai no naka ni seisisite ita yuugen-nagasa no heiban ga, aru zikoku kara ittei no sokudo de ugokidasita to suru. Kono toki, heiban no usirohasi kara wa uzusou ga hagare, ryuutai no naka ni nobi, sono saki wa makiagatte iku (Zu 2.1). Sosite, zyuubun zikan ga tatta noti, heiban no mawari niwa Kutta zyouken kara kimaru ookisa no meguri o motu nagare ga dekiagaru.

Kono gensyou o uzuito-kinzi de toriatukai, uzusou no hattatu no arisama o tuisekisi, heiban no mawari no meguri no zikanteki henka o siraberu no ga kono setu no mokuteki de aru.

Zu 2.1

Ippan ni, z -men ($z = x + iy$) no ten a ni aru, tuyosa κ no uzuito wa, hucusosokudo-potensyal $-i\kappa \log(z - a)$

no nagare o umidasu. Sokode, heiban kara nagare no naka ni deta uzusou o z_j ($j = 1, 2, \dots, n$) ni aru tuyosa κ_j no uzuito de okikaeta to sureba, heiban no nagasa o 2, itiyou-nagare no sokudo o 1, heiban no mukaekaku o α to site, nagare no ba wa tugi no hukusosokudo-potensyal

$$f = \frac{1}{2} e^{-i\alpha} \left(\zeta + \frac{e^{2i\alpha}}{\zeta} \right) - i \sum_{j=1}^n \kappa_j \log \frac{\zeta - \zeta_j}{\zeta - \zeta'_j} \quad (2.1)$$

ni yotte ataerareru. Tadasi

$$z = \frac{1}{2} e^{-i\alpha} \left(\zeta + \frac{1}{\zeta} \right), \quad \zeta'_j = 1/\bar{\zeta}_j$$

de aru. (Zu 2.2). Nao, Kutta zyouken — heiban no usirohasi

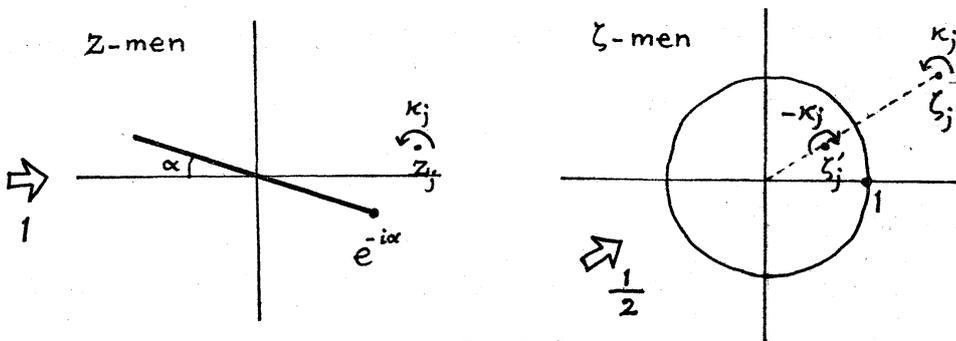
$z = e^{-i\alpha}$ de nagare no sokudo wa yuugen — kara

$$\left(\frac{df}{d\zeta} \right)_{\zeta=1} = 0,$$

sunawati

$$\sin \alpha + \sum_{j=1}^n \kappa_j \left(\frac{1}{1-\zeta_j} - \frac{1}{1-\zeta'_j} \right) = 0 \quad (2.2)$$

to iu zyouken ga kaserarete iru.



Zu 2.2

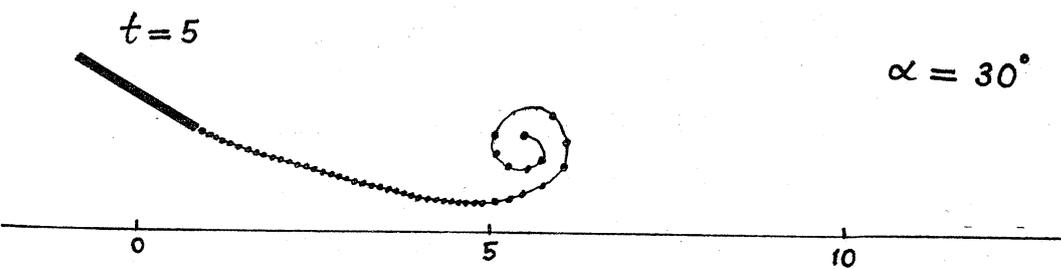
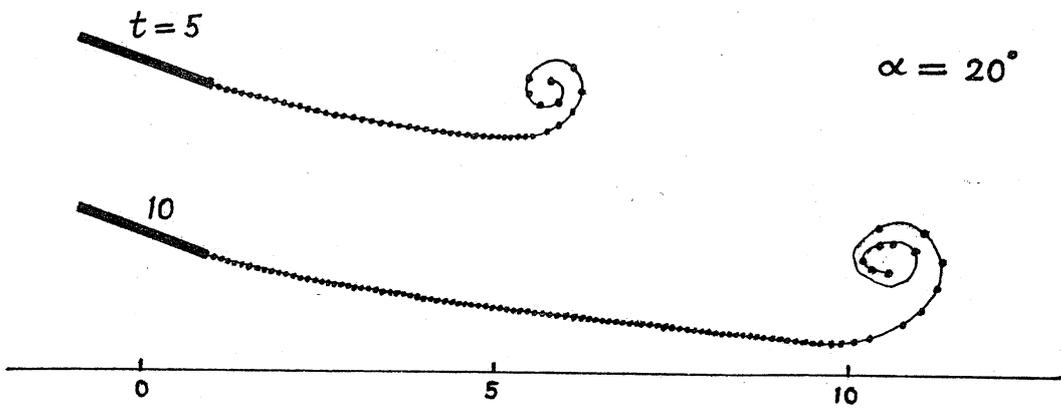
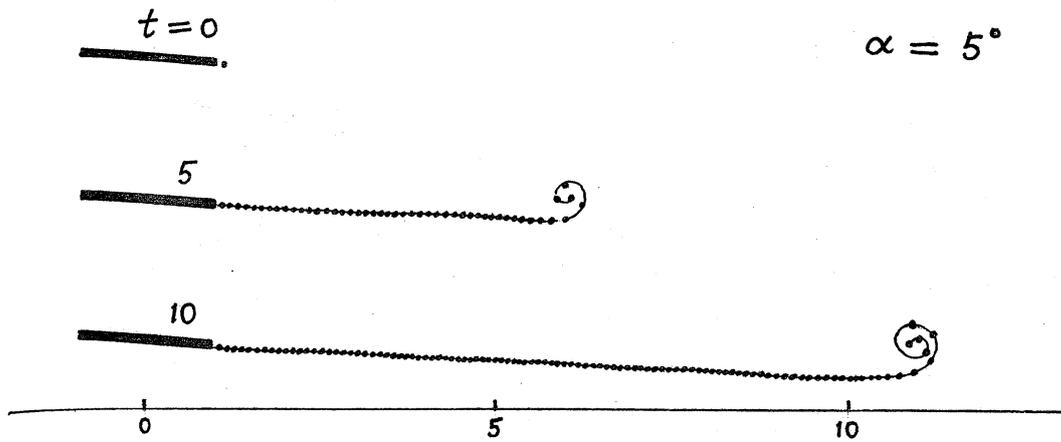
Sate, uzusou ga nagare no naka ni nobite iku katei o, uzuito-kinzi dewa tugi no youni toriatukau koto ga dekiru. Sunawati, tiisai zikan-kankaku Δt gotoni, ittei no iti (tatoeba $z = e^{-i\alpha}(1 + \varepsilon)$, $0 < \varepsilon \ll 1$) ni atarasii uzuito ga arawareru to site, uzuito no kazu o sidaini huyasite iku no de aru. Kono youni sureba, aru zikoku $t = t_n$ ni n -ko no uzuito ga nagare no naka ni aru to sita baai, zikoku $t = t_n + \Delta t$ niwa, uzuito $1 - n$ wa sokudoba $(df/dz)_{t=t_n}$ ni yotte sorezore aru kimatta iti made nagasarete ori, atarasiku arawareru $(n + 1)$ -banme no uzuito no tuyosa κ_{n+1} wa siki (2.2) kara kimaru koto ni naru.

Kono youna tezyun ni sitagatte, $\alpha = 5^\circ, 10^\circ, 20^\circ, 30^\circ$ no baai ni, $\varepsilon = 0.1, 0.05$, $\Delta t = 0.1, 0.05$ nado to erande suutizikken o okonatta. Ikutuka no kekka o Zu 2.3 - 2.4 ni simesu.

Uzusou no makiagatta bubun o nagai zikan no noti made komakaku tusekisuru koto wa dekinai ga, sono bubun o nozoita bubun ni tuite no keisan wa, huantei o okosu koto naku susumerareru. Kono mondai no genmitukai wa sirarete inai node, koko de no keisan wa, uzuito-kinzi ga, uzusou no nobite iku mondai nimo tukau koto ga dekiru koto o teiseiteki ni simesite iru ni todomaru.

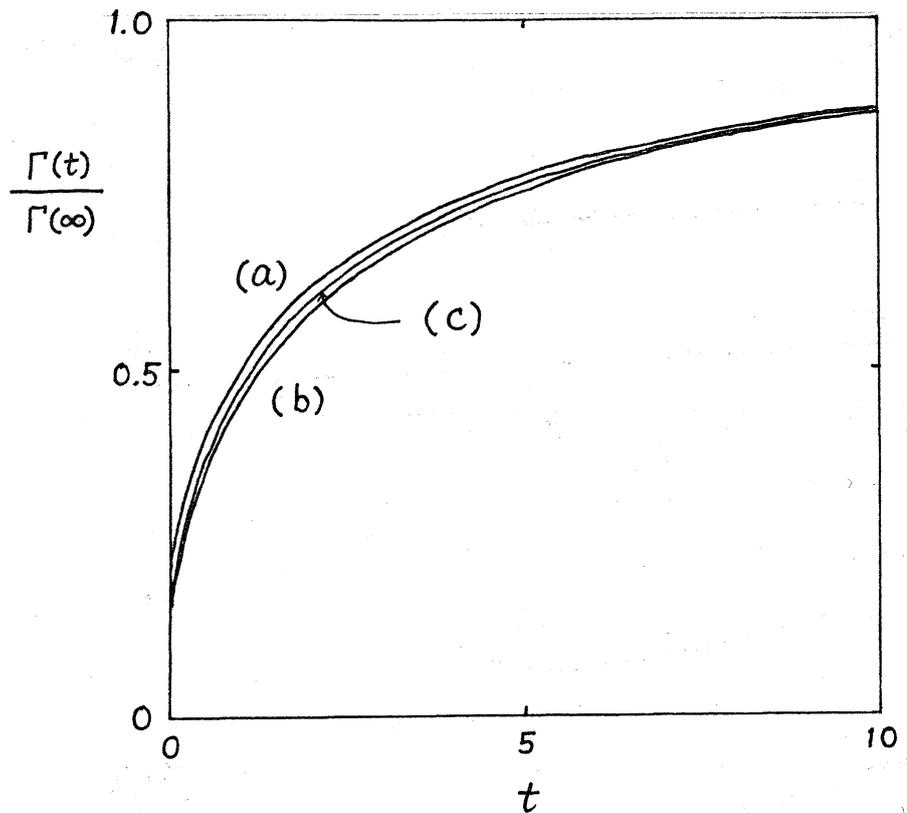
§ 3 Uzukuda no kaiten.

Uzuito-kinzi ni yotte koremade ni nasareta keisan no kekka wa subete, tekitou ni kaisyakusureba, tyokkan aruiwa



Zu 2.3

Mukaekaku α de ugokidasita heiban (nagasa 2) kara deru uzusou. Heiban ni mottomo tikai uzuito wa, heiban no usirohasi kara sono entyou-zyou ni tuneni kyori $\varepsilon = 0.1$ no iti ni aru to suru. Sabunkinzi no zikan-kankaku: $\Delta t = 0.1$.



Zu 2.4

Heiban no mawari ni dekiru meguri no ookisa.

$$\Gamma(t) = \sum \kappa_j, \quad \Gamma(\infty) = 2\pi \sin \alpha.$$

$$(a) \quad \varepsilon = 0.1, \quad \Delta t = 0.1,$$

$$(b) \quad \varepsilon = 0.05, \quad \Delta t = 0.1,$$

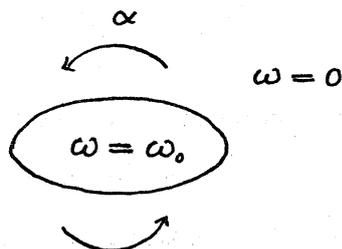
$$(c) \quad \varepsilon = 0.05, \quad \Delta t = 0.05.$$

keiken kara yosousareru mono to yoku ittisite ita to iu koto ga dekiru. Sikasi, sono itti wa akumademo teiseiteki de aru ni suginai. Sokode, mosi genmitukai no sirarete iru nagare ga areba, sore to kuraberu koto ni yotte, uzuito-kinzi no seido no teiryouteki na meyasu o eru koto ga dekiru de arou.

Sate, 2-zigen ni okeru

uzu-undou no genmitukai to site tugi no mono gas sirarete iru (Lamb, 1932). Ima, tyokkaku-zahyou (x, y) o tori, (x, y) -men no naka no daen-ryouiki D:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \quad (0 < b \leq a)$$



Zu 3.1

o kangaeru. Nagare-kansuu o Ψ to site, tugi no youna nagare o kangaeyou:

$$\Psi = \begin{cases} \Psi_{naka} = -\frac{1}{2} \omega_0 (Ax^2 + By^2), & (\text{D no naka}) \\ \Psi_{soto} = -\frac{1}{2} ab \omega_0 \left(\frac{1}{2} e^{-2\xi} \cos 2\eta + \xi \right), & (\text{D no soto}) \end{cases}$$

tadasi

$$x + iy = \sqrt{a^2 - b^2} \cosh(\xi + i\eta)^*$$

* $\xi = \xi_0 \equiv \tanh^{-1}(b/a)$ ga ryouiki D no kyoukai ni taiousuru.

Ψ_{naka} wa uzudo ga ittei ($= \omega_0$) no nagare o, Ψ_{soto} wa uzunasi no nagare o sorezore arawasite iru. Mata, D no kyoukai de Ψ oyobi Ψ no doukansuu wa renzoku de aru. Sikamo, kono Ψ ni yotte arawasareru nagare ga ittan okotta to suru to, uzunasi no ryouiki to uzuari no ryouiki to no kyoukai wa, katati o kaezu ni sonomama ittei no kakusokudo

$$\alpha = \frac{ab}{(a+b)^2} \omega_0 \quad (3.1)$$

de kaitensite iku koto ga simesareru (Zu 3.1).

Kono nagare ni tuite, itiyou na uzudo ω_0 o motu uzukuda o uzuito no retu matawa taba de kinzishite, sono undou o tusekisita kekka o tugi ni nobeyou.

Suutizikken wa, (1) uzusou, (2) daenkei-uzukuda, (3) enkei-uzukuda, no 3-toori no baai ni tuite okonatta.

(1) Uzusou. $(a, b) = (1, 0)$, sunawati, daen ga tubureta baai de aru. Itiyou na uzudo o motu daenkei-uzukuda no kyokugen to site no, nagasa 2 no uzusou o, 1-retu ni naranda n -ko ($n = 41, 81$) no uzuito de kinzishuru. Sono kinzi no sikata ni tuite wa, tugi no 2-toori no houhou o kokoromita: (a) uzusou o hitosii tuyosa no uzuito ni wakeru (sitagatte, uzuito dousi no kankaku wa hitosiku nai), (b) uzusou o hitosii kankaku no uzuito ni wakeru. Mata, keisan o anteikasuru tame ni kari no nensei* o ire, nenseiritu no atai o 0, 0.0005, 0.001, 0.002, nado ni kaete keisan o

* Kyakutyuu wa tugi no peizi ni.

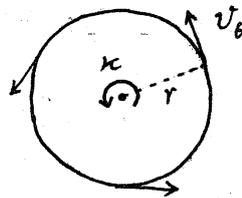
okonatta. Kekka o Zu 3.2 - 3.3 oyobi Hyou 3.1 ni simesu. Zu kara wakaru youni, uzusou no heri ni ataru uzuito no kaiten no okure ga medatu. Heri no tikaku dewa uzuito no bunpu ga mabara ni narisugite iru tame de arou to kangae-rareru. Nao, (a) to (b) no kekka no aida niwa medatta tigi wa mirarenakatta.

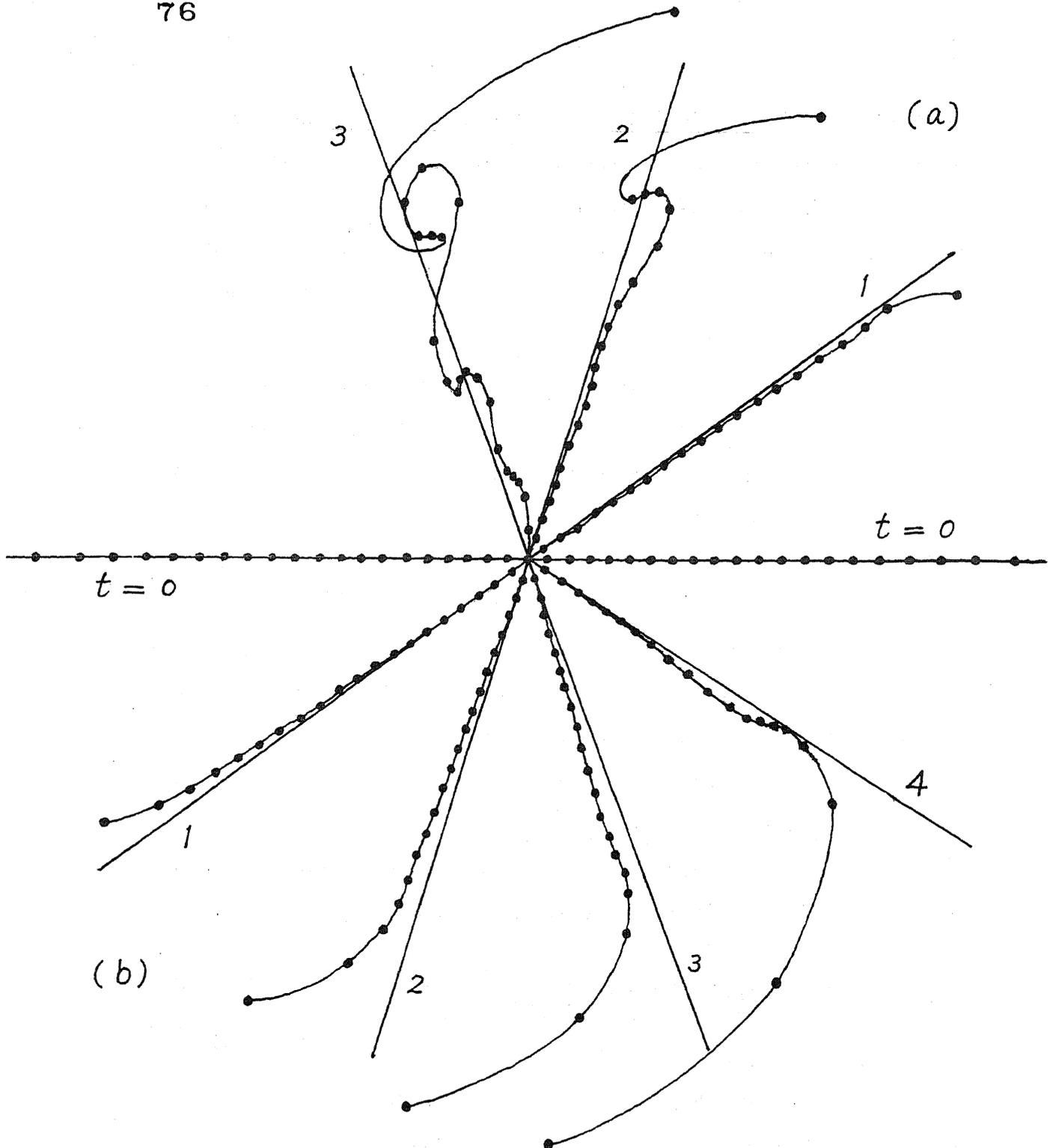
Kono keisan-rei ni oite mo, kari-nensei ga keisan o anteikasuru hataraki wa itizirusii. Sikasi, $\nu > 0$ to toru koto wa uzuito-retu no kaiten o sidaini okuraseru kekka ni naru. Kaitenkaku no okureru wariai $(\theta - \bar{\theta}_k)/\theta$, oyobi 1-ko no uzuito o tyuusin to site nensei no eikyou ga oyonde iru kyori r_0 mo onazi hyou ni simesita. Kono hyou de, tatoeba $n = 81$, $\nu = 0.001$, $t = 2.0$ dewa $(\theta - \bar{\theta}_k)/\theta \lesssim 4\%$, $r_0 \sim 0.09$ de aru. Sunawati, sudeni kono zikoku niwa, 1-ko no uzuito no nensei no eikyou ga 10-ko tikai kazu no uzuito ni oyonde iru. Sorenimo kakawarazu kaiten no okure ga wazuka

*Tuyosa κ no uzuito ni yotte okosareru nagare no sokudo wa $v_\theta = \kappa/r$ de aru ga, sono kawari ni

$$v_\theta = \frac{\kappa}{r} \{ 1 - \exp(-r^2/4\nu t) \}, \quad \nu > 0$$

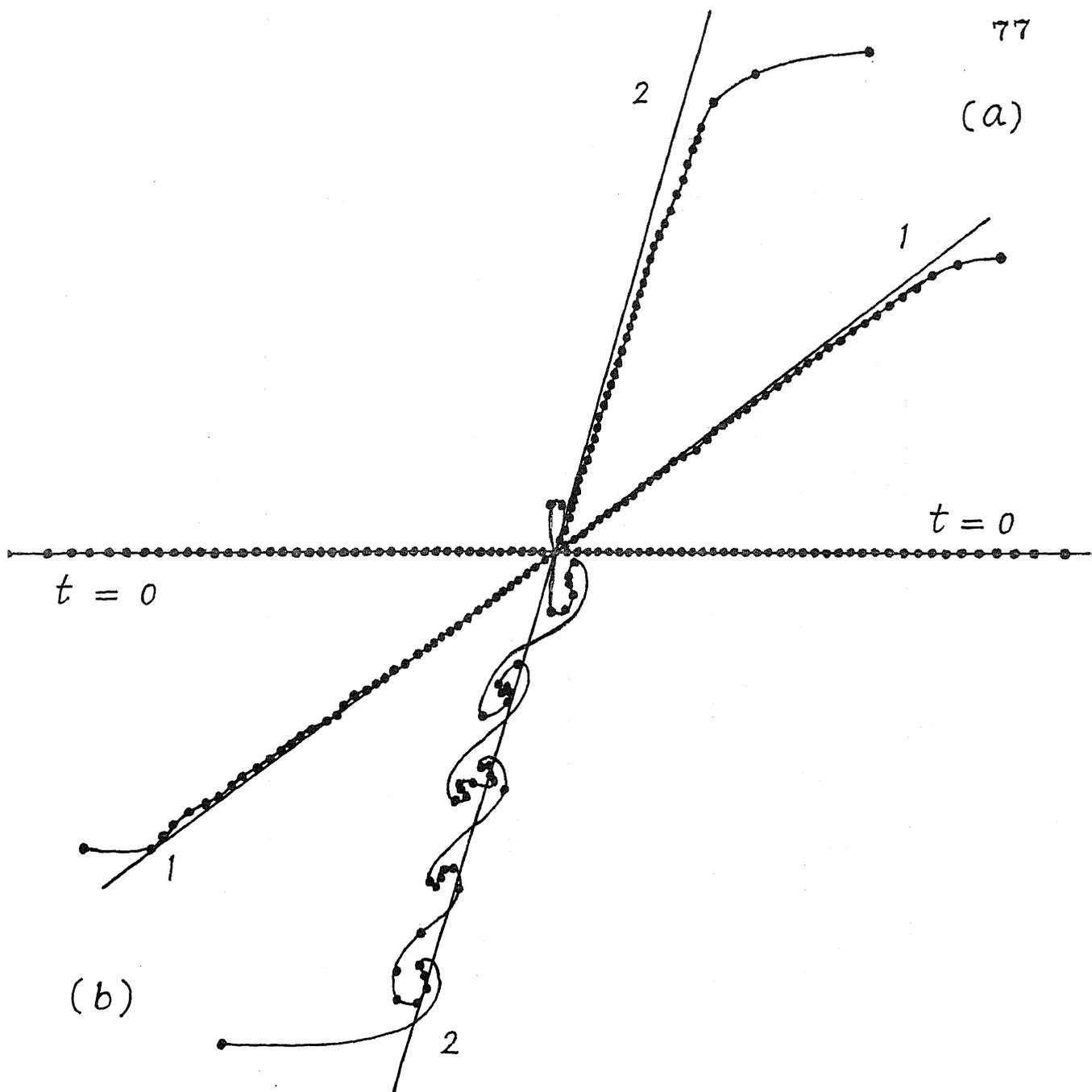
de ataerareru sokudo o motiiru. Uzuito ga tada 1-ko sika nai baai niwa, kono sokudoba wa, nenseiritu ν o motu ryuutai ni tuite no Navier-Stokes houteisiki no genmitukai de aru. ν no atai o tiisai ga zero de naku toru koto ni yotte, suutikeisan o itizirusiku anteikasuru koto ga dekiru (Kuwahara, 1970).





Zu 3.2

Ittei no kakusokudo $\alpha = 36.4756\dots$ ($^{\circ}$ /zikan) de kaiten suru heimenkei-uzusou ni taisuru uzuito-kinzi. $n = 41$; uzusou o hitosii tuyosa no uzuito ni wakeru. (a) $\nu = 0.001$, (b) $\nu = 0.002$.



Zu 3.3

Ittei no kakusokudo $\alpha = 36.4756\dots$ ($^{\circ}/\text{zikan}$) de kaiten suru heimenkei-uzusou ni taisuru uzuito-kinzi. $n = 81$; uzusou o hitosii tuyosa no uzuito ni wakeru. (a) $\nu = 0.001$, (b) $\nu = 0.0005$.

Hyou 3.1

Ittei no kakusokudo $\alpha = 36.4756\dots$ ($^{\circ}$ /zikan) de kaiten suru heimenkei-uzusou ni taisuru uzuuto-kinzi. Uzusou o hitosii tuyosa no uzuuto ni waketa baai; $\Delta t = 0.005$.

n	ν	t	θ	$\bar{\theta}_k$	$\frac{\theta - \bar{\theta}_k}{\theta}$	r_ν
41		0.2	7.295°	7.367°	-1.0%	0.028
	0.001	1.0	36.48	35.57	2.5	0.063
		2.0	72.95	70.26	3.7	0.089
		0.2		7.330	-0.5	0.040
	0.002	1.0		35.12	3.7	0.089
		2.0		69.16	5.2	0.126
81		0.2		7.403	-1.5	0.020
	0.0005	1.0		35.92	1.5	0.045
		0.2		7.375	-1.1	0.028
	0.001	1.0		35.57	2.5	0.063
		2.0		70.26	3.7	0.089

n : uzuuto no kazu.

ν : kari no nenseiritu.

t : kaitenkaku o keisansita zikoku.

θ : genmitukai ni yoru uzusou no kaitenkaku.

$\bar{\theta}_k$: kaitenkaku no heikin (uzuuto 1-ko atari).

r_ν : kari-nensei no eikyou ga oyonde iru kyori ($= \sqrt{4\nu t}$).

4 % ni suginai koto o kangaeru to, kari-nensei o ireru houhou ga, $\nu = 0$ no nagare no keisanhou to site, zituyouteki ni kiwamete manzokusubeki mono de aru to iu koto ga dekiru de arou.

(2) Daenkei-uzukuda. Danmen no kyoukai no katati ga $(a, b) = (1, 0.5)$ no daen de atte, naibu de itiyou na tuyosa ω_0 o motu uzukuda o, hitosii kankaku de naranda hitosii tuyosa no uzuito de okikaeru. Suutikeisan wa, uzuito no kazu $n = 8, 28, 56, 160$ no baai ni tuite, $\omega_0 = 2/\pi$ ($\pi ab\omega_0 = (\pi/2)\omega_0 = n \cdot 2\pi\kappa$ de aru kara $\kappa = 1/(2\pi n)$), $\nu = 0$ to site okonatta. Uzuito no iti ga kawatte iku arisama o Zu 3.4 - 3.7 ni simesu.

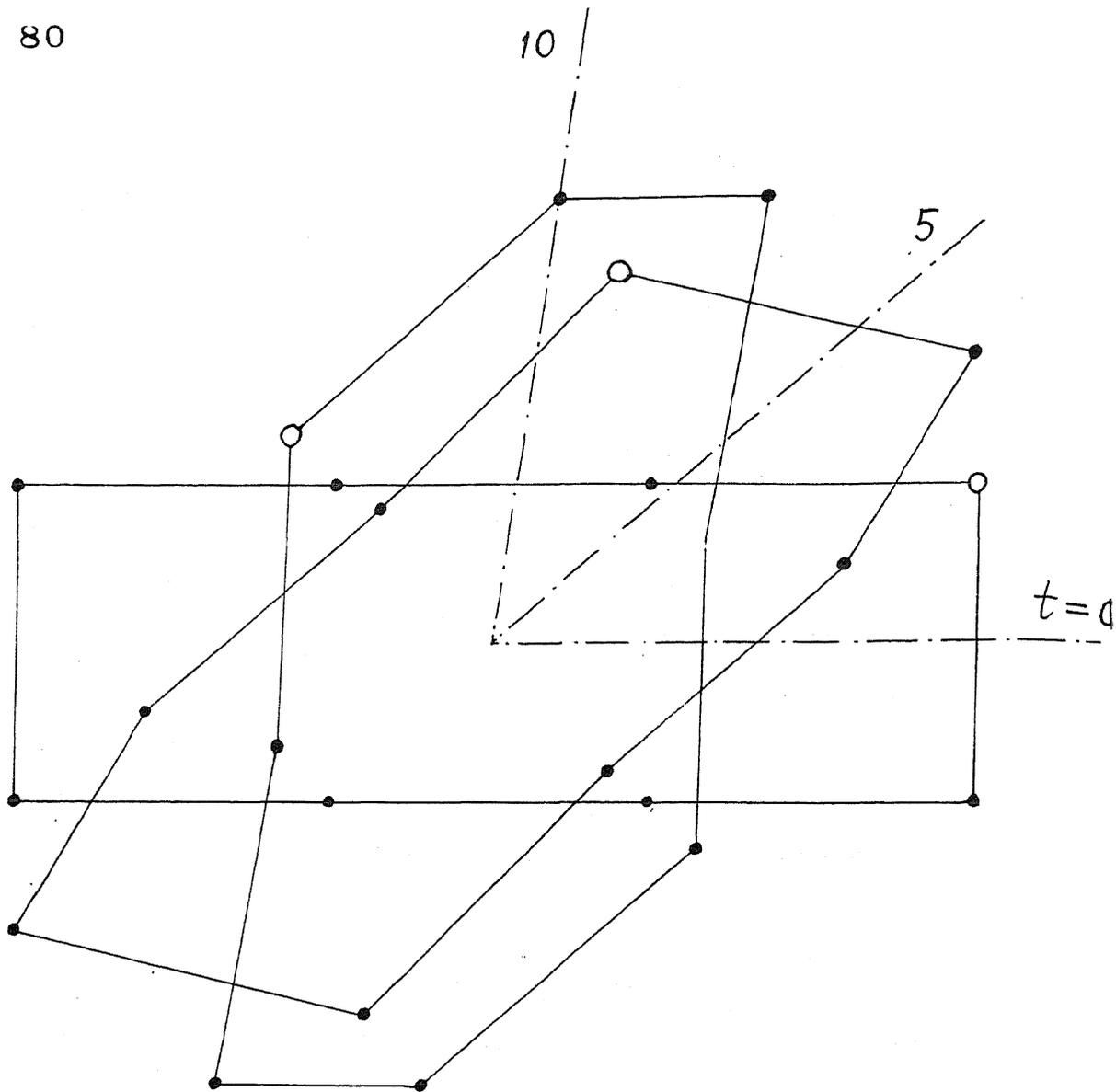
Kyokugen $n \rightarrow \infty$ ni soutousuru, itiyou na tuyosa no daenkei-uzukuda no kyoukaimen no kakusokudo α wa (3.1) ni yotte ataerareru kara, $a = 1, b = 0.5, \omega_0 = 2/\pi$ to site

$$\alpha = 80/\pi^2 = 8.10569\dots \text{ (}^\circ/\text{zikan)}$$

de aru. Keisan no kekka wa, uzuito no kazu o huyasu ni turete nagare no yousu ga kono zyoukyou ni tikazuite iku koto o simesite iru.

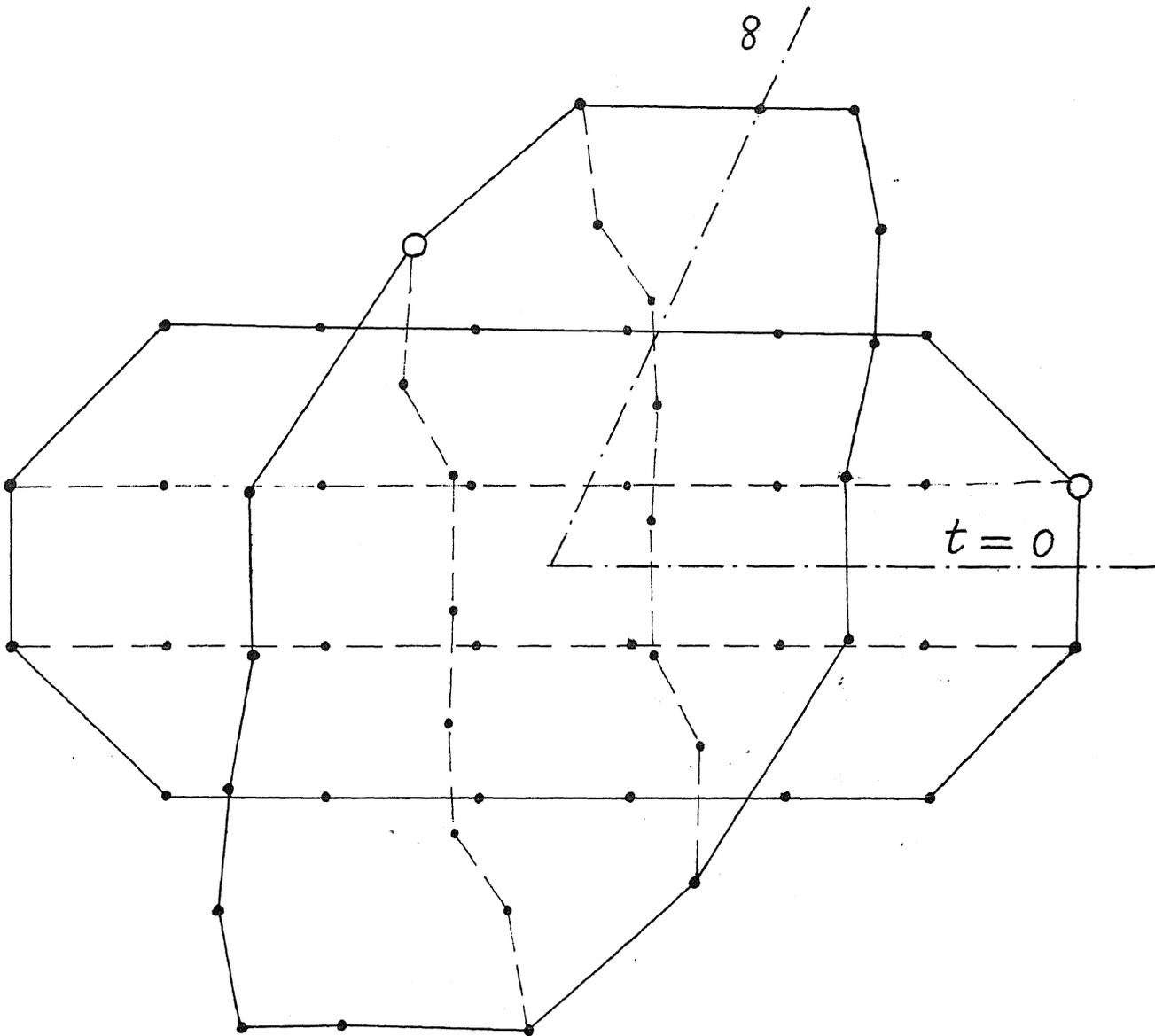
(3) Enkei-uzukuda. Daenkei-uzukuda no baai to mattaku douyouni site, tuyosa ga itiyou na enkei-uzukuda $(a, b) = (1, 1)$ no undou o sirabeta.

Suutikeisan wa, $n = 4, 12, 52, 80, 316, 616, 716, 1264$ no baai ni tuite, $\omega_0 = 2/\pi$ ($\kappa = 1/(\pi n)$), $\nu = 0$ to site okonatta. Kekka o Hyou 3.2 ni simesu.



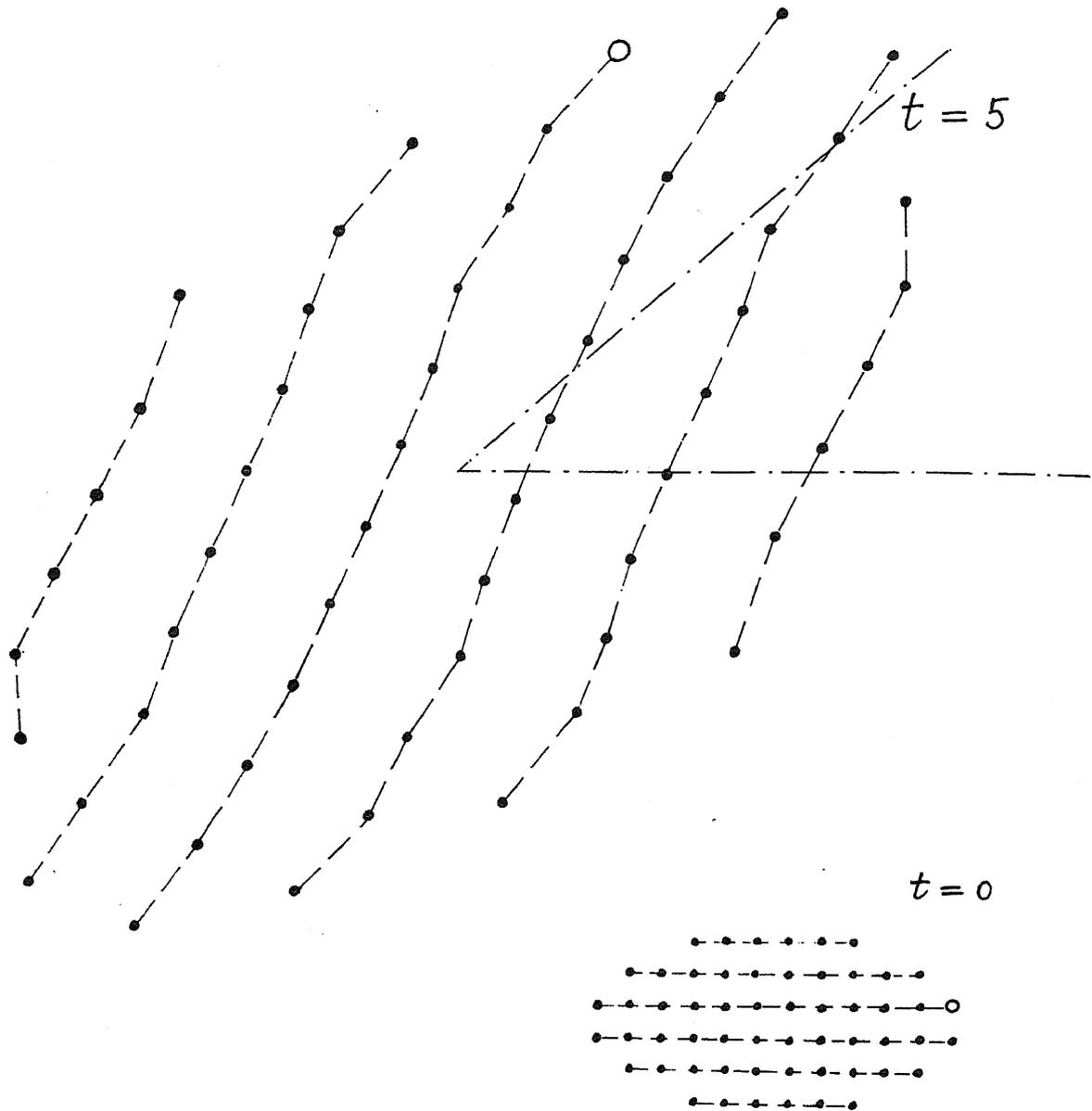
Zu 3.4

Ittei no kakusokudo $\alpha = 8.10569\dots$ ($^{\circ}/\text{zikan}$) de kaiten suru daenkei-uzukuda ni taisuru uzuito-kinzi. Daen no tateyoko-hi wa $1/2$; $n = 8$, $\Delta t = 0.01$.



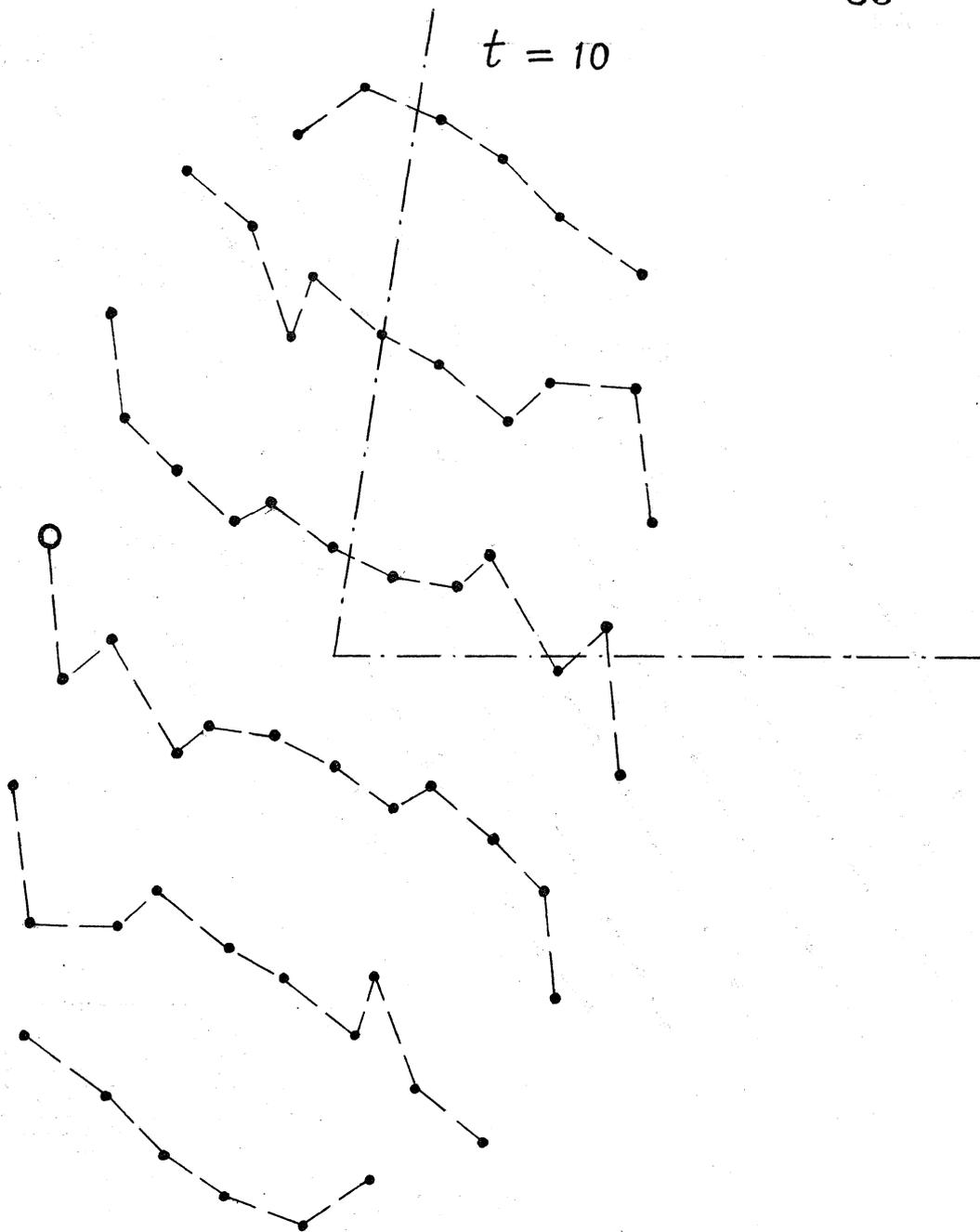
Zu 3.5

Ittei no kakusokudo $\alpha = 8.10569\dots$ ($^{\circ}/\text{zikan}$) de kaiten
 suru daenkei-uzukuda ni taisuru uzuito-kinzi. Daen no
 tateyoko-hi wa $1/2$; $n = 28$, $\Delta t = 0.01$.

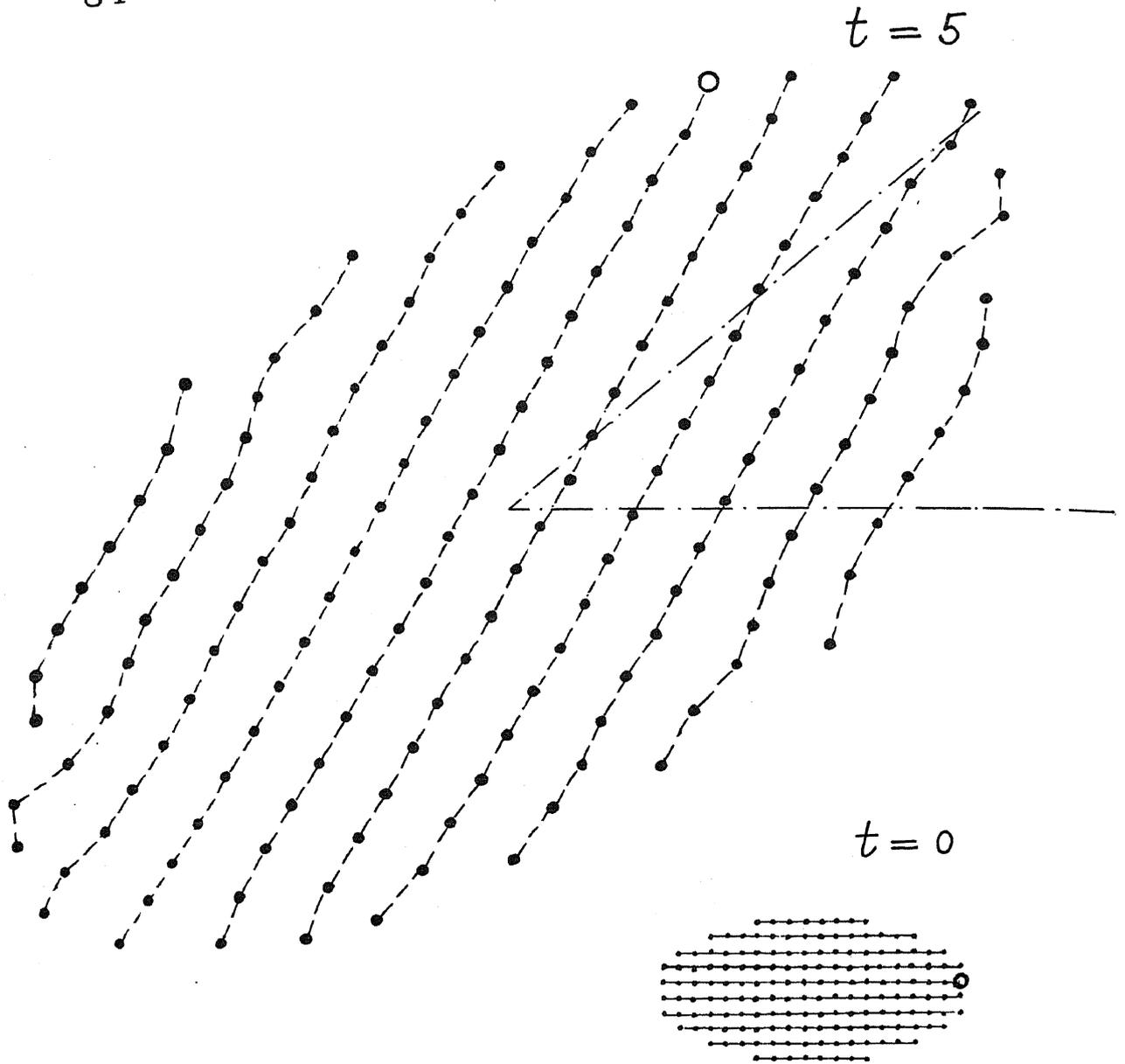


Zu 3.6 (a)

Ittei no kakusokudo $\alpha = 8.10569\dots$ ($^{\circ}$ /zikan) de kaiten suru daenkei-uzukuda ni taisuru uzuito-kinzi. Daen no tateyoko-hi wa $1/2$; $n = 56$, $\Delta t = 0.05$.

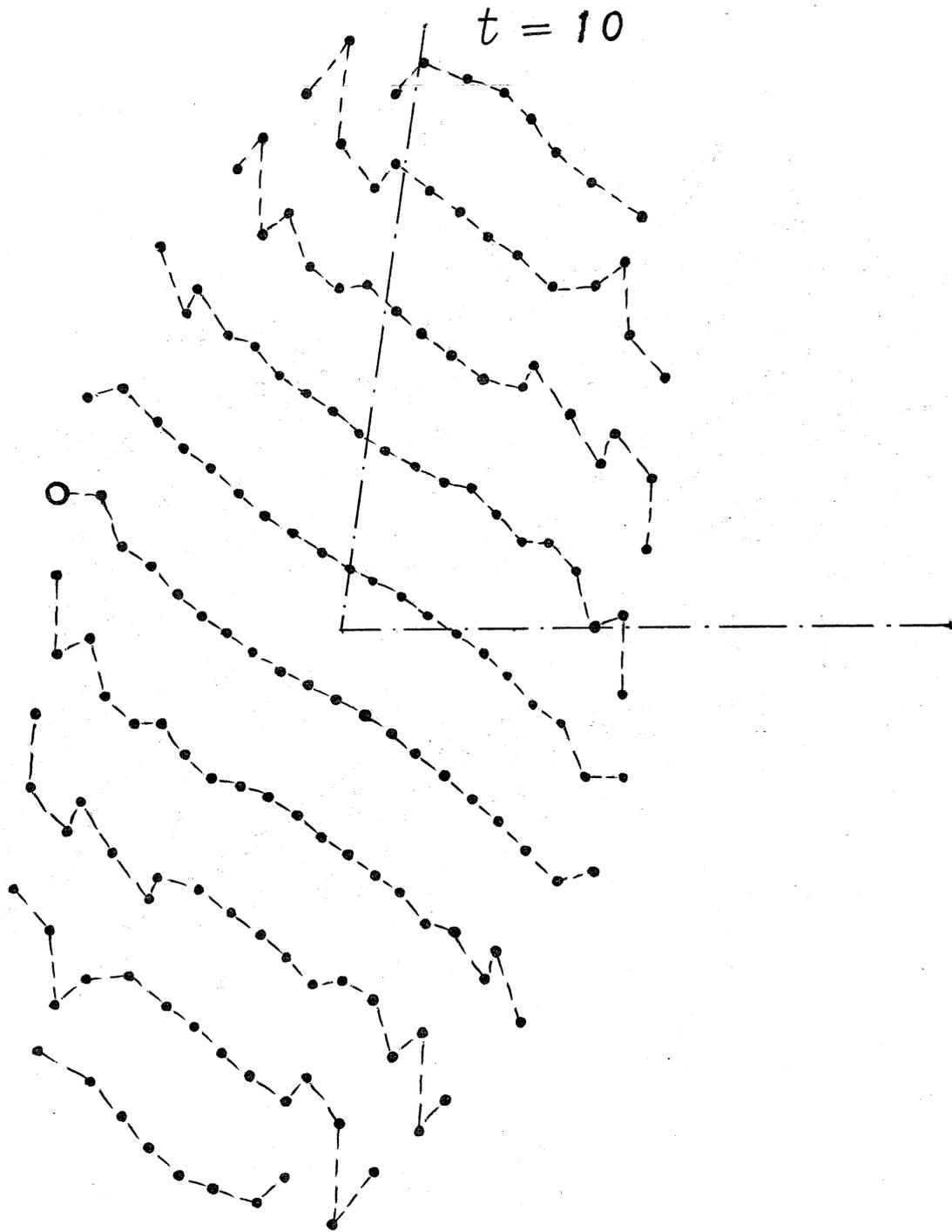


Zu 3.6 (b)

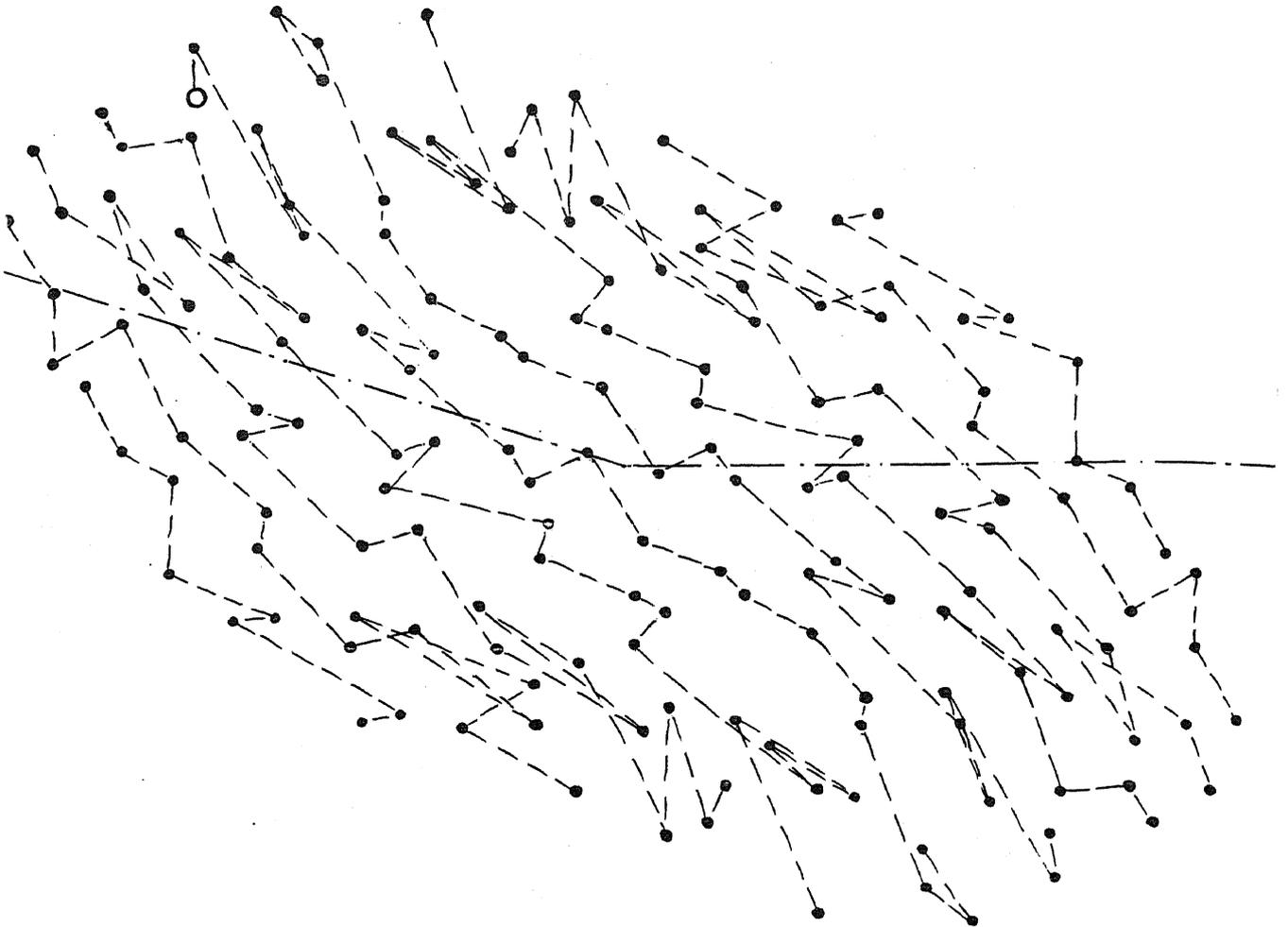


Zu 3.7 (a)

Ittei no kakusokudo $\alpha = 8.10569\dots$ ($^{\circ}/\text{zikan}$) de kaiten suru daenkei-uzukuda ni taisuru uzuito-kinzi. Daen no tateyoko-hi wa $1/2$; $n = 160$, $\Delta t = 0.05$.



Zu 3.7 (b)

$t = 20$ 

Zu 3.7 (c)

Hyou 3.2

Ittei no kakusokudo $\alpha = 9.1189\dots$ (%/zikan) de kaiten suru enkei-uzukuda ni taisuru uzuuto-kinzi ($\nu = 0$).

n	Δt	t	$\bar{\alpha}_h$	α_h	$\frac{\alpha - \alpha_h}{\alpha}$
4	0.1	1.0	6.635 ^{%/zikan}	8.702 ^{%/zikan}	4.6%
	0.01	1.0	6.839	8.707	4.5
	0.001	1.0	6.839	8.707	4.5
12	0.1	1.0	9.270	8.852	2.9
	0.01	0.9	9.280	8.862	2.8
52	0.01	1.0	8.769	9.072	0.5
		5.0	8.765	9.067	0.6
80	0.01	1.0	8.904	9.070	0.5
		5.0	8.899	9.065	0.6
316	0.01	1.0	9.052	9.105	0.2
		3.0	9.050	9.103	0.2
616	0.01	0.1	9.109	9.113	0.07
		1.0	9.108	9.112	0.08
716	0.01	0.2	8.997	9.113	0.07
1264	0.01	0.2	9.062	9.115	0.04

$\bar{\alpha}_h$: zikoku t made no kaitenkaku no heikin (uzuuto 1-ko atari) kara keisansita heikin-kakusokudo.

α_h : uzuuto no simeru yuukou-menseki ga seikaku niwa π ni hitosiku nai koto ni yoru hosei o $\bar{\alpha}_h$ ni hodokosita mono. Sunawati, uzuuto no kankaku o h to sureba

$$\alpha_h = (nh^2/\pi) \bar{\alpha}_h.$$

Uzukuda no kakusokudo (enkei-danmen no baai niwa uzukuda zentai ga goutaiteki ni kaiten o okonau) wa

$$\alpha = 90/\pi^2 = 9.11890\dots \quad (^\circ/\text{zikan})$$

de aru. Uzuito no kazu \mathcal{N} o masu ni turete, ue no genmitukai no atai ni kiwamete tikai atai ga erareru koto ga wakaru. Tadasi, kono baai nimo, (1) no toki to douyou ni, heri no tikaku ni aru uzuito no kaiten wa okureru.

§4 Musubi.

Heiban no usirohasi kara deru uzusou no hattatu, oyobi daenkei-danmen o motu uzukuda no kaiten, to iu hutatu no uzu-undou o uzuito-kinzi o motiite suutiteki ni sirabeta. Uzusou ga buttai kara wakidasite iku youna baai nimo uzuito-kinzi ga sonomama tukaeru koto ga simesareta. Mata, genmitukai to kuraberu koto ni yotte, uzuito-kinzi no seido no teiryouteki na meyasu ga erareta. Douzini, kari no nensei o ireru keisanhou no yuukou na koto ga akiraka ni natta.

Nao, kono keisan no daibubun wa Toukyou Daigaku Oogata Keisanki Sentaa oyobi Toukyou Daigaku Utyuu-Koukuu Kenkyuuzyo no HITAC 5020E o motiite okonatta. Mata, uzusou no makiagari no keisan no itibu wa, wareware no hitori ga, Kyouto Daigaku Suurikaiseki Kenkyuuzyo de no tanki-kenkyuu no aida ni onazi kenkyuuzyo no TOSBAC-3400 o motiite okonatta mono de aru.

Bunken

- 1) Abernathy, F. H. and R. E. Kronauer: The formation of vortex streets.
J. Fluid Mech. 13 (1962) 1-20.
- 2) Kuwahara, K.: Vortex sheet no makiagari ni tuite.
Kyouto Daigaku Suurikaiseki Kenkyuuzyo Koukyuuroku 101 (1970) 40-43.
- 3) Lamb, H.: Hydrodynamics (sixth edition), Cambridge University Press, 1932, p.232.
- 4) Rosenhead, L.: The formation of vortices from a surface of discontinuity.
Proc. Roy. Soc. A 134 (1931) 170-192.
- 5) Westwater, F. L.: The rolling up of the surface of discontinuity behind an aerofoil of finite span.
British A. R. C. Reports and Memoranda No. 1692 (1936) 116-131.