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Supplement to 'Minimal class generated by open compact and perfect mappings'

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This is a supplementary note on my lecture[4] at Kyoto Symposium on Topological Spaces, Dec. 2, 1971. A space is called a weak p-space if it is regular($T_1 + T_3$) and has a sequence $\{U_i\}, i=1,2,\ldots$, of open coverings of $X$ satisfying:

If $x \notin U_1 \notin \bigcup_i U_i$, then $i \leq n \Rightarrow \bigcap_{i=1}^{\infty} U_i \text{ is compact and}$

$i_1) \bigcap_{i=1}^{\infty} U_i \subset U$ with $U$ open implies $\bigcap_{i=1}^{\infty} U_i \subset \bigcap_{i=1}^{n} U_i \subset U$

for some $n$. This sequence $\{U_i\}$ is called a defining one.

This condition was introduced by Burke[2], Theorem 1.3, where he proved that a completely regular space is a p-space in the sense of Arhangel'skiî[1], Definition 5, if and only if it is a weak p-space. However the space $T$ in Engelking[3], Example 2.4.4, p.35, is not a p-space but a weak p-space, which proof is left to the reader. Since every Moore space is evidently a weak p-space, the class of weak p-spaces offers a class containing all p-spaces and all Moore spaces. The property to be a weak p-space is inherited under the operations taking $G_\delta$ sets, closed sets, perfect preimages and countable products.

The aim of this paper is to note that wherever 'p-spaces' are in [4] is replaced by 'weak p-spaces'. That can easily be verified with the aid of the following Lemmas 1 and 2 and
with trivial minor changes of the original proofs in [4]. For undefined terminologies refer to [4].

**LEMMA 1.** Let $X$ be a weak p-space with a derining sequence

\[ \{ U_i : a \in A_i \} : i=1,2, \ldots \] \[ \cup_{i=1}^{\infty} U_{ia} \neq \emptyset. \]

Let $B_i$ be a finite subset of $A_i$ and $\varphi_i^{i+1}: B_{i+1} \rightarrow B_i$ a transformation such that $\varphi_i^{i+1}(a) = \beta$ implies $U_{i+1, a} \subset U_{i\beta}$ and such that $\langle a \rangle \in \text{inv} \lim \{ B_i \}$.

Then $K$ is compact and $\{ U_i \}$ forms a neighborhood base of $K$ in $X$.

**Proof.** It suffices to consider the case: $K \neq \emptyset$. Let $\mathcal{F}$ be a maximal filter of subsets of $K$. Set

\[ C_i = \{ a \in B_i : U_{ia} \cap K \in \mathcal{F} \}. \]

Then $C_i \neq \emptyset$ and $\{ C_i \}$ forms an inverse subsystem of $\{ B_i \}$.

Pick an element $\langle a \rangle$ from $\text{inv} \lim C_i$. Set

\[ L = \bigcap_{i=1}^{\infty} U_{ia}. \]

Then $L$ is a non-empty compact set with $L \subset K$. Let $F$ be an arbitrary element of $\mathcal{F}$. Assume $F \cap L = \emptyset$. Then $F \cap U_{i\alpha_j} = \emptyset$ for some $j$, a contradiction. Thus a point of $L$ adheres $\mathcal{F}$, which proves that $K$ is compact.

To prove $\{ U_i \}$ forms a neighborhood base of $K$ in $X$ assume the contrary. Let $U$ be an open set of $X$ with $K \subset U$ and with $U_i - U \neq \emptyset$ for any $i$. Set

\[ D_i = \{ a \in B_i : U_{ia} - U \neq \emptyset \}. \]

Then $D_i \neq \emptyset$ and $\{ D_i \}$ forms an inverse subsystem of $\{ B_i \}$. 

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Pick an element \( \langle \beta_i \rangle \) from \( \text{inv lim } D_i \). Set
\[
M = \bigcap_{i=1}^{\infty} U_i \beta_i.
\]
Since \( M \subset K \), then \( U_k \beta_k \subset U \) for some \( k \), a contradiction. The proof is finished.

**LEMMA 2.** A weak p-space \( X \) is of countable type.

**Proof.** Let \( \mathcal{Q} \) be a non-empty compact set of \( X \) and \( \{ \mathcal{U}_i \} \) a defining sequence of open coverings of \( X \). Let \( \mathcal{U}_i = \{ U_{i\alpha} : \alpha \in A_i \} \) be an open covering of \( X \) such that i) \( \mathcal{U}_i \) refines \( \mathcal{V}_i \), ii) \( \mathcal{U}_{i+1} \) refines \( \mathcal{U}_i \), and iii) all but a finite number of elements of \( \mathcal{U}_i \) do not meet \( \mathcal{Q} \). Then \( \{ \mathcal{U}_i \} \) is also a defining one. Let \( \varphi_i^{i+1} : A_i+1 \to A_i \) be a transformation such that
\[
\varphi_i^{i+1}(a) = \beta \implies \overline{U_{i+1},a} \subset U_i \beta_i.
\]
Set
\[
B_i = \{ \alpha \in A_i : U_{i\alpha} \cap \mathcal{Q} \neq \emptyset \}.
\]
Then \( B_i \neq \emptyset \) and \( \{ B_i \} \) forms an inverse subsystem of \( \{ A_i ; \varphi_i^{i+1} \} \).
Since the condition of Lemma 1 is satisfied, \( K \) defined in Lemma 1 is a compact set of countable character. Since \( \mathcal{Q} \subset K \), \( X \) is of countable type and the proof is finished.

**References**

