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# Supplement to 'Minimal class generated by open compact and perfect mappings'

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This is a supplementary note on my lecture[4] at Kyoto Symposium on Topological Spaces, Dec.2, 1971. A space is called a <u>weak p-space</u> if it is regular($T_1 + T_3$) and has a sequence $\mathcal{U}_i$, i=1,2,···, of open coverings of X satisfying: If $x \in U_i \in \mathcal{U}_i$, i=1,2,···, then i) $\bigcap_{i=1}^{\infty} \overline{U_i}$ is compact and ii) $\bigcap_{i=1}^{\infty} \overline{U_i} \subset U$ with U open implies $\bigcap_{i=1}^{\infty} \overline{U_i} \subset \bigcap_{i=1}^{n} \overline{U_i} \subset U$ for some n. This sequence $\{\mathcal{U}_i\}$ is called a <u>defining</u> one. This condition was introduced by Burke[2], Theorem 1.3, where he proved that a completely regular space is a p-space in the sense of Arhangel'skiǐ[1], Definition 5, if and only if it is a weak p-space. However the space T in Engelking[3], Example 2.4.4, p.85, is not a p-space but a weak p-space, which proof is left to the reader. Since every Moore space is evidently a weak p-space, the class of weak p-spaces offers a class containing all p-spaces and all Moore spaces. The property to be a weak p-space is inherited under the operations taking $G_\delta$ sets, closed sets, perfect preimages and countable products. The aim of this paper is to note that wherever 'p-spaces' are in [4] is replaced by 'weak p-spaces'. That can easily be verified with the aid of the following Lemmas 1 and 2 and with trivial miner changes of the original proofs in [4]. For undefined terminologies refer to [4].

LEMMA 1. *Let* $X$ *be a weak p-space with a defining sequence* $\{ \mathfrak{U}_i = \{ U_{i\alpha} : \alpha \in A_i \} : i=1,2,\cdots \}$. *Let* $B_i$ *be a finite subset of* $A_i$ *and* $\varphi_i^{i+1} : B_{i+1} \longrightarrow B_i$ *a transformation such that* $\varphi_i^{i+1}(\alpha) = \beta$ *implies* $\overline{U_{i+1,\alpha}} \subset U_{i\beta}$ *and such that* $\langle \alpha_i \rangle \in \text{inv lim} \{ B_i ; \varphi_i^{i+1} \}$ *implies* $\bigcap_{i=1}^{\infty} U_{i\alpha_i} \neq \emptyset$. *Set*

$$U_i = \cup \{ U_{i\alpha} : \alpha \in B_i \} , \quad K = \cap U_i .$$

*Then* $K$ *is compact and* $\{ U_i \}$ *forms a neighborhood base of* $K$ *in* $X$.

Proof. It suffices to consider the case: $K \neq \emptyset$. Let $\mathfrak{F}$ be a maximal filtre of subsets of $K$. Set

$$C_i = \{ \alpha \in B_i : U_{i\alpha} \cap K \in \mathfrak{F} \}.$$

Then $C_i \neq \emptyset$ and $\{ C_i \}$ forms an inverse subsystem of $\{ B_i \}$. Pick an element $\langle \alpha_i \rangle$ from inv lim $C_i$. Set

$$L = \bigcap_{i=1}^{\infty} U_{i\alpha_i} .$$

Then $L$ is a non-empty compact set with $L \subset K$. Let $F$ be an arbitrary element of $\mathfrak{F}$. Assume $\overline{F} \cap L = \emptyset$. Then $\overline{F} \cap U_{j\alpha_j} = \emptyset$ for some $j$, a contradiction. Thus a point of $L$ adheres $\mathfrak{F}$, which proves that $K$ is compact.

To prove $\{ U_i \}$ forms a neighborhood base of $K$ in $X$ assume the contrary. Let $U$ be an open set of $X$ with $K \subset U$ and with $U_i - U \neq \emptyset$ for any $i$. Set

$$D_i = \{ \alpha \in B_i : U_{i\alpha} - U \neq \emptyset \}.$$

Then $D_i \neq \emptyset$ and $\{ D_i \}$ forms an inverse subsystem of $\{ B_i \}$.

Pick an element $\langle \beta_i \rangle$ from inv lim $D_i$. Set

$$M = \bigcap_{i=1}^{\infty} U_{i\beta_i}.$$

Since $M \subset K$, then $U_{k\beta_k} \subset U$ for some k, a contradiction. The proof is finished.

LEMMA 2. <u>A weak p-space X is of countable type</u>.

Proof. Let Q be a non-empty compact set of X and $\{ \mathfrak{V}_i \}$ a defining sequence of open coverings of X. Let $\mathfrak{U}_i = \{ U_{i\alpha} : \alpha \in A_i \}$ be an open covering of X such that i) $\mathfrak{U}_i$ refines $\mathfrak{V}_i$, ii) $\overline{\mathfrak{U}_{i+1}}$ refines $\mathfrak{U}_i$, and iii) all but a finite number of elements of $\mathfrak{U}_i$ do not meet Q. Then $\{ \mathfrak{U}_i \}$ is also a defining one. Let $\varphi_i^{i+1} : A_{i+1} \longrightarrow A_i$ be a transformation such that $\varphi_i^{i+1}(\alpha) = \beta$ implies $\overline{U_{i+1,\alpha}} \subset U_{i\beta}$. Set

$$B_i = \{ \alpha \in A_i : U_{i\alpha} \cap Q \neq \emptyset \}.$$

Then $B_i \neq \emptyset$ and $\{ B_i \}$ forms an inverse subsystem of $\{ A_i ; \varphi_i^{i+1} \}$. Since the condition of Lemma 1 is satisfied, K defined in Lemma 1 is a compact set of countable character. Since $Q \subset K$, X is of countable type and the proof is finished.

## References

[1] A. V. Arhangel'skiĭ, On a class of spaces containing all metric and all locally compact spaces, Dokl. Akad. Nauk SSSR 151(1963)751-754; Soviet Math Dokl. 4(1963)1051-1055.

[2] Dennis K. Burke, On p-psaces and w$\Delta$-spaces, Pacific J. Math. 35(1970)285-296.

[3] R. Engelking, 'Outline of general topology', North-Holland

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[4] K. Nagami, Minimal class generated by open compact and perfect mappings, forthcoming.