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Note on shape theory

Yukihiro Kodama

Department of Mathematics, Tokyo University of Education

§1. Shape of compacta.

In [2], [3] K.Borsuk introduced the notion of shapes of metric compacta. Let X and Y be compacta lying in the Hilbert cube Q ($= \prod_{n=1}^{\infty} I_n$, I_n a copy of the interval I = [0,1], $n \in N$, where N is the set of positive integers). A sequence $\underline{f} = \{f_n\}$ of maps (= continuous maps) $f_n : Q \to Q$, $n \in N$, is said to be a <u>fundamental sequence</u> of X to Y if for each neighborhood V of Y (in Q) there is a neighborhood U of X such that $f_n|U \simeq f_{n+1}|U$ in V for almost all n, that is, there is a homotopy $H : U \times I \to V$ such that $H(x,0) = f_n(x)$ and $H(x,1) = f_{n+1}(x)$ for $x \in U$. We write $\underline{f} : X \to Y$. Setting $i_n(x) = x$ for each $x \in Q$, for each compactum $X \subset Q$ $\underline{i}_X = \{i_n\} : X \to X$ is a fundamental sequence which is called the <u>fundamental identity sequence</u> of X.

Two fundamental sequences $\underline{f}, \underline{g} : X \to Y$ are said to be <u>homotopic</u> if for each neighborhood V of Y there is a neighborhood U of X such that $f_n|U \simeq g_n|U$ in V for almost all n. We denote it by $\underline{f} \simeq \underline{g}$. The collection of all fundamental sequences homotopic to a given fundamental sequence $\underline{f}$ is said to be the fundamental class with the representative $\underline{f}$ and it is denoted by $[\underline{f}]$.

The composition $\underline{h} = \underline{gf} : X \to Z$ of fundamental sequences $\underline{f} : X \to Y$ and $\underline{g} : Y \to Z$ is defined as the fundamental sequence consisting of maps $h_n = g_n f_n : Q \to Q$. If $\underline{f} \simeq \underline{f}' : X \to Y$ and $\underline{g} \simeq \underline{g}' : Y \to Z$, then $\underline{gf} \simeq \underline{g}'\underline{f}' : X \to Z$.

Each map $f : X \to Y$ defines a fundamental sequence $\underline{f} : X \to Y$ as follows. Take any extension $h : Q \to Q$ of f and put $f_n = h$ for each n. Then $\underline{f} = \{f_n\}$ is a fundamental sequence of X to Y. We call $\underline{f}$ the fundamental sequence induced by f.

Proposition 1. *Let X and Y be 0-dimensional compacta. Then every fundamental sequence $\underline{f} : X \to Y$ is induced by a map $f : X \to Y$ and f is uniquely determined by $\underline{f}$.*

Proof. Let $\underline{f} = \{f_n\} : X \to Y$. From the definition of a fundamental sequence and the compactness of Q the sequence $\{f_n(x)\}$, $x \in X$, converges some point $f(x)$ of Y. Obviously the correspondence $x \to f(x)$, $x \in X$, defines a map $f : X \to Y$ and it induces $\underline{f}$.

Compacta X and Y in Q are said to be fundamentally equivalent if there exist two fundamental sequences $\underline{f} : X \to Y$ and $\underline{g} : Y \to X$ such that $\underline{gf} \simeq \underline{i}_X$ and $\underline{fg} \simeq \underline{i}_Y$. Then we write $X \simeq_F Y$. If we assume only that the relation $\underline{gf} \simeq \underline{i}_X$ holds, then we say that Y fundamentally dominates X and we write $Y \geq_F X$. If X and Y are homeomorphic, then $X \simeq_F Y$, because if $\underline{f} : X \to Y$ and $\underline{g} : Y \to X$ are fundamental sequences induced by f and $g = f^{-1}$ then

$\underline{gf} \simeq \underline{i}_X$ and $\underline{fg} \simeq \underline{i}_Y$. Also, if X and Y are homotopically equivalent, then $X \simeq_F Y$.

It is known that the relation of the fundamental equivalence and the relation of the fundamental domination have an absolute character, that is, they do not depend on locations of compacta X and Y in Q. Since the relation of the fundamental equivalence is equivalence relation, the class of all compacta decomposes into mutually disjoint classes of compacta, called _shapes_. We denote by Sh(X) the class containing X and we call it the _shape_ of X. Also we write $Sh(X) \geq Sh(Y)$ if $X \geq_F Y$. If X and Y are ANR's (= compact ANR's for metric spaces), then it is known that $Sh(X) \geq Sh(Y)$ if and only if X dominates homotopically Y and S(X) = Sh(Y) if and only if X and Y have same homotopy type. The shape of a space consisting of only one point is said to be _trivial_ and denote by Sh(1). If X is contractible, then it is obvious Sh(X) = Sh(1).

Let X be a compactum contained in a compactum Y. A fundamental sequence $\underline{f} = \{f_n\} : Y \to X$ is said to be a _fundamental retraction_ if $\underline{f}\underline{j}_X \simeq \underline{i}_X$, where $\underline{j}_X : X \to Y$ is a fundamental sequence induced by the inclusion $j_X : X \subset Y$. If there is a fundamental retraction $\underline{f} : Y \to X$, then we call X a _fundamental retract_ of Y. If there is a fundamental retraction $\underline{f} : Y \to X$ such that $\underline{f} \simeq \underline{i}_Y$, then X is a _fundamental deformation retract_ of Y. A compactum X is said to be a _fundamental absolute retract_ (a _fundamental absolute neighborhood retract_), written as FAR (FANR), if X is a fundamental retract of an AR (an ANR).

The following theorem characterizes a compactum with trivial shape ([4],[8],[18]).

Theorem 1. (K.Borsuk, D.M.Hyman, S.Mardešić) _For a compactum X the followings are equivalent._

(1) _X is of trivial shape._

(2) _X is an FAR._

(3) _For a certain imbedding_ $X \subset Q$ _there is a sequence_ $\{X_n\}$ _of neighborhoods of X such that each_ X_n _is homeomorphic to_ Q, $X_{n+1} \subset \text{Interior } X_n$, $n \in N$, _and_ $\bigcap_n X_n = X$.

For a compactum X, Borsuk defined Fd(X), the _fundamental dimension_ of X, as the minimum of dimensions of all compacta Y with $Sh(Y) \geq Sh(X)$:

$$Fd(X) = \min_{Sh(Y) \geq Sh(X)} \dim Y$$

Obviously it holds that if $Sh(X) \leq Sh(Y)$ then $Fd(X) \leq Fd(Y)$ and if X and Y are compacta and $Y \neq \emptyset$ then $Fd(X) \leq Fd(X \times Y) \leq Fd(X) + Fd(Y)$.

Let X be a compactum. A closed subset Y of X is said to be a _fundamental_ k-_skeleton_ of X if $\dim Y \leq k$ and the homomorphisms : $\check{H}_n(Y:G) \rightarrow \check{H}_n(X:G)$ and $\underline{\pi}_n(Y,y_0) \rightarrow \underline{\pi}_n(X,y_0)$ induced by the inclusion $(Y,y_0) \subset (X,y_0)$, y_0 a point of Y, are isomorphisms for $0 \leq n < k$ and an epimorphism for $n = k$, where $\check{H}_n(X:G)$ is the n-dimensional Čech homology group of X with coefficients in G and $\underline{\pi}_n(X,y_0)$ is the n-dimensional fundamental group of (X,y_0) defined by Borsuk [3].

We do not know whether every compactum has a fundamental 0-skeleton or not. If X is a solenoid of Van Dantzig, then X has a fundamental 0-skeleton which homeomorphic to a Cantor discontinuum. (See Corollary of Theorem 5).

§2. Approach to shapes by Mardesic and Segal.

By an ANR-sequence we imply an inverse sequence $\underline{X} = \{X_n, \pi_{n n+1}\}$ over the set of positive ingers N, where X is an ANR and $\pi_{n n+1}$: $X_{n+1} \to X_n$ is a map, $n \in N$ ($\pi_{nm} = \pi_{n n+1} \cdots \pi_{m-1 m}$ for $n < m$). Let $X = \varprojlim \underline{X}$ and let $\pi_n : X \to X_n$ be the projection. A map $\underline{f} : \underline{X} \to \underline{Y} = \{Y_n, \mu_{n n+1}\}$ consists of an increasing function $f : N \to N$ and of a collection of maps $f_n : X_{f(n)} \to Y_n$ such that

$$f_n \pi_{f(n) f(n')} \simeq \mu_{nn'} f_{n'} \qquad \text{for } n \leq n',\ n, n' \in N.$$

Two maps $\underline{f}, \underline{g} : \underline{X} \to \underline{Y}$ are said to be homotopic, $\underline{f} \simeq \underline{g}$, if for each $n \in N$ there is an $n' \in N$, $n' \geq f(n), g(n)$, such that

$$f_n \pi_{f(n) n'} \simeq g_n \pi_{g(n) n'} .$$

The composite $\underline{gf} : \underline{X} \to \underline{Z}$ of $\underline{f} : \underline{X} \to \underline{Y}$ and $\underline{g} : \underline{Y} \to \underline{Z} = \{Z_n, \nu_{n n+1}\}$ is a map of sequences $\underline{h} : \underline{X} \to \underline{Z}$, where $h = fg : N \to N$ and $h_n = g_n f_{g(n)} : X_{fg(n)} \to Z_n$. The identity map of sequences $\underline{i_X} : \underline{X} \to \underline{X}$ is given by the identity $1_N : N \to N$ and the map $i_{X_n} : X_n \to X_n$ $n \in N$. Two compacta X and Y are said to be of the same shape in the sense of ANR-systems, written as $\overline{Sh}(X) = \overline{Sh}(Y)$, provided there exist ANR-systems $\underline{X}$ and $\underline{Y}$ with $X = \varprojlim \underline{X}$ and $Y = \varprojlim \underline{Y}$ and maps $\underline{f} : \underline{X} \to \underline{Y}$ and $\underline{g} : \underline{Y} \to \underline{X}$ such that $\underline{gf} \simeq \underline{i_X}$ and $\underline{fg} \simeq \underline{i_Y}$.

Mardešić and Segal [15,16] gave the following useful characterization of shapes.

Theorem 2. (S.Mardešić and J.Segal) Let X and Y be compacta. Then $Sh(X) = Sh(Y)$ if and only if $\overline{Sh}(X) = \overline{Sh}(Y)$.

§ 2. Shape of decomposition spaces.

According to Borsuk [4,p.266], a compactum X is said to be approximatively k-connected if for a certain imbedding $X \subset Q$ and for every neighborhood V of X in Q there is a neighborhood U of X such that every map of a k-sphere S^k into U is null homotopic in V. It is known that the approximative k-connectedness is the shape invariant.

Theorem 3. (Kodama) Let f be a map of a compactum X onto a compactum Y with $\dim Y \leq n$ such that for each $y \in Y$ $f^{-1}(y)$ is approximatively k-connected, $k = 0,1,..,n$. Then $Sh(X) \geq Sh(Y)$. Moreover, if $\dim X \leq n$ then $Sh(X) = Sh(Y)$.

In the proof of Theorem 3 ([11]) an argument in the proof of Theorem of [9] is used essentially.

The following corollary is a generalization of Borsuk [4, Theorem (6.1)].

Corollary 1. An n-dimensional compactum X is of trivial shape if and only if X is approximatively k-connected for $k = 0,1,..,n$.

For the proof it is sufficient to apply Theorem 3 to the case where Y is a space consisting of one point.

Corollary 2. (R.B.Sher) If X and Y are finite dimensional and f is a map of X onto Y such that $f^{-1}(y)$ is of trivial shape

This is an immediate consequence of Theorems 1 and 3.

For a compactum X, denote by $\square(X)$ the set of all components of X. We consider $\square(X)$ as the decomposition space of X. Then it is a compactum. As an application of Theorem 3, we obtain the following theorem by Borsuk [3, Theorem (8.1)].

Corollary 3. (Borsuk) *Let* X, Y *be compacta in* Q. *Then for every fundamental sequence* $\underline{f} : X \to Y$ *there is a unique (continuous) map* $\Lambda_f : \square X \to \square Y$ *such that for each component* X_0 *of* X $\underline{f} : X_0 \to \Lambda_f(X_0)$ *is a fundamental sequence. Moreover* Λ_f *depends only on the fundamental class* $\underline{f}$ *and this dependence is covariant, that is, if* $\underline{g} : Y \to Z$ *is a fundamental sequence then* $\Lambda_{gf} = \Lambda_g \Lambda_f$.

Proof. Let $\pi_X : X \to \square X$ and $\pi_Y : Y \to \square Y$ be the decomposition maps. Since $\pi_X^{-1}(x)$ is a continuum for each $x \in \square X$, it is approximatively 0-connected. Since $\dim \square X = 0$, by Theorem 3 there is a fundamental sequence $\underline{h} : \square X \to X$ such that $\underline{\pi}_X \underline{h} \simeq \underline{i}_{\square X}$. Consider $\underline{\pi}_Y \underline{fh} : \square X \to \square Y$. By Proposition 1, $\underline{\pi}_Y \underline{fh}$ is induced by a map $\Lambda_f : \square X \to \square Y$. It is obvious that Λ_f satisfies Corollary 3.

The following generalizes Sher [21, Theorem 12] and it is given by a similar method as in a proof of Theorem 3 (cf. [11]).

Corollary 4. *Let* (X, x_0) *and* (Y, y_0) *be pointed compacta. Let* f *be a map of* (X, x_0) *onto* (Y, y_0). *If* $f^{-1}(y)$ *is approximatively* k-*connected for each* $y \in Y$ *and* $k = 0, 1, .., n$, *then the induced homomorphism* $f_* : \underline{\pi}_k(X, x_0) \to \underline{\pi}_k(Y, y_0)$ *is an isomorphism for* $k = 0, 1, .., n$, *where* $\underline{\pi}_k$ *is the* k-*dimensional fundamen-*

tal group of Borsuk [3].

Corollary 5. *Let f be a map of a compactum X onto an n-dimensional compactum Y such that $f^{-1}(y)$ is approximatively k-connected for each $y \in Y$ and $k = 0,1,..,n$. Then $Fd(X) \geq Fd(Y)$.*

4. Δ-spaces and fundamental dimension.

A compactum X is said to be a *Δ-space* if there is an inverse sequence $\{K_n, \pi_{n\,n+1}\}$ of finite simplicial complexes such that $X = \varprojlim \{K_n\}$ and each bonding map $\pi_{n\,n+1}: K_{n+1} \to K_n$ is simplicial.

Theorem 4. (Kodama) (1) *Every 0-dimensional compactum and every finite polytope are Δ-spaces.*

(2) *There is a 1-dimensional AR with property (Δ) which is not a Δ-space.*

(3) *Every Δ-space is dimensionally full-valued for paracompact spaces* (cf. [14]).

In the shape category every compactum has a Δ-space as its representative as shown by the following.

Theorem 5. (Kodama) *For each compactum X there is a Δ-space X' such that $Sh(X) = Sh(X')$ and $Fd(X) = Fd(X')$.*

Corollary. *For every compactum X there is a compactum X' such that X' contains X as a fundamental deformation retract and X' has a fundamental k-skeleton for each $k = 0,1,2,..$.*

We only give a proof of Corollary. For a given compactum X, find a Δ-space Y by Theorem 5 such that $Sh(X) = Sh(Y)$. By Moszyńska [20] there is a compactum X' such that both X and Y are fundamental deformation retracts of X'. Let $\{K_n, \pi_{n n+1}\}$ be an inverse sequence of finite simplicial complexes such that $Y = \varprojlim \{K_n\}$ and each $\pi_{n n+1}$ is simplicial. Denote by K_n^i the i-skeleton of K_n, $i = 0,1,\ldots$. Then $\{K_n^i, \pi_{n n+1}\}$ forms an inverse sequence. Put $Y_i = \varprojlim \{K_n^i\}$, $i = 0,1,\ldots$. Then it is obvious that Y_i is a fundamental i-skeleton of X'.

As shown in the above, every Δ-space has a fundamental k-skeleton for each $k = 0,1,2,\ldots$. On the other hand, consider the 2-dimensional continuum $Q(\sigma)$ constructed in [10,p.390]. It is easy to know that $Q(\sigma)$ has no fundamental 1-skeleton. Also, we can see that every ANR has a fundamental i-skeleton for $i =$ 0, 1. The following example is a trivial modification of the example constructed by Borsuk [1].

Example. There is an infinite dimensional ANR X which does not have a fundamental k-skeleton for each $k = 2,3,\ldots$.

To find such an ANR X, let S^2 be a 2-sphere and let A be an arc in S^2. Take a map f from A onto the Hilbert cube Q and let X be the adjunction space obtained by S^2, Q and f. Then X is an infinite dimensional ANR. If X_k is a fundamental k-skeleton of X for $k \geq 2$, then X_k has to contain a subset S^2-A of X. Since S^2-A is dense in X, we have $X_k = X$. Thus there is no fundamental k-skeleton of X, $k = 2,3,\ldots$.

Proposition 2. *If* X *is a compactum in an* n-*dimensional euclidean space* R^n, *then* $Fd(X) \leq n - 1$.

Proof. Take a sequence $\{K_k\}$ of triangulable neighborhoods of X in R^n such that $K_{k+1} \subset K_k$, $k = 1,2,\ldots$, and $\bigcap_k K_k = X$. Since K_k is an n-dimensional polyhedron in R^n, there is a subplyhedron L_k of K_k such that L_k is a strong deformation retract of K_k and $\dim L_k \leq n-1$. By induction, we can find a simplicial subdivision $\tilde{L}_k$ of L_k and a simplicial map $\pi_{kk+1}: \tilde{L}_{k+1} \to \tilde{L}_k$ such that $j_k \pi_{kk+1} \simeq i_{k+1} j_{k+1} | \tilde{L}_{k+1}$ in K_k for $k = 1,2,\ldots$, where $j_k : L_k \to K_k$, $j_{k+1} : L_{k+1} \to K_{k+1}$ and $i_{k+1} : K_{k+1} \to K_k$ are the inclusions. Consider the inverse sequence $\{\tilde{L}_k, \pi_{kk+1}\}$ and $X' = \varprojlim \tilde{L}_k$. It is known by Theorem 2 that $Sh(X) = Sh(X')$. Since $\dim X' \leq n-1$, we know $Fd(X) \leq n-1$.

Let $\mathcal{C} = \{X_\alpha | \alpha \in \Lambda\}$ be a collection of compacta. A member X_0 of $\mathcal{C}$ is said to be <u>majorant</u> for the shapes of members of $\mathcal{C}$ if $Sh(X_0) \geq Sh(X_\alpha)$ for each $X_\alpha \in \mathcal{C}$. For example, let $\mathcal{C}$ be the collection of all 0-dimensional compacta Y such that $Sh(Y) \leq Sh(X)$ for a given compactum X. Then the decomposition space $\square X$ of X consisting of all components of X is majorant for the shapes of members of $\mathcal{C}$. This follows from Corollary 3 of Theorem 3.

Proposition 3. (Watanabe) <u>For the collection $\mathcal{R}$ of all compacta in R^1 a Cantor discontinuum is majorant for the shapes of members of $\mathcal{R}$.</u>

This is a consequence of Proposition 2.

Theorem 6. (1) (S.Spież) <u>There is a compactum in R^2 which is majorant for the shapes of all compacta in R^2.</u>

(2) (K.Borsuk and W.Holsztyński) <u>For the collection of</u>

<u>all solenoids</u> $\mathcal{T}$ <u>no compactum</u> X_0 <u>satisfies the condition</u> $Sh(X) \leq Sh(X_0)$ <u>for every</u> $X \in \mathcal{T}$. <u>Hence, if</u> $\mathcal{C}$ <u>is the collection of all compacta in</u> R^3, <u>then there is no compactum which is majorant for the shapes of members of</u> $\mathcal{C}$.

Problem 1. Let X be a compactum and let $\mathcal{C}_X$ be the collection of all compacta Y such that $Sh(X) > Sh(Y)$. Does there exist a compactum which is majorant for the shapes of members of $\mathcal{C}_X$?

The following problem is raised by Borsuk [3].

Problem 2. (Borsuk) Let X and Y be compacta. If $Fd(Y) > 0$, then does it hold $Fd(X \times Y) \geq Fd(X) + 1$?

It is likely true that the following holds. However it does not know yet.

Problem 3. For every compactum X, does it hold that $Fd(X \times S^1) = Fd(X) + 1$? Here S^1 is a 1-sphere.

§5. Movable compacta.

According to Borsuk [3,5], a compactum X in Q is said to be <u>movable</u> if for each neighborhood U of X there is a neighborhood V of X such that for every neighborhood W of X there is a homotopy $H : V \times I \to U$ satisfying the condition:

$$H(x,0) = x \quad \text{and} \quad H(x,1) \in W \quad \text{for each } x \in V.$$

A compactum X is said to be k-<u>movable</u> if for every neighborhood U of X there is a neighborhood V of X such that for every compactum $A \subset V$ with $\dim A \leq k$ and for every neighborhood W of X there is a homotopy $H : A \times I \to U$ satisfying the condition:

$H(x,0) = x$ and $H(x,1) \in W$ for $x \in A$.

Mardešić and Segal [17] gave a characterization of movable compacta in terms of ANR sequences.

Theorem 7. (Mardešić and Segal) A compactum X is movable if and only if there is an ANR sequence $\{X_n, \pi_{n\,n+1}\}$ satisfying the following condition: $X = \varprojlim \{X_n\}$ and for each $n \in N$ there is an n', $n' \geq n$, such that every $n'' \geq n$ there is a map $\mu_{n''n'}: X_{n'} \to X_{n''}$ satisfying the homotopy relation $\mu_{n''n'} \pi_{nn''} \simeq \pi_{nn'}$.

For movable compacta, the followings are known.

Theorem 8. (Borsuk) (1) Let X and Y be compacta with $Sh(X) \geq Sh(Y)$. If X is movable (k-movable), then Y is movable (k-movable).

(2) If X is movable (k-movable), then the suspension ΣX of X is movable (k-movable).

(3) Every compactum in R^2 is movable.

(4) If X_i is a movable compactum for $i = 1,2,\ldots$, then $\prod_i X_i$ is movable.

(5) Every FANR is movable.

Theorem 9. (Kodama and Watanabe) An n-dimensional and n-movable compactum is movable.

Theorem 10. (1) (Mardešić) An n-dimensional LC^{n-1} compactum is movable.

(2) (Borsuk) An LC^{n-1} compactum is n-movable.

Let X be a Δ-space. As we know from the proof of Corollary of Theorem 5, for each $k = 0,1,\ldots$, there is a fundamental k-skeleton X_k of X. It is easy to see X_k is i-movable for i = 0,1,..,k-1, if X is movable.

Problem 4. Let X be a movable Δ-space. For each k = 1, 2,.. , does there exist a fundamental k-skeleton X_k of X which is movable ?

K.Borsuk[5] raised the following problems:

(1) Is it true that if X is m-movable and Y is n-movable then $X \times Y$ is (m+n)-movable ?

(2) Does there exist, for each n = 1,2,.. , a continuum which is n-movable, but is not (n+1)-movable ?

(3) Does there exist a non-movable compactum which is n-movable for every n = 1,2,.. ?

These were solved by Kodama and Watanabe [12].

Theorem 11. (Kodama and Watanabe) (1) For compacta X and Y, $X \times Y$ is k-movable if and only if both X and Y are k-movable.

(2) If X is k-movable, then ΣX is (k+1)-movable.

(3) There is a continuum X such that X is k-movable for every k = 1,2,.. , but not movable.

To show (3) of Theorem 11, we remark that there is a non-movable continuum X_0 such that ΣX_0 is homeomorphic to X_0 [7]. Since an n-fold suspension of a compactum X is (n-1)-movable by (2) of Theorem 11, the continuum X_0 mensioned above is k-movable for every k = 1,2,.. . Borsuk proved every solenoid is not 1-movable. It is known that a suspension of a solenoid is 1-movable but not 2-movable.

It is known that every 2-dimensional ANR is dimensionally full-valued (cf.[14]).

Problem 5. Is every 2-dimensional movable compactum dimensionally full-valued ?

Let X be a compactum with metric d. Let K be a finite simplicial complex and let V(K) be the set of vertices of K. For a map $f : V(K) \to X$, we mean by mesh f the maximum of diameters of $f(s \cap V(K))$ for every simplex s of K. Let $\epsilon > 0$. For maps $f,g : V(K) \to X$ with max (mesh f,mesh g) $< \epsilon$, by $f \sim_{\epsilon} g$ we imply that there is a sequence of maps $h_i : V(K) \to X$, $i = 0,1,..,n$, such that $f = h_o$, $g = h_n$, mesh $h_i < \epsilon$, $i = 0,1,..,n$, and $\max \{d(h_i(v),h_{i+1}(v)) : v \in V(K)\} < \epsilon$, $i = 0,1,..,n-1$.

Proposition 4. <u>A compactum</u> X <u>is movable if and only if for every</u> $\epsilon > 0$ <u>there is a</u> $\delta > 0$ <u>satisfying the following conditions</u>: <u>For every finite simplicial complex</u> K, <u>every map</u> $f : V(K) \to X$ <u>with</u> mesh $f < \delta$ <u>and every</u> $\nu > 0$ <u>there is a subdivision</u> K' <u>of</u> K <u>and a map</u> $g : V(K') \to X$ <u>such that</u> mesh $g < \nu$ <u>and</u> $f\pi \sim_{\epsilon} g$, <u>where</u> $\pi : V(K') \to V(K)$ <u>is a map defined by any projection of</u> K' <u>to</u> K.

This proposition gives a simple proof of Theorem 10. In a similar form to Proposition 4 we can obtain a necessary and sufficient condition for a compactum X in order that X be an FANR.

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