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Optimum State-Regulator of Time-Lag System

by

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The optimum solution of the following problem will be studied:

Time-Lag System;

$$\frac{dx(t)}{dt} = Ax(t) + Bx(t-\theta) + Cu(t), \qquad (1)$$
$$\theta = (\theta_1, \theta_2, \cdots \theta_n)'$$

Performance Index;

$$J(u) = \frac{1}{2}\int_{T-\theta}^{T} \| x(t)\|^2 dt + \frac{h}{2}\int_0^T \| u(t)\|^2 dt \qquad (2)$$

This is usually called as the state-regulator problem.

At first, for the state vector $\mathcal{X}(t,\tau)$,

$$\mathcal{X}(t,\tau) = x(t+\tau\theta), \qquad -1 < \tau < 0 \qquad (3)$$

the system (1) will be transformed into the following equivalent partial differential equation:

$$\left.\begin{aligned} &\frac{\partial \mathcal{X}(t,\tau)}{\partial t} = \Theta \frac{\partial \mathcal{X}(t,\tau)}{\partial \tau}, \quad \Theta = \underset{i}{\mathrm{diag}}\ \frac{1}{\theta_i}, \\ &\Theta \frac{\partial \mathcal{X}(t,\tau)}{\partial \tau}\Big|_{\tau=0} = A\mathcal{X}(t,0) + B\mathcal{X}(t,-1) + Cu(t), \end{aligned}\right\} \qquad (4)$$

and the input-state relation will be written explicitly by the semi-group. Furthermore the performance index (2) will be written equivalently as follows:

$$J(u) = \frac{1}{2}\| \mathcal{X}(T,\cdot)\|^2 + \frac{h}{2}\int_0^T \| u(t)\|^2 dt, \qquad (5)$$

Then the optimum input for the system (4) is obtained uniquely in terms of its initial value and time-derivative by Functional Analysis approach.

The results obtained in this paper are different from Pontriyagin's one which depends on $x(t)$, $x(t-\theta)$ and Maximum Principle.

Ref.[4] is the extension of this paper.

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1. Introduction

In this paper the state-regulator problem of the linear time-lag system is studied. Since the state-space of the time -lag system is the infinite-dimensional space, the problem considered in this paper is different from the usual output -regulator problem, and by Functional Analysis approach the solution of it will be obtained very clearly.

In Sec. 2 the time-lag system will be transformed into its equivalent partial differential system in order to show explicitly that the state-space of the time-lag system in the function space, that is, the infinite-dimensional space. This partial differential equation is the first-order hyperbolic one. After this transformation is done, the input-function becomes the so -called boundary input function which exists in the boundary condition of the partial differential equation.

The input-state relation of this partial differential system with boundary input has not yet been described with the semi -group which characterizes the free mation of the system. Therefore the boundary input function will be transformed into its equivalent spatially distributed input function, and then the input-state relation can be described in the convolution form of the semi-group and input-function. Since it is necessary to differentiate the input function when the boundary input function

is transformed into its equivalent spatially distributed input function, the family of input functions will be restricted to differentiable functions.

In Sec. 3 the optimum regulator input will be obtained. The representation of optimum input is obtained in terms of its initial value and its time derivative.

2. System and Input-State Relation

The systems are described by the following linear differential -difference equations:

$$\left.\begin{array}{l} \dfrac{dx}{dt} = Ax(t) + Bx(t-\theta) + Cu(t), \\ x(t);\ n\times 1,\quad u(t);\ r\times 1, \\ x(t-\theta) = [x_1(t-\theta_1), x_2(t-\theta_2), \cdots, x_n(t-\theta_n)]',\ \theta_i \geq 0 \\ A;\ n\times n,\quad B;\ n\times n,\quad C;\ n\times r \end{array}\right\} \qquad (1)$$

These are called commonly time-lag systems in control engineering, $x(t-\theta)$ are the outputs of time-lag elements and u(t) is the control vector.

Generally not the all θ_i, $i=1,2,\cdots,n$, are positive. Therefore it is supposed that if $\theta_k = 0$, the kth row of B is put into 0 and θ_k into $\max_{1\leq i\leq n} \theta_i$.

The following system can be given as an example.

The actual system;

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t-2) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad (2)$$

The transformed system; $\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(t-2) \\ x_2(t-2) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$, (3)

In the following of this section, the input-state relation of the system (1) will be derived. This is the preparation for the optimization of the system (1).

The solution of Eq. (1) is assured to exist uniquely in $t \in (t_0, \infty)$ for the forcing function $u(t_0, \infty)$ and the initial function $x_0(\cdot)$[1]

$$x_0(\cdot) = \begin{bmatrix} x_{01}(\theta_1\tau), & \tau \in (-1, 0] \\ x_{02}(\theta_2\tau), & \tau \in (-1, 0] \\ \vdots & \\ x_{0n}(\theta_n\tau), & \tau \in (-1, 0] \end{bmatrix}, \tag{4}$$

so the state of the system (1) at t_0 which parametrizes the bejavior of it in (t_0, ∞) can be taken as the vector function (4) defined on $(-1, 0]$.

From this view point, the Eq. (1) is considered as the mapping on the function space $X(-1, 0)$ to itself, and can be changed to the more convinient one, that is, partial differential equation, for control problem.

The function $x(t, \tau)$ is defined on $\tau \in (-1, 0]$ as follows:

$$x(t, \tau) = \begin{bmatrix} x_1(t+\theta_1\tau) \\ x_2(t+\theta_2\tau) \\ \vdots \\ x_n(t+\theta_n\tau) \end{bmatrix}, \quad \tau \in (-1, 0], \tag{5}$$

where $(x_1(t+\theta_1\tau), x_2(t+\theta_2\tau), \cdots, x_n(t+\theta_n))$ is the solution of the vector differential equation (1).

Then from Eq. (5), the partial differential equation

$$\frac{\partial \mathcal{X}(t,\tau)}{\partial t} = \frac{\partial}{\partial t}\begin{bmatrix} x_1(t+\theta_1\tau) \\ x_2(t+\theta_2\tau) \\ \vdots \\ x_n(t+\theta_n\tau) \end{bmatrix}$$

$$= \Theta \frac{\partial \mathcal{X}(t,\tau)}{\partial \tau} \quad , \quad \Theta = \operatorname*{diag}_i \frac{1}{\theta_i} \qquad (6-1)$$

can be obtained.

Futhermore since $x(t) = \mathcal{X}(t,0)$, $x(t-\theta) = \mathcal{X}(t,-1)$,

and $\frac{dx(t)}{dt} = \frac{\partial \mathcal{X}(t,\tau)}{\partial t}\Big|_{\tau=0} = \Theta \frac{\partial \mathcal{X}(t,\tau)}{\partial \tau}\Big|_{\tau=0}$,

then the boundary condition

$$\Theta \frac{\partial \mathcal{X}(t,0)}{\partial \tau} = A\mathcal{X}(t,0) + B\mathcal{X}(t,-1) + Cu(t) \quad , \qquad (6-2)$$

is derived.

The dynamical system (6) is the one which has the so-called boundary control.

The linear operator $\mathfrak{A}$ is defined as follows:

$$\mathfrak{A}\phi(\tau) = \psi(\tau), \quad \tau \in (-1,0] \quad , \qquad (7)$$

implies that

$$\psi(\tau) = \Theta \frac{\partial \phi(\tau)}{\partial \tau} \quad , \quad \tau \in (-1,0) \qquad (8)$$

$$\Theta \frac{\partial \phi}{\partial \tau}\Big|_{\tau=0} = A\phi(0) + B\phi(-1) \quad , \qquad (9)$$

and the domain $\mathfrak{D}(\mathfrak{A})$ of $\mathfrak{A}$ is the subset of $C^1(-1,0)$ whose functions satisfy Eq. (9). The domain $\mathfrak{D}(\mathfrak{A})$ is dense

in $L_2^n(-1,0)$ (the n product of $L_2(-1,0)$).

The zero-input response $x_f(t,\cdot)$ of Eq. (1) for any initial state $x_0(\cdot)\in L_2^n(-1,0)$ can be assured to exist. Evidently $x_f(t,\cdot)$ belongs to $L_2^n(-1,0)$. Then the operator S(t) which corresponds $x_0(\cdot)$ to $x_f(t,\cdot)$ can be defined, and is linear bounded operator on $L_2^n(-1,0)$ into itself .

$$x_f(t,\cdot) = S(t)x_0(\cdot) \quad , \quad x_0(\cdot)\in L_2^n(-1,0), \qquad (10)$$

The following semi-group property of $\{S(t)\}_{t\geq 0}$ can be easily seen.

$$\text{(i)} \quad S(0) = I \quad , \qquad (11\text{-}1)$$

$$\text{(ii)} \quad S(t+s) = S(t)S(s) \quad , \quad t,s \geq 0 \quad , \qquad (11\text{-}2)$$

Since the $x_f(t,0)$ is the solution of Eq. (1) for the zero forcing term and the initial condition $x_0(\cdot)$, then

$$s\text{-}\lim_{t\downarrow 0} S(t)x(\cdot) = \lim_{t\downarrow 0} x_f(t,\cdot) = x_0(\cdot) \quad , \text{ for any } x_0(\cdot)\in L_2^n(-1,0), \qquad (12)$$

This Eq. (12) implies that $\{S(t)\}_{t\geq 0}$ is the strongly continuous semi-group.

It is supposed that $x(0,\cdot)\in \mathcal{D}(\mathfrak{A})$ and $x(\Delta,\cdot)$, $\Delta>0$ is defined as follows:

$$x(\Delta,\cdot) = S(\Delta)x(0,\cdot) \quad ,$$

that is,

$$x(\Delta,\tau) = \begin{cases} x(0,\Delta+\tau) \quad , \quad -1<\tau<-\Delta & (13\text{-}1) \\ e^{(\Delta+\tau)A}x(0,0) + \int_0^{\Delta+\tau} e^{(\Delta+\tau-\sigma)A} B\, x(0,\sigma-1)\,d\sigma \quad , \quad -\Delta\leq\tau\leq 0 & (13\text{-}2) \end{cases}$$

If Δ is very small, Eq(13.2) can be rewritten as follows:

$$(I+\overline{\Delta+\tau}A)x(0,0) + \overline{\Delta+\tau}\, B\, x(0,-1) \quad .$$

Then

$$\underset{\Delta \downarrow 0}{s\text{-}\lim} \frac{S(\Delta) - I}{\Delta} x(0,\cdot) = \mathfrak{A} x(0,\cdot) \tag{14}$$

This concludes that the operator $\mathfrak{A}$ is the infinitesimal generator of the semi-group $\{S(t)\}_{t \geq 0}$

In order to obtain the input-state relation of Eq. (6) in the convolution integral form of forcing function in u(•) and the semi-group $\{S(t)\}_{t \geq 0}$, the system (6) will be transformed into the equivalent system which has the spatially distributed control. For this purpose the following dummy vector $\underline{x}(t,\tau)$ is introduced:

$$\underline{x}(t,\tau) = \tau \Theta^{-1} C u(t) \text{ (*1)} . \tag{15}$$

The variable $\tilde{x}(t,\tau)$ which is equivalent to $x(t,\tau)$ is defined as follows:

$$\tilde{x}(t,\tau) = x(t,\tau) - \underline{x}(t,\tau) \tag{16}$$

By making use of Eqs. (15) and (16), Eq. (6) can be transformed into the following equation of $\tilde{x}(t,\cdot)$:

$$\begin{cases} \dfrac{\partial \tilde{x}(t,\tau)}{\partial t} = \Theta \dfrac{\partial \tilde{x}(t,\tau)}{\partial \tau} + C u(t) - \tau \Theta^{-1} C \dfrac{du}{dt} , & (17\text{-}1) \\ \Theta \dfrac{\partial \tilde{x}(t,0)}{\partial \tau} = A \tilde{x}(t,0) + B \tilde{x}(t,-1) , & (17\text{-}2) \end{cases}$$

where it is supposed that u(•) is differentiable.

If the linear bounded operators $\mathcal{B}$ and $\mathcal{L}$ from E_r into $L_2^n(-1,0)$ are defined as follows:

(*1) It is assumed that $u(0) = 0$,since $u(\cdot)$ is defined on $(0, \infty)$

$$\beta(\tau)\,u(t) = C\,u(t) - \tau\Theta^{-1}C\frac{du}{dt} \qquad , (18)$$

$$\mathcal{L}(\tau)\,u(t) = \tau\Theta^{-1}C\,u(t) \qquad , (19)$$

then Eq. (6) will be transformed into the following abstract dynamical system in $L_2^n(-1,0)$:

$$\frac{d}{dt}\tilde{x}(t) = \mathcal{A}\tilde{x}(t) + \beta\,u(t)\,, \qquad (20\text{-}1)$$

$$\tilde{x}(0) = x(0), \qquad (20\text{-}2)$$

$$x(t) = \tilde{x}(t) + \mathcal{L}u(t) \quad . \qquad (20\text{-}3)$$

By integrating the Eq. (20-1) under the initial condition (20-2), the following relation holds.

$$\tilde{x}(t) = S(t)\,x(0) + \int_0^t S(t-\sigma)\beta\,u(\sigma)d\sigma \qquad (21)$$

With Eqs. (20-3) and (21), the input-state relation of the system (6) can be written in the form of Eq. (22).

$$x(t) = S(t)\,x(0) + \int_0^t S(t-\sigma)\beta\,u(\sigma)d\sigma + \mathcal{L}u(t) \qquad (22)$$

Since u(t) can be written as follows:

$$u(t) = u(0+) + \int_0^t \dot{u}(\sigma)\,d\sigma, \qquad (23\text{-}1)$$

$$\dot{u}(\sigma) = \frac{du(\sigma)}{d\sigma}\,. \qquad (23\text{-}2)$$

then Eq. (22) can be rewritten by Eq. (23).

$$\mathcal{X}(t) = S(t)\mathcal{X}(0) + \left[\mathcal{L}u(0+) + \int_0^t S(t-\sigma)\mathcal{C}u(0+)d\sigma\right]$$

$$+\int_0^t \left\{\mathcal{L}\dot{u}(\sigma) - S(t-\sigma)\mathcal{L}\dot{u}(\sigma) + S(t-\sigma)\mathcal{C}\int_0^\sigma \dot{u}(\mu)d\mu\right\}d\sigma, \quad (24)$$

If the second term of Eq. (24) will be simply represented by $L_t\, u(0+)$ and the third one by $\dot{L}_t\, \dot{u}(\cdot)$, then the Eq. (24) can be rewritten by

$$\mathcal{X}(t) = S(t)\mathcal{X}(0) + L_t u(0+) + \dot{L}_t \dot{u} \ . \quad (25)$$

3. Optimum Regulator Input

The performance index J(u) is defined by

$$J(u) = \frac{1}{2}\int_{-1}^{0} \| \mathcal{X}(T,\tau)\|^2 d\tau + \frac{h}{2}\int_0^T \| u(t)\|^2 dt$$

$$= \frac{1}{2}\| \mathcal{X}(T)\|^2 + \frac{h}{2}\| u(\cdot)\|^2 \ , \quad (26)$$

where T, h are positive constants.

The state $\mathcal{X}(T)$ is written by

$$\mathcal{X}(T) = S(T)\mathcal{X}(0) + \mathcal{L}u(0+) + \dot{\mathcal{L}}\dot{u} \ , \quad (27)$$

where $\mathcal{L} = L_T$, $\dot{\mathcal{L}} = \dot{L}_T$, (28)

$\mathcal{L}: E_r \to L_2^n(-1,0)$,

$\dot{\mathcal{L}}: L_2^r(0,T) \to L_2^n(-1,0)$.

The performance index J(u) is rewritten by

$$J(u(0+), \dot{u}) = \frac{1}{2}\| \mathcal{X}(T)\|^2 + \frac{h}{2}\| u(0+) + \mathcal{J}\dot{u}\|^2 \ , \quad (29)$$

where $\mathcal{J}(t)\dot{u} = \int_0^t \dot{u}(\sigma)d\sigma$. (30)

If the Eq. (27) is substituted into Eq. (29), the latter becomes

$$J(u(0+),\dot{u})$$
$$= \frac{1}{2}\| S(T)x(0)+\mathcal{L}u(0+)+\dot{\mathcal{L}}\dot{u}\|^2+\frac{k}{2}\| \mathbb{1}u(0+)+\mathcal{J}\dot{u}\|^2, \tag{31}$$

where $\mathbb{1}(t)=1$, $t\in(0,T]$. (32)

Then the optimum regulator input can be calculated by the Frechet derivative of $J(u(0+),\dot{u})$ with $u(0+)$ and $\dot{u}$. These Frechet derivatives are

$$\frac{\nabla J}{\nabla u(0+)} = \mathcal{L}^*[S(T)x(0)+\mathcal{L}u(0+)+\dot{\mathcal{L}}\dot{u}]+k\mathbb{1}^*[\mathbb{1}u(0+)+\mathcal{J}\dot{u}], \tag{33}$$

$$\frac{\nabla J}{\nabla \dot{u}} = \dot{\mathcal{L}}^*[S(T)x(0)+\mathcal{L}u(0+)+\dot{\mathcal{L}}\dot{u}]+k\mathcal{J}^*[\mathbb{1}u(0+)+\mathcal{J}\dot{u}], \tag{34}$$

where * denotes the normed conjugate operators.

The optimum input $u_0(0+)$ and $\dot{u}_0$ must satisfy the equations

$$\left.\frac{\nabla J}{\nabla u(0+)}\right|_{u(0+)=u_0(0+),\,\dot{u}=\dot{u}_0} = 0 , \tag{35}$$

$$\left.\frac{\nabla J}{\nabla \dot{u}}\right|_{u(0+)=u_0(0+),\,\dot{u}=\dot{u}_0} = 0 . \tag{36}$$

Then $u_0(0+)$ and $\dot{u}_0$ are the solutions of the following system of integral equations.

$$\left.\begin{aligned}&[\mathcal{L}^*\mathcal{L}+k\mathbb{1}^*\mathbb{1}]u(0+)+[\mathcal{L}^*\dot{\mathcal{L}}+k\mathbb{1}^*\mathcal{J}]\dot{u}=-\mathcal{L}^*S(T)x(0)\\&[\dot{\mathcal{L}}^*\mathcal{L}+k\mathcal{J}^*\mathbb{1}]u(0+)+[\dot{\mathcal{L}}^*\dot{\mathcal{L}}+k\mathcal{J}^*\mathcal{J}]\dot{u}=-\dot{\mathcal{L}}^*S(T)x(0)\end{aligned}\right\}, \tag{37}$$

where $\mathcal{L}^*;\ L_2^n(-1,0)\longrightarrow E_n$, $\dot{\mathcal{L}}^*;\ L_2^n(-1,0)\longrightarrow L_2^r(0,T)$,

$\mathbb{1}^*$; $L_2^r(0,T) \to E_r$.

The operator $\mathcal{K}$ is defined by

$$\mathcal{K}\begin{bmatrix} u(0+) \\ \dot{u} \end{bmatrix} = \begin{bmatrix} [\mathcal{L}^*\mathcal{L} + k\mathbb{1}^*\mathbb{1}]u(0+) + [\mathcal{L}^*\dot{\mathcal{L}} + k\mathbb{1}^*\mathcal{J}]\dot{u} \\ [\dot{\mathcal{L}}^*\mathcal{L} + k\mathcal{J}^*\mathbb{1}]u(0+) + [\dot{\mathcal{L}}^*\dot{\mathcal{L}} + k\mathcal{J}^*\mathcal{J}]\dot{u} \end{bmatrix} . \tag{38}$$

Take any $u(0+)$, $u'(0+) \in E_r$ and $\dot{u}, \dot{u}' \in L_2^r(0,T)$.

Then

$$\begin{aligned} &\left\langle \mathcal{K}\begin{bmatrix} u(0+) \\ \dot{u} \end{bmatrix}, \begin{bmatrix} u'(0+) \\ \dot{u}' \end{bmatrix} \right\rangle \\ &= \langle [\mathcal{L}^*\mathcal{L} + k\mathbb{1}^*\mathbb{1}]u(0+) + [\mathcal{L}^*\dot{\mathcal{L}} + k\mathbb{1}^*\mathcal{J}]\dot{u}, u'(0+) \rangle \\ &\quad + \langle [\dot{\mathcal{L}}^*\mathcal{L} + k\mathcal{J}^*\mathbb{1}]u(0+) + [\dot{\mathcal{L}}^*\dot{\mathcal{L}} + k\mathcal{J}^*\mathcal{J}]\dot{u}, \dot{u}' \rangle \\ &= \langle u(0+), [\mathcal{L}^*\mathcal{L} + k\mathbb{1}^*\mathbb{1}]u'(0+) + [\mathcal{L}^*\dot{\mathcal{L}} + k\mathbb{1}^*\mathcal{J}]\dot{u}' \rangle \\ &\quad + \langle \dot{u}, [\dot{\mathcal{L}}^*\mathcal{L} + k\mathcal{J}^*\mathbb{1}]u'(0+) + [\dot{\mathcal{L}}^*\dot{\mathcal{L}} + k\mathcal{J}^*\mathcal{J}]\dot{u}' \rangle \\ &= \left\langle \begin{bmatrix} u(0+) \\ \dot{u} \end{bmatrix}, \mathcal{K}\begin{bmatrix} u'(0+) \\ \dot{u}' \end{bmatrix} \right\rangle . \end{aligned} \tag{39}$$

Therfore the operator $\mathcal{K}$ is self adjoint.

$$\mathcal{K} = \mathcal{K}^* \tag{40}$$

Futhermore

$$\begin{aligned} &\left\langle \mathcal{K}\begin{bmatrix} u(0+) \\ \dot{u} \end{bmatrix}, \begin{bmatrix} u(0+) \\ \dot{u} \end{bmatrix} \right\rangle \\ &= \langle \mathcal{L}u(0+) + \dot{\mathcal{L}}\dot{u}, \mathcal{L}u(0+) + \dot{\mathcal{L}}\dot{u} \rangle + \langle \mathbb{1}u(0+) + \mathcal{J}\dot{u}, k[\mathbb{1}u(0+) + \mathcal{J}\dot{u}] \rangle \\ &\geqslant \langle \mathbb{1}u(0+) + \mathcal{J}\dot{u}, k[\mathbb{1}u(0+) + \mathcal{J}\dot{u}] \rangle \end{aligned} \tag{41}$$

and $[\mathbb{1}u(0+)+\mathcal{J}\dot{u}]_t = u(0+)+\int_0^t \frac{du}{d\sigma}d\sigma = u(t)$. (42)

Since h is a positive constant,

$$< \mathbb{1}u(0+)+\mathcal{J}\dot{u}, h[\mathbb{1}u(0+)+\mathcal{J}\dot{u}] > > 0 ,$$

when $u(\cdot) \neq 0$.

So the null space $\mathcal{N}(\mathcal{K})$ of the operator $\mathcal{K}$ is the only zero function in $L_2^r(0,T)$ .. That is

$$\mathcal{N}(\mathcal{K}) = 0 . \tag{43}$$

It is concluded that the bounded inverse $\mathcal{K}^{-1}$ of $\mathcal{K}$ exists, and the optimum input can be calculated from Eq. (37) as follows:

$$\begin{bmatrix} u_0(0+) \\ \dot{u}_0 \end{bmatrix} = -\mathcal{K}^{-1} \begin{bmatrix} \mathcal{L}^* S(T)\mathcal{X}(0) \\ \dot{\mathcal{L}} S(T)\mathcal{X}(0) \end{bmatrix} . \tag{44}$$

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