Optimal Pricing and Diseconomies of Scale in Public Water Utilities

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武藤　幸雄：公営水道事業における最適料金制と規模の不経済性

水道事業体から消費者が水道水を購入するとき、メーター計測費や料金徴収費などが消費者の実際の水購入量の大きさと関わりなく発生する。水道事業において消費者の水購入量の大きさと関わりなく発生するこのような費用項目は、まとめて需要家費と呼ばれる。水道事業体は、需要家費を回収するために、水道契約に加入する消費者に対して固定料金や接続料金を課すのが一般的であり、これらの料金額が比較的高いと、水道契約に加入しないことを選ぶ消費者が現れるようになる。本研究は、水道事業体の水道料金収入と総費用を一致させる制約のもとで消費者余剰を最大化するような水料金制を、最適な水料金制としてみなす。そして、消費者の水に関する選好に関していくつか緩やかな制約条件を仮定し、水道契約に加入しないことを選択する消費者が発生する状況を想定しながら、最適な水料金制の特性について分析する。

水の限界価格とは、消費者が水購入量を追加的に１単位増やすとき、消費者が支払うべき水料金額がどれだけ増えるかを表す。ファーストベストの状態で水供給に際し規模の不経済性（あるいは、規模の経済性）が生じる場合、最適な水料金制の限界価格は水購入量が大きいときほど高くなり（あるいは、低くなり）、最適な水料金制のもとでの水供給量と水道契約加入者数がそれぞれのファーストベストの水準よりも大きくなる（あるいは、小さくなる）ことが、本研究では示される。

1. Introduction

A water utility service incurs customer costs according to the number of customers with a connection to the water service system. For example, if a customer connects to a pipedwater system, then the costs of pipe installation, meter reading, and revenue collecting are incurred irrespective of the actual amount of the customer's water consumption. To allay those customer costs, fixed or minimum fees are usually charged to customer-subscribers of a water service system. Such fees can influence customers' subscription decisions if the fees are high in relation to customers' willingness to pay for water or income levels. For instance, it is shown by McPhail [18] that in Tunisia, the cash down-payments that municipal water utilities charge to allay the connection costs discourage households from connecting to piped water systems.[1)]

In the present study, following Littlechild [16], the long run costs of an enterprise are divided into customer and production costs: the sum of the customer costs is expressed as a function of the number of customers purchasing the product or service provided by the enterprise, whereas the sum of the production costs is expressed as a function of the output of the product or service. The rise in the production cost resulting from a unit increment in the output is called the marginal production cost; the increase in the customer cost when the enterprise supplies the product or service to one additional customer is called the marginal customer cost.

Because of *external* diseconomies of scale in supplying water, which often accrue from the scarcity of water resources,[2)] it is possible that the average production cost of a water utility (i.e. the production cost per unit of water supply) increases with water supply, even when the water utility is a natural monopoly. For instance, large plant setup costs that are required for water purification and chemical treatment allow a water utility to enjoy *internal* economies of scale and to act as a natural monopoly.[3)] However, a large city like New York or Los Angeles usually uses readily available local sources first; it then gradually reaches out to ever more distant and expensive supplementary sources to satisfy growing water demand (Hirshleifer et al. [13, chap. 5]). In such a case, if the marginal production cost of supplying water from distant sources is much higher than that from the local sources, then the average production cost rises as the water supply from the distant sources increases.[4)]

Littlechild [16] studied optimal pricing for a monopoly in a simplified case where the customer cost and production cost functions are both linear. That study showed that if some potential customers do not purchase the product or service provided by the monopoly, then the total surplus (the sum of consumer and producer surpluses) is maximized when the monopoly employs a two-part tariff in which the marginal price equals the marginal production cost and the fixed charge equals the marginal customer cost.[5)]

This conclusion is applicable to optimal pricing for a monopolistic water utility in cases where the customer cost function is linear, and where, for relevant levels of water supply, the average production cost increases with water supply because of external diseconomies of scale. In such cases, if some potential customers disconnect from the water service system, then the first-best solution can be realized by means of a two-part tariff in which the fixed charge and marginal price are respectively set to be equal to the marginal customer and production costs.

When adopting such a two-part tariff, the water utility obtains a positive level of profit because the marginal production cost is greater than the average production cost at the first-best water supply. However, when a local government operates the water utility, it might be viewed as socially unacceptable for the water utility to secure such a positive profit. In this circumstance, if the water utility employs the two-part tariff described above, and if the profit accrued by means of the two-part tariff is redistributed to potential customers, including both those connecting to the system and those disconnected from the system, in a lump-sum fashion, then the water utility can avoid excess profits and simultaneously achieve the first-best solution. However, this redistribution is unfair for those customers connecting to the system: under such a redistribution program, a part of their payments to the water utility is transferred directly to those who are disconnected. Therefore, if those customers connecting to the system form a majority of the population, then they might well choose to politically block such redistribution.

If, as argued above, monetary transfers between the water utility and potential customers disconnected are not allowed, then the water utility will be required to satisfy the break-even constraint that the tariff revenue it collects from the customers connecting to the system should match the total cost of water services.[6)] In such situations, what rate schedule should the water utility employ to maximize consumer surplus? As a solution, one might propose adopting a two-part tariff in which the marginal price is set to be equal to the marginal production cost at the first-best level of water supply, and in which the fixed charge is adjusted below the marginal customer cost so as to satisfy the break-even constraint. However, if, at the first-best solution, there are potential customers not provided with water services, then such a two-part tariff encourages those excluded customers to subscribe to water services. As a result, the tariff increases both the total water consumption and the number of customers subscribing at greater than their respective first-best levels, thus engendering an efficiency loss. This argument indicates that if the first-best solution entails customer exclusions, then the water utility cannot meet the break-even constraint without introducing an efficiency loss. Nonlinear tariff schedules are known to present the advantage that the marginal price is adjustable depending on the quantity purchased. In those cases where the efficiency loss associated with customer exclusions is unavoidable, nonlinear tariff schedules will thereby offer the water utility the maximum scope for minimizing the efficiency loss, as suggested in Roberts [20, p. 66]. Therefore, if, at the first-best solution, there are potential customers who disconnect from the water service system, then it will be generally optimal for the water utility to adopt a nonlinear water tariff in order to maximize consumer surplus under the break-even constraint.

The literature regarding nonlinear pricing has so far not fully investigated the problem of designing a nonlinear tariff schedule to address the above situation. Willig [25, p. 68] noted that marginal cost pricing may be viewed as undesirable for a public utility service if marginal cost pricing induces a level of production at which there are locally decreasing returns to scale and a positive level of vendor profit that is viewed as socially unacceptable.[7)] Subsequently, few attempts have been made at clarifying the properties of optimal tariffs in such a situation while accounting for the influences of customer costs and exclusions.[8)]

In this paper, we consider a water market wherein a monopolistic municipal water utility provides water services to customers while incurring customer costs. Assuming that monetary transfers between the utility and customers disconnected from the water service system are infeasible, we characterize an optimal water tariff that maximizes consumer surplus in the market subject to the break-even constraint that the tariff revenue the utility collects from customers connecting to the system should match the total cost of water services. Specifically, we investigate how the rate structure and efficiency of the optimal water tariff are affected by diseconomies of scale in water production.

Under certain conditions, this paper demonstrates that the marginal price in the optimal water tariff becomes a monotone increasing (resp. decreasing) function of the quantity of water purchased if diseconomies (resp. economies) of scale exist in producing water in the first-best situation. It is thereby shown that the presence or absence of such diseconomies of scale can affect whether public water utilities should use quantity premiums or discounts in water pricing.

In both developing and developed countries, water utility price regulators now often employ increasing block tariffs (IBTs), in which the marginal price of water increases stepwise with the quantity of water purchased.[9)] Notwithstanding their popularity, the economic rationale for employing IBTs has not been fully explored in the literature on water pricing; theoretical research on the rationale has remained underdeveloped.[10)] By illustrating situations in which quantity premiums are optimal for public water utilities, this study aims to bridge the gap that separates theory and practice in water pricing.

This paper is organized as follows: The next section presents a model of a water utility and customers' preference for water, and studies the first-best water allocation. Section 3 formulates a water pricing problem when the water utility maximizes consumer surplus under the break-even constraint. It then examines the conditions for the optimality of the problem. The rate structure and efficiency of the optimal water tariff are analyzed in section 4. Finally, section 5 describes concluding remarks.

2. The Model

Consider a water market with a municipal water utility and a continuum population of potential customers. Let N denote the size of the population of potential customers. The long-run cost of the water utility comprises customer and production costs. The customer cost is given as vN^s, where N^s signifies the number of customers connecting to the water service system, and where v is a positive constant, representing the marginal (average) customer cost. The production cost is given as $C(Y)$, where Y is the water supply and C is a twice-differentiable, increasing, and convex function ($C'(Y) > 0$ and $C''(Y) \geq 0$). The long-run cost is therefore given as $vN^s + C(Y)$.[11)]

Differences among potential customers are measured using a taste type parameter t. If a customer of type t purchases x units of water for P dollars, then the customer's utility U is given as

$$U(x, t, P) = \int_0^x \rho(y, t)dy - P,$$

where the function ρ represents the marginal willingness to pay for an additional unit of water.[12)] In the study by Timmins [24], the marginal willingness to pay for an additional unit of water is assumed to be linear in the logarithm of water consumption; the water demand function is specified in a semilog form. Timmins [24] argued that semilog specifications provide a reasonable representation of municipal water demand functions given high storage costs of water and legal prohibition of water resale. The present study follows that approach, and includes the assumption that differences among customers arise from different levels of satiation in water consumption: The marginal willingness to pay is specified as $\rho(x, t) = (\ln t - \ln x)/\gamma$, where γ is a positive constant, and where t is distributed over an interval $(0, M)$ according to the distribution function $F(t)$.[13)] In this setting, the marginal willingness to pay $\rho(x, t)$ diverges to infinity as water consumption x approaches zero; on the other hand, it becomes zero when $x = t$, showing that the satiation level for type-t customers equals t. The dollar benefit for a type-t customer from purchasing x units of water is then given as

$$b(x,t) \equiv \int_0^x \rho(y,t)dy = \frac{x}{\gamma}\{1+\ln(t/x)\}. \tag{1}$$

The benefit function b satisfies $b_x(x,t) = \rho(x,t)$, $b_t(x,t) = x/\gamma t$, and $b_{xt}(x,t) = 1/\gamma t > 0$.[14)]

The water utility cannot distinguish any particular customer type, but knows that t is distributed according to the distribution F. The density function of t, $f(t) \equiv F'(t)$, is assumed to be positive and continuously differentiable on the interval $(0, M)$. We denote the reciprocal of the hazard rate function of t as $I(t) \equiv \bar{F}(t)/f(t)$, where $\bar{F}(t) \equiv 1 - F(t)$, and make the following assumption:

Assumption 1 $\{I(t)/t\}' = I'(t)/t - I(t)/t^2 < 0$ *for* $t \in (0, M)$.

For instance, if $\ln f(t)$ is a strictly concave function, then, as verified in Prekova [19], $I(t)$ is a decreasing function, thereby fulfilling Assumption 1.

The remainder of this section examines the water allocation in the first-best optimum. Let $q(t) \in [0,1]$ denote the probability that a type-t customer connects to the water service system. Let $x(t)$ denote the water consumption of a type-t customer when connecting to the system. The population size of the customers connecting to the system is then expressible as $N\int_0^M q(t)f(t)dt$. The first-best optimum for the present model can be found by solving the following social surplus maximization problem:

$$\max_{q(t)\in[0,1],\ x(t)>0,\ Y} \quad N\int_0^M q(t)\{b(x(t),t) - v\}f(t)dt - C(Y)$$
$$s.\ t. \quad Y = N\int_0^M q(t)x(t)f(t)dt, \tag{2}$$

where Eq. (2) is the requirement that the water supply be equal to the total water consumption.

The Hamiltonian for the above control problem is represented as

$$\Phi[q(t), x(t), Y, t] = \Big[Nq(t)\{b(x(t),t) - \mu x(t) - v\} + \mu Y - C(Y)\Big]f(t),$$

where μ is the multiplier of constraint (2). By the maximum principle, the following conditions pertain at the optimum:

$$\{q(t), x(t)\} \in \arg\max_{(q,\, x)}\ q\{b(x, t) - \mu x - v\}\ \text{ subject to }\ 0 \leq q \leq 1 \text{ and } x > 0, \tag{3}$$

$$\int_0^M \Phi_Y[q(t), x(t), Y, t]dt = \mu - C'(Y) = 0. \tag{4}$$

Let $Y^{fb} > 0$ denote the value of Y at the first-best optimum. We define a function S as $S(t) \equiv \max_{x>0} b(x, t) - xC'(Y^{fb}) - v$. The solution for x to this maximization problem is deduced as $x = x^{fb}(t) \equiv t\exp\{-\gamma C'(Y^{fb})\}$, because the first-order condition implies $b_x(x, t) = C'(Y^{fb})$. As a result, we obtain $S(t) = \gamma^{-1}t\exp\{-\gamma C'(Y^{fb})\} - v$. Conditions (3) and (4) together imply that if $S(t) >$ (or $<$) 0, then $q(t) = 1$ [or $q(t) = 0$]. Accordingly, if $t >$ (or $<$) $\tau^{fb} \equiv \gamma v\exp\{\gamma C'(Y^{fb})\}$, then $q(t) = 1$ [or $q(t) = 0$]. As this result indicates, providing water services to a customer with type higher (or lower) than τ^{fb} generates a positive (or negative) surplus in the first-best case. The first-best solution requires those customers with types higher (or lower) than τ^{fb} to connect to (or disconnect from) the system. Consequently, τ^{fb} represents the marginal customer type under the first-best solution. The first-best water consumption of a customer with type $t \geq \tau^{fb}$ is given as $x^{fb}(t)$. The minimum water consumption of all connected customers equals $x^{fb}(\tau^{fb}) = \gamma v > 0$ according to the first-best solution.

The reasoning presented above suggests that $\{x^{fb}(t), \tau^{fb}, Y^{fb}\}$ is determined by solving the following system of equations:

$$Y^{fb} = N\int_{\tau^{fb}}^{M} x^{fb}(t)f(t)dt, \tag{5}$$

$$b_x(x^{fb}(t), t) = C'(Y^{fb}) \ \text{ for } t \in [\tau^{fb}, M), \tag{6}$$

$$b(x^{fb}(\tau^{fb}), \tau^{fb}) = v + x^{fb}(\tau^{fb})C'(Y^{fb}). \tag{7}$$

If the marginal production cost equals $C'(Y^{fb})$, and if a type-τ^{fb} customer starts connecting to the system and consumes $x^{fb}(\tau^{fb})$ units of water, then the water utility incurs a cost of $v + x^{fb}(\tau^{fb})C'(Y^{fb})$ in serving the customer. The right-hand side (RHS) of Eq. (7) accordingly measures the cost for providing an additional marginal customer with water services in the first-best case. Equation (7) implies that the benefit of water to a marginal customer should equal that cost. On the other hand, Eq. (6) indicates that the marginal benefit of water to a customer connecting to the system should equal the marginal production cost.

Consider a situation in which the water utility employs the following two-part tariff for water:

$$T^{fb}(x) = v + xC'(Y^{fb}) \quad \text{for } x \geq x^{fb}(\tau^{fb}), \tag{8}$$

where x denotes the quantity of water purchased, $T^{fb}(x)$ is the payment, and $x^{fb}(\tau^{fb})$ represents the minimum purchase of water in the tariff. The minimum charge and marginal price in this tariff are given, respectively, as $T^{fb}(x^{fb}(\tau^{fb})) = v + x^{fb}(\tau^{fb})C'(Y^{fb})$ and $dT^{fb}/dx = C'(Y^{fb})$. When a type-t customer chooses to connect to the system under this tariff, the customer's water purchase is determined by solving the problem: $\max_{x>0} b(x,t) - xC'(Y^{fb}) - v$. According to the definitions of S and x^{fb}, the customer then gains a surplus of $S(t) = b(x^{fb}(t), t) - x^{fb}(t)C'(Y^{fb}) - v$ through purchasing $x^{fb}(t)$ units of water. As presented above, we have $S(t) \gtreqless 0$ for $t \gtreqless \tau^{fb}$. Therefore, given that the potential customers' reservation utility is zero, the marginal customer type equals τ^{fb} under the tariff T^{fb}. The customers whose taste types lie between 0 and τ^{fb} choose to disconnect from the system under the tariff T^{fb} because the minimum charge $T^{fb}(x^{fb}(\tau^{fb}))$ is greater than the benefits that they can derive from purchasing water. Consequently, the first-best optimum studied above is realized when the water utility employs the tariff T^{fb}.

It is implied by Eq. (8) that when adopting the tariff T^{fb}, the water utility obtains a tariff revenue of $vN\overline{F}(\tau^{fb}) + C'(Y^{fb})Y^{fb}$. In that case, the water utility incurs a total cost of $vN\overline{F}(\tau^{fb}) + C(Y^{fb})$. The water utility's profit under the tariff T^{fb} equals $C'(Y^{fb})Y^{fb} - C(Y^{fb})$, which becomes positive (or negative) if diseconomies (or economies) of scale exist in producing water at the first-best optimum. Subsequent sections investigate a water pricing problem when it is not socially permissible for the water utility to generate such excess profits or losses.

3. The Water Pricing Problem under a Break-Even Constraint

This section describes a situation in which the water utility must satisfy the break-even constraint that the tariff revenue it collects from customers connecting to the system should match the total cost of water services. We formulate a water pricing problem for the water utility and derive conditions for optimality of the problem.

Assume a case in which the water utility introduces a water tariff schedule $\{x(t), P(t)\}$ that induces a type-t customer to purchase $x(t)$ units of water at a given tariff $P(t)$. In this case, the incentive compatibility constraint requires that

$$b(x(t), t) - P(t) \geq b(x(\tilde{t}), t) - P(\tilde{t}) \text{ for all } (t, \tilde{t}) \in (0, M) \times (0, M). \tag{9}$$

Let $w(t)$ denote the surplus of a type-t customer under the tariff schedule $\{x(t), P(t)\}$:

$$w(t) = b(x(t), t) - P(t) = \max_{\tilde{t} \in (0,\, M)} b(x(\tilde{t}), t) - P(\tilde{t}).$$

Because $b_{xt}(x, t) > 0$, the incentive compatibility constraint (9) is equivalent to the conjunction of the following two conditions: (IC1) $x(t)$ is nondecreasing in t; and (IC2) $w'(t) = b_t(x(t), t)$ (see Fudenberg and Tirole [8, chap. 7]). Condition (IC1) assures the existence of a tariff function $P(t)$ such that $\{x(t), P(t)\}$ satisfies the incentive compatibility constraint (9) (see Guesnerie and Laffont [10]). Condition (IC2), which is deduced from the envelope theorem, implies that the rate at which the surplus changes with t equals $b_t(x(t), t)$. Under condition (IC2), we have $w'(t) = b_t(x(t), t) > 0$ when $x(t) > 0$, which means that the surplus that a customer gains from water purchasing is increasing concomitantly with the customer's type. With asymmetric information, the water utility allows higher-type customers to earn higher information rents because higher-type customers might mimic the behaviors of lower-type customers.

Monetary transfers between the water utility and customers disconnected from the water service system are assumed to be infeasible because of political or other constraints such as those discussed in Section 1. The surpluses of disconnected customers are uncontrollable for the utility and are assumed to be zero. The following analysis addresses a situation in which the customer cost is sufficiently high that the water utility must impose a minimum charge on customer-subscribers to allay the customer cost. We assume that under the relevant water tariff schedules, potential customers whose taste types are sufficiently close to zero choose to disconnect from the system because of the minimum charge, as in the case in which the water utility employs the tariff T^{fb}. Let $\tau \in (0, M)$ signify the marginal customer type under the tariff schedule $\{x(t), P(t)\}$. In this setting, on the interval $0 < t < \tau$, the surplus and the tariff schedule are given as $w(t) = x(t) = P(t) = 0$; both (IC1) and (IC2) are satisfied there because $x(t)$ is constant at zero, and because we have $w'(t) = 0 = x(t)/\gamma t = b_t(x(t), t)$. Type-$\tau$ customers gain zero surplus if connecting to the system. Accordingly, condition $w(\tau) = b(x(\tau), \tau) - P(\tau) = 0$ must hold true. On the other hand, on the interval $\tau < t < M$, the water purchase $x(t)$ is positive, and, with conditions (IC2) and $w(\tau) = 0$, the surplus function is expressible as

$$w(t) = \int_{\tau}^{t} b_t(x(s), s)ds. \tag{10}$$

In the circumstance described above, the number of customers connecting to the system is given as $N^s = N\bar{F}(\tau)$, which is decreasing in τ. The water utility's profit, Π, is the difference between the tariff revenue it receives from the customers connecting to the system and the total cost of water services. Using $P(t) = b(x(t), t) - w(t)$ and Eq. (10), and integrating by parts, we can represent the profit as

$$\Pi[x(\cdot), \tau, Y] \equiv N \int_{\tau}^{M} \left\{ b(x(t), t) - \int_{\tau}^{t} b_t(x(s), s)ds \right\} f(t)dt - vN\bar{F}(\tau) - C(Y) \tag{11}$$

$$= N \int_{\tau}^{M} \left[\{b(x(t), t) - v\} f(t) - b_t(x(t), t)\bar{F}(t) \right] dt - C(Y). \tag{12}$$

Furthermore, integration by parts enables us to express the aggregate of customers' surpluses, W, as

$$W[x(\cdot), \tau] \equiv N \int_{\tau}^{M} f(t) \int_{\tau}^{t} b_t(x(s), s)dsdt = N \int_{\tau}^{M} b_t(x(t), t)\bar{F}(t)dt. \tag{13}$$

Because the water supply must be greater than or equal to the total water consumption, the following constraint is imposed:

$$Z[x(\cdot), \tau, Y] \equiv Y - N \int_{\tau}^{M} x(t)f(t)dt \geq 0. \tag{14}$$

Assume that it is viewed as socially unacceptable for the water utility to secure a positive level of profit and that the water utility must satisfy the break-even constraint: $\Pi[x(\cdot), \tau, Y] = 0$. The problem for the water utility, denoted herein as (WP), is to maximize the consumer surplus function $W[\cdot]$ subject to (IC1), the output constraint (14), and the break-even constraint. This can be formulated as the following optimization problem:

$$(WP) \qquad \max_{x(t),\, \tau,\, Y} W[x(\cdot), \tau] = N \int_{\tau}^{M} b_t(x(t), t)\bar{F}(t)dt \;\text{ subject to (14), and}$$

$$(IC1): \quad x(t) \text{ is nondecreasing in } t \text{ on the interval } \tau \leq t < M,$$

$$\Pi[x(\cdot), \tau, Y] = N \int_{\tau}^{M} \Big[\{b(x(t), t) - v\} f(t) - b_t(x(t), t)\bar{F}(t)\Big] dt - C(Y) = 0. \qquad (15)$$

The rest of this section presents derivation of the conditions for the optimality of this problem. In this analysis, for simplicity, we first ignore constraint (IC1) in (WP). The relaxed problem is denoted as (WP'):

$$(WP') \qquad \max_{x(t),\, \tau,\, Y} W[x(\cdot), \tau] \;\text{ subject to (14), (15).}$$

Deriving the necessary conditions for optimality of (WP'), we examine when an optimal solution to (WP') satisfies constraint (IC1).

The Lagrangian for the problem (WP') is defined as

$$L[x(t), t] \equiv (1 - \lambda)Nb_t(x(t), t)\bar{F}(t) + \lambda N\{b(x(t), t) - v\} f(t) - \mu N x(t) f(t), \qquad (16)$$

where the multipliers μ and λ respectively measure the shadow prices of the output constraint (14) and the break-even constraint (15), and where μ is nonnegative. The first-order condition for $x(t)$ yields

$$L_x[x(t), t] = N f(t) [(1 - \lambda)b_{xt}(x(t), t)I(t) + \lambda b_x(x(t), t) - \mu] = 0 \;\text{ for } t \in [\tau, M). \qquad (17)$$

On the other hand, differentiating $W[x(\cdot), \tau] + \lambda\Pi[x(\cdot), \tau, Y] + \mu Z[x(\cdot), \tau, Y]$ with respect to τ and Y, we deduce the first-order conditions for τ and Y, respectively, as follows:

$$\begin{aligned} W_\tau + \lambda\Pi_\tau + \mu Z_\tau &= N f(\tau) [(\lambda - 1)b_t(x(\tau), \tau)I(\tau) - \lambda\{b(x(\tau), \tau) - v\} + \mu x(\tau)] \\ &= 0, \qquad (18) \\ \lambda\Pi_Y + \mu Z_Y &= \mu - \lambda C'(Y) = 0. \qquad (19) \end{aligned}$$

Because of Eq. (19) and because $C'(Y) > 0$, λ has the same sign as μ and is nonnegative. If we were to obtain $\lambda = \mu = 0$, we would have $L_x[x(t), t] = N f(t)b_{xt}(x(t), t)I(t) > 0$ for

$t \in [\tau, M)$, which contradicts Eq. (17). Consequently, we have $\lambda > 0$ and $\mu > 0$ at the optimum of (WP').

Let us examine when an optimal solution to (WP') meets constraint (IC1). The solution for $x(t)$ to Eq. (17) is unique and is given as $x(t) = t\exp[\{(1-\lambda)H(t) - \gamma\mu\}/\lambda]$, where $H(t) \equiv I(t)/t$. Its derivative is expressible as

$$\frac{d}{dt}x(t) = x(t)\left\{\frac{1}{t} + \frac{1-\lambda}{\lambda}H'(t)\right\}. \tag{20}$$

Assumption 1 implies that $H'(t) < 0$ for $t \in (0, M)$. Therefore, the solution for $x(t)$ to Eq. (17) satisfies $dx(t)/dt \geq 0$ on the interval $\tau \leq t < M$ if and only if

$$\frac{1}{tH'(t)} \leq \frac{\lambda - 1}{\lambda} \quad \text{for all } t \in [\tau, M). \tag{21}$$

Condition (21) is necessarily satisfied if $\lambda \geq 1$. From these arguments, we conclude the following:

Proposition 1 *Assume that $\{x(t), \tau, Y\}$ is a solution to problem (WP'), and that the shadow price of the break-even constraint (15) in problem (WP') equals λ for the solution $\{x(t), \tau, Y\}$. In this case, $\{x(t), \tau, Y\}$ satisfies constraint (IC1) and is a solution to problem (WP) (i) if $\lambda \geq 1$, or (ii) if $\lambda < 1$ and condition (21) holds true for λ and τ.*

At the optimum of (WP'), we have $\mu > 0$, so that the water supply equals the total water consumption:

$$Y = N\int_{\tau}^{M} x(t)f(t)dt. \tag{22}$$

On the other hand, insertion of Eq. (19) into Eqs. (17) and (18) yields

$$b_x(x(t), t) = \alpha b_{xt}(x(t), t)I(t) + C'(Y) \quad \text{for} \quad t \in [\tau, M), \tag{23}$$

$$b(x(\tau), \tau) = \alpha b_t(x(\tau), \tau)I(\tau) + v + x(\tau)C'(Y), \tag{24}$$

where $\alpha \equiv (\lambda-1)/\lambda$ represents the Ramsey number. If condition (21) is satisfied at the optimum of problem (WP'), then Eqs. (15) and (22) – (24) together represent the necessary conditions for the optimality of problem (WP). Otherwise, derivation of the optimal solution for $x(t)$ in problem (WP) requires bunching of taste types, and the necessary conditions for the optimality of (WP) become more complicated than those presented above.

Equations (23) and (24) can be interpreted in the following manner. Equation (10) implies that if the water purchase $x(t)$ is raised marginally for a given $t \in [\tau, M)$, the information rents earned by the customers with types higher than t increase. The resultant increases in the information rents shift the consumer surplus W upward [see Eq. (13)], but engender reductions in the water utility's profit [see Eq. (11)], thereby affecting the break-even constraint negatively. In Eq. (17), the term $(1-\lambda)b_{xt}(x(t), t)I(t)$ captures the impacts that those increases in the information rents have on the objective function value in problem (WP'). Equation (23) requires that the marginal benefit of water to a type-t customer equal this term multiplied by $-1/\lambda$ plus the marginal production cost.

Equation (10) implies also that if the marginal customer type τ decreases by one unit, the information rents accrued to the infra-marginal customers increase by $b_t(x(\tau), \tau)$. In a similar manner to the above, the resultant increases in the information rents shift the consumer surplus W upward, but negatively affect the water utility's profit and the break-even constraint. In Eq. (18), the term $(\lambda-1)b_t(x(\tau), \tau)I(\tau)$ captures the impacts of those increases in the information rents on the objective function value in problem (WP'); this term, multiplied by $1/\lambda$, equals the first term in the RHS of Eq. (24). On the other hand, the sum of the second and third terms in the RHS of Eq. (24) reflects the cost for providing an additional marginal customer with water services under the optimal tariff (see the arguments following Eqs. (5) – (7)). Equation (24) represents that the sum of these three terms is necessary to equal the benefit of water to a marginal customer.[15)]

4. The Rate Structure and Efficiency of the Optimal Water Tariff

This section presents an analysis of the rate structure and efficiency of the optimal water tariff that is determined from the necessary conditions derived in the preceding section.

Assume that (i) the solution to problem (WP') is given as $\{x(t), \tau, Y\} = \{x^*(t), \tau^*, Y^*\}$, and that (ii) $x^*(t)$ is strictly increasing on the interval $\tau^* \leq t < M$, i.e. the strict inequality pertains in condition (21):

$$\frac{1}{tH'(t)} < \frac{\lambda^* - 1}{\lambda^*} \quad \text{for all } t \in [\tau^*, M), \tag{25}$$

where λ^* denotes the shadow price of the break-even constraint (15) in problem (WP'). In this case, the results in the preceding section indicate that the solution to problem (WP) is also given as $\{x(t), \tau, Y\} = \{x^*(t), \tau^*, Y^*\}$, and that $\{x(t), \tau, Y, \alpha\} = \{x^*(t), \tau^*, Y^*, \alpha^*\}$ must satisfy Eqs. (15) and (22) – (24), where $\alpha^* \equiv (\lambda^* - 1)/\lambda^*$.

The first part of this section introduces several functions and characterizes them to facilitate analysis of the optimal water tariff. Functions $\bar{x}$ and $\bar{\alpha}$ are defined respectively as

$$\bar{x}[t, Y, \alpha] \equiv t\exp\{-\alpha H(t) - \gamma C'(Y)\}, \tag{26}$$

$$\bar{\alpha}(\tau, Y) \equiv \{\ln(\tau/v\gamma) - \gamma C'(Y)\}/H(\tau). \tag{27}$$

If Eq. (23) is solved with respect to $x(t)$ for a given Y and α, the solution is unique and is obtained as $x(t) = \bar{x}[t, Y, \alpha]$. Furthermore, if $x(\tau) = \bar{x}[\tau, Y, \alpha]$ is substituted into Eq. (24) and the equation is solved with regard to α for a given τ and Y, the solution is unique; it is derived as $\alpha = \bar{\alpha}(\tau, Y)$. Therefore, for a given τ and Y, we can solve the system of Eqs. (23) and (24) uniquely with respect to α and $x(t)$, and the solutions are given as $\alpha = \bar{\alpha}(\tau, Y)$ and $x(t) = \bar{x}[t, Y, \bar{\alpha}(\tau, Y)]$.

Inserting $x(t) = \bar{x}[t, Y, \bar{\alpha}(\tau, Y)]$ into Eq. (22), we transform Eq. (22) into the equation $\bar{Z}(\tau, Y) = 0$, where the function $\bar{Z}$ is defined as

$$\bar{Z}(\tau, Y) \equiv Y - N\int_{\tau}^{M} \bar{x}[t, Y, \bar{\alpha}(\tau, Y)]\, f(t)dt. \tag{28}$$

Assumption 1 implies that $H(t)/H(\tau) < 1$ for $t > \tau$. The partial derivative $\bar{Z}_Y(\tau, Y)$ is therefore evaluated as

$$\bar{Z}_Y(\tau, Y) = 1 - N\gamma C''(Y)\int_{\tau}^{M} \left\{\frac{H(t)}{H(\tau)} - 1\right\} \bar{x}[t, Y, \bar{\alpha}(\tau, Y)]\, f(t)dt > 0, \tag{29}$$

which means that $\bar{Z}(\tau, Y)$ is strictly increasing in Y. Given τ, the value of Y that fulfills the equation $\bar{Z}(\tau, Y) = 0$ is therefore unique; we denote the value as $Y = \hat{Y}(\tau)$ to express the functional dependence on τ. Substitution of $Y = \hat{Y}(\tau)$ into $\bar{\alpha}(\tau, Y)$ permits us to define a function $\hat{\alpha}$ as $\hat{\alpha}(\tau) \equiv \overline{\alpha}(\tau, \hat{Y}(\tau))$. By construction, if the system of Eqs. (22) – (24) is solved with respect to Y, α, and $x(t)$ for a given τ, then the solutions are unique, and are derived as $Y = \hat{Y}(\tau)$, $\alpha = \hat{\alpha}(\tau)$, and $x(t) = \overline{x}[t, \hat{Y}(\tau), \overline{\alpha}(\tau, \hat{Y}(\tau))] = \overline{x}[t, \hat{Y}(\tau), \hat{\alpha}(\tau)]$. Given that $\tau = \tau^*$, Eqs. (22) – (24) are satisfied for $Y = Y^*$, $\alpha = \alpha^*$, and $x(t) = x^*(t)$. Consequently, we have $\hat{Y}(\tau^*) = Y^*$, $\hat{\alpha}(\tau^*) = \alpha^*$, and $\overline{x}[t, \hat{Y}(\tau^*), \hat{\alpha}(\tau^*)] = x^*(t)$. On the other hand, Eqs. (5) – (7) indicate that, given $\tau = \tau^{fb}$, Eqs. (22) – (24) are satisfied for $Y = Y^{fb}$, $\alpha = 0$, and $x(t) = x^{fb}(t)$. Therefore, we also have $\hat{Y}(\tau^{fb}) = Y^{fb}$, $\hat{\alpha}(\tau^{fb}) = 0$, and $\overline{x}[t, \hat{Y}(\tau^{fb}), \hat{\alpha}(\tau^{fb})] = x^{fb}(t)$.

If the solutions for $x(t)$ and Y derived above (respectively, $x(t) = \overline{x}[t, \hat{Y}(\tau), \hat{\alpha}(\tau)]$ and $Y = \hat{Y}(\tau)$) are substituted into the profit function $\Pi[x(\cdot), \tau, Y]$ shown in (11), then a function $\hat{\Pi}$ is definable as

$$\begin{aligned} \hat{\Pi}(\tau) \equiv\ & N \int_{\tau}^{M} \left\{ b(\overline{x}[t, \hat{Y}(\tau), \hat{\alpha}(\tau)], t) - \int_{\tau}^{t} b_t(\overline{x}[s, \hat{Y}(\tau), \hat{\alpha}(\tau)], s) ds \right\} f(t) dt \\ & - vN\overline{F}(\tau) - C(\hat{Y}(\tau)). \end{aligned} \tag{30}$$

The value of $\hat{\Pi}(\tau)$ reflects the water utility's profit level when the water allocation is determined by solving the system of Eqs. (22) – (24) with respect to Y, α, and $x(t)$ given τ. The optimal solution for τ, τ^*, satisfies $\hat{\Pi}(\tau^*) = 0$ because $\overline{x}[t, \hat{Y}(\tau^*), \hat{\alpha}(\tau^*)] = x^*(t)$ and $\hat{Y}(\tau^*) = Y^*$, and because the break-even constraint $\Pi[x(\cdot), \tau, Y] = 0$ holds true for $x(t) = x^*(t)$, $\tau = \tau^*$, and $Y = Y^*$. On the other hand, it is noteworthy that $b(x^{fb}(t), t)$ can be transformed as follows:

$$\begin{aligned} b(x^{fb}(t), t) &= b(x^{fb}(\tau^{fb}), \tau^{fb}) + \int_{\tau^{fb}}^{t} \frac{d}{ds} b(x^{fb}(s), s) ds \\ &= b(x^{fb}(\tau^{fb}), \tau^{fb}) + \int_{\tau^{fb}}^{t} \left\{ b_x(x^{fb}(s), s) \frac{d}{ds} x^{fb}(s) + b_t(x^{fb}(s), s) \right\} ds. \end{aligned}$$

Using this equality and Eqs. (6) and (7), it can be deduced that

$$\begin{aligned} & b\left(\overline{x}[t, \hat{Y}(\tau^{fb}), \hat{\alpha}(\tau^{fb})], t\right) - \int_{\tau^{fb}}^{t} b_t\left(\overline{x}[s, \hat{Y}(\tau^{fb}), \hat{\alpha}(\tau^{fb})], s\right) ds \\ =\ & b(x^{fb}(t), t) - \int_{\tau^{fb}}^{t} b_t(x^{fb}(s), s) ds = b(x^{fb}(\tau^{fb}), \tau^{fb}) + \int_{\tau^{fb}}^{t} b_x(x^{fb}(s), s) \frac{d}{ds} x^{fb}(s) ds \\ =\ & v + C'(Y^{fb}) x^{fb}(\tau^{fb}) + C'(Y^{fb}) \int_{\tau^{fb}}^{t} \frac{d}{ds} x^{fb}(s) ds \\ =\ & v + C(Y^{fb}) x^{fb}(t) = T^{fb}(x^{fb}(t)). \end{aligned} \tag{31}$$

Incorporating the above into Eq. (30) when $\tau = \tau^{fb}$, we can obtain $\hat{\Pi}(\tau^{fb}) = C'(Y^{fb})Y^{fb} - C(Y^{fb})$.

We now present the results of the impacts of changes in τ on the function values of $\hat{Y}$, $\hat{\alpha}$, and $\hat{\Pi}$. As described later, the results help us clarify how the size relationships between τ^* and τ^{fb}, between Y^* and Y^{fb}, and between λ^* and 1 are determined. Differentiating Eq. (27) with respect to Y, we have $\bar{\alpha}_Y(\tau, Y) = -\gamma C''(Y)/H(\tau) \leq 0$; that is, $\bar{\alpha}(\tau, Y)$ is nonincreasing in Y. Furthermore, if $1/\tau H'(\tau) < \hat{\alpha}(\tau)$, then, differentiating Eq. (27) with respect to τ, and incorporating $Y = \hat{Y}(\tau)$, we have:

$$\bar{\alpha}_\tau(\tau, \hat{Y}(\tau)) = \frac{H'(\tau)}{H(\tau)} \left\{ \frac{1}{\tau H'(\tau)} - \hat{\alpha}(\tau) \right\} > 0. \tag{32}$$

In the Appendix, the following theorem is established using these properties of $\bar{\alpha}$.

Theorem 1 *Assume that the following inequality is satisfied for a given* τ:

$$1/\tau H'(\tau) < \hat{\alpha}(\tau) < 1. \tag{33}$$

Then, we have $\hat{\Pi}'(\tau) > 0$, $\hat{Y}'(\tau) < 0$, *and* $\hat{\alpha}'(\tau) > 0$.

As implied by the theorem, if the system of Eqs. (22) – (24) is solved with respect to $\{x(t), Y, \alpha\}$ for a given τ, and if the solution for α, $\hat{\alpha}(\tau)$, satisfies condition (33), then the water utility's profit level determined from the solution for $\{x(t), Y, \alpha\}$, $\hat{\Pi}(\tau)$, increases with τ. In addition, $\hat{\alpha}(\tau)$ is increasing in τ, whereas the solution for Y, $\hat{Y}(\tau)$, is decreasing in τ.

Application of this theorem enables characterization of τ^*, Y^*, and λ^* in the following manner.

Proposition 2 *Assume that the solution to problem (WP') is given as* $\{x(t), \tau, Y\} = \{x^*(t), \tau^*, Y^*\}$, *that the shadow price of constraint (15) in (WP') equals* λ^* *for the solution* $\{x^*(t), \tau^*, Y^*\}$, *and that these satisfy condition (25). Then, one of the following three cases can occur*:

(I) *The solution and the shadow price fulfill* $\tau^* = \tau^{fb}$, $Y^* = Y^{fb}$, *and* $\lambda^* = 1$. *Under the first-best situation, there are constant returns to scale in producing water*: $C'(Y^{fb}) = C(Y^{fb})/Y^{fb}$.

(II) *The solution and the shadow price fulfill* $\tau^* < \tau^{fb}$, $Y^* > Y^{fb}$, *and* $0 < \lambda^* < 1$. *Under the first-best situation, there are decreasing returns to scale in producing water*: $C'(Y^{fb}) > C(Y^{fb})/Y^{fb}$.

(III) *The solution and the shadow price fulfill* $\tau^* > \tau^{fb}$, $Y^* < Y^{fb}$, *and* $\lambda^* > 1$. *Under the first-best situation, there are increasing returns to scale in producing water*: $C'(Y^{fb}) < C(Y^{fb})/Y^{fb}$.

Proof. We have $\alpha^* = (\lambda^* - 1)/\lambda^* < 1$ because $\lambda^* > 0$. A number π^{fb} is defined as $\pi^{fb} \equiv C'(Y^{fb})Y^{fb} - C(Y^{fb})$. First, assume that $\tau^* = \tau^{fb}$. In this case, we have $\pi^{fb} = \hat{\Pi}(\tau^{fb}) = \hat{\Pi}(\tau^*) = 0$ and $Y^* = \hat{Y}(\tau^*) = \hat{Y}(\tau^{fb}) = Y^{fb}$. We also have $\alpha^* = \hat{\alpha}(\tau^*) = \hat{\alpha}(\tau^{fb}) = 0$, which implies that $\lambda^* = 1$.

Second, assume that $\tau^* < \tau^{fb}$. Condition (33) is satisfied for $\tau = \tau^*$ because $\hat{\alpha}(\tau^*) = \alpha^* < 1$ and because of (25). By Theorem 1, for a sufficiently small number $\epsilon > 0$, we obtain $\alpha^* = \hat{\alpha}(\tau^*) < \hat{\alpha}(\tau^* + \epsilon) < 1$. This inequality and (25) together imply that $1/(\tau^* + \epsilon)H'(\tau^* + \epsilon) < \alpha^* < \hat{\alpha}(\tau^* + \epsilon) < 1$, which indicates that condition (33) is satisfied for $\tau = \tau^* + \epsilon$. Because of Theorem 1, for a sufficiently small number $\epsilon' > 0$, we have $\hat{\alpha}(\tau^* + \epsilon) < \hat{\alpha}(\tau^* + \epsilon + \epsilon') < 1$. Combining this result with (25), we obtain $1/(\tau^*+\epsilon+\epsilon')H'(\tau^*+\epsilon+\epsilon') < \alpha^* < \hat{\alpha}(\tau^*+\epsilon+\epsilon') < 1$, which shows that condition (33) is satisfied also for $\tau = \tau^* + \epsilon + \epsilon'$. Repeating this argument, we can prove that as τ increases from τ^* to τ^{fb}, $\hat{\alpha}(\tau)$ increases monotonically and reaches $\hat{\alpha}(\tau^{fb}) = 0$, and that condition (33) is satisfied on the interval $\tau^* \leq \tau \leq \tau^{fb}$. We therefore obtain $\alpha^* = \hat{\alpha}(\tau^*) < \hat{\alpha}(\tau^{fb}) = 0$, showing that $\lambda^* < 1$. Theorem 1 indicates that $Y(\tau)$ (resp. $\Pi(\tau)$) is decreasing (resp. increasing) on the interval $\tau^* \leq \tau \leq \tau^{fb}$. Consequently, we also obtain $Y^* = \hat{Y}(\tau^*) > \hat{Y}(\tau^{fb}) = Y^{fb}$ and $\pi^{fb} = \hat{\Pi}(\tau^{fb}) > \hat{\Pi}(\tau^*) = 0$.

Finally, assume that $\tau^* > \tau^{fb}$. Because $\hat{\alpha}(\tau^{fb}) = 0$, condition (33) holds true for $\tau = \tau^{fb}$. Theorem 1 implies that, for a sufficiently small number $\epsilon > 0$, we have $0 = \hat{\alpha}(\tau^{fb}) < \hat{\alpha}(\tau^{fb} + \epsilon) < 1$. Therefore, $1/(\tau^{fb} + \epsilon)H'(\tau^{fb} + \epsilon) < 0 < \hat{\alpha}(\tau^{fb} + \epsilon) < 1$, which shows that condition (33) is satisfied also for $\tau = \tau^{fb} + \epsilon$. By virtue of Theorem 1, we know that $\hat{\alpha}(\tau^{fb} + \epsilon) < \hat{\alpha}(\tau^{fb} + \epsilon + \epsilon') < 1$ for a sufficiently small number $\epsilon' > 0$. Repetition of this argument shows that, as τ rises from τ^{fb} to τ^*, $\hat{\alpha}(\tau)$ increases monotonically and reaches $\hat{\alpha}(\tau^*) = \alpha^* < 1$, and that condition (33) is satisfied on the interval $\tau^{fb} \leq \tau \leq \tau^*$. Accordingly, we have $0 = \hat{\alpha}(\tau^{fb}) < \hat{\alpha}(\tau^*) = \alpha^*$, which implies that $\lambda^* > 1$. It follows from Theorem 1 that $Y^* = \hat{Y}(\tau^*) < \hat{Y}(\tau^{fb}) = Y^{fb}$ and that $\pi^{fb} = \hat{\Pi}(\tau^{fb}) < \hat{\Pi}(\tau^*) = 0$.

We therefore established that one of the three cases described in the proposition can occur under the assumption of the proposition, thereby completing the proof. Q.E.D.

Proposition 2 suggests that given condition (25), if diseconomies (or economies) of scale exist in producing water under the first-best situation, then both the water supply and the number of customers connecting to the system are greater (or less) in the solution of problem (WP) than under the first-best situation. When condition (25) is satisfied, the shadow price λ^* can be either larger or smaller than 1 because the left-hand side (LHS) of the inequality in (25) is negative. Proposition 2 indicates that, in such a case, whether or not the shadow price λ^* is larger than 1 depends on the presence or absence of scale economies in producing water at the first-best optimum.

Having examined the size relationship between λ^* and 1, we can derive properties of the optimal water tariff. Let $T(x)$ denote the optimal charge for x units of water as defined from the solution $\{x^*(t), \tau^*, Y^*\}$. We consider a case where the consumption of a marginal customer, $x^*(\tau^*)$, gives the minimum purchase in the optimal water tariff, and where it is sold as a block for the minimum charge $T(x^*(\tau^*))$, in a way similar to that seen for the tariff T^{fb}.[16] We have $T(x^*(\tau^*)) = b(x^*(\tau^*), \tau^*)$ because the marginal customers gain zero surplus. On the other hand, let X_M denote the limit of the optimal water consumption as taste type approaches M, i.e., $X_M \equiv \lim_{t \to M} x^*(t)$. For $x \in [x^*(\tau^*), X_M)$, let $t^*(x)$ denote the value of t that satisfies condition $x^*(t) = x$; in other words, $t^*(x)$ stands for the type of customer who purchases x units of water under the optimal tariff. (Note that such a type is unique because $x^*(t)$ is strictly increasing on the interval $\tau^* \leq t < M$.) The first-order condition for maximizing customer utility implies that the optimal marginal price at consumption level $x \in [x^*(\tau^*), X_M)$ is given as $T'(x) = b_x(x, t^*(x))$.

If $C'(Y^{fb}) = C(Y^{fb})/Y^{fb}$, then $\tau^* = \tau^{fb}$, $Y^* = Y^{fb}$, and $x^*(t) = \bar{x}[t, Y^{fb}, 0] = x^{fb}(t)$; the optimal payment by a customer of type $t \geq \tau^{fb}$ is given as $T(x^*(t)) = T^{fb}(x^{fb}(t))$ because of equality (31). That is, the optimal tariff $T(\cdot)$ coincides with $T^{fb}(\cdot)$, realizing the first-best water allocation studied in Section 2. Its marginal price and minimum charge respectively equal the marginal production cost, $C'(Y^{fb})$, and the cost for providing an additional marginal customer with water services, $v + C'(Y^{fb})x^{fb}(\tau^{fb})$.

On the other hand, if $C'(Y^{fb}) >$ (or $<$) $C(Y^{fb})/Y^{fb}$, then $\alpha^* <$ (or $>$) 0, and Eq. (24) indicates that $T(x^*(\tau^*)) <$ (or $>$) $v + C'(Y^*)x^*(\tau^*)$: the optimal minimum charge is lower (or higher) than the cost for providing water services to an additional marginal customer. Using Eq. (23), the percentage profit margin at consumption level $x \in [x^*(\tau^*), X_M)$ is expressible as

$$\frac{T'(x) - C'(Y^*)}{T'(x)} = \frac{\alpha^* b_{xt}(x, t^*(x))I(t^*(x))}{b_x(x, t^*(x))} = \frac{\alpha^*}{\eta(x)}, \tag{34}$$

where $\eta(x) \equiv b_x(x, t^*(x))/b_{xt}(x, t^*(x))I(t^*(x))$ is the price elasticity of the water demand for an increment of consumption at consumption level x.[17)] If $C'(Y^{fb}) >$ (or $<$) $C(Y^{fb})/Y^{fb}$, then $\alpha^* <$ (or $>$) 0, and $T'(x) <$ (or $>$) $C'(Y^*)$, which indicates that the optimal marginal price is distorted below (or above) the marginal production cost. With the help of Eq. (23), η is expressible as $\eta(x) = \alpha^* + \gamma C'(Y^*)/H(t^*(x))$.[18)] On the interval $x^*(\tau^*) \leq x < X_M$, the elasticity $\eta(x)$ increases with x because $t^*(\cdot)$ is increasing, and because $H'(t) < 0$. Therefore, Eq. (34) implies that $T''(x) >$ (or $<$) 0 when $C'(Y^{fb}) >$ (or $<$) $C(Y^{fb})/Y^{fb}$. We thus establish the following proposition:

Proposition 3 *Assume the same conditions as those in Proposition 2. Let $T(x)$ denote the optimal charge for x units of water as defined from the solution $\{x^*(t), \tau^*, Y^*\}$. Then, we have*:

(I) *If constant returns to scale exist in producing water under the first-best situation (i.e. $C'(Y^{fb}) = C(Y^{fb})/Y^{fb}$), then the optimal water tariff $T(\cdot)$ coincides with the tariff $T^{fb}(\cdot)$ shown in Eq. (8).*

(II) *If decreasing returns to scale exist in producing water under the first-best situation (i.e. $C'(Y^{fb}) > C(Y^{fb})/Y^{fb}$), then (i) the optimal minimum charge satisfies $T(x^*(\tau^*)) < v + C'(Y^*)x^*(\tau^*)$, and (ii) the optimal marginal price $T'(x)$ is less than $C'(Y^*)$ and is increasing in water purchase x on the interval $x^*(\tau^*) \leq x < X_M$.*

(III) *If increasing returns to scale exist in producing water under the first-best situation (i.e. $C'(Y^{fb}) < C(Y^{fb})/Y^{fb}$), then (i) the optimal minimum charge satisfies $T(x^*(\tau^*)) > v +$*

$C'(Y^*)x^*(\tau^*)$, and (ii) the optimal marginal price $T'(x)$ is higher than $C'(Y^*)$ and is decreasing in water purchase x on the interval $x^*(\tau^*) \leq x < X_M$.

The rate structure of the optimal water tariff is demonstrably determined according to whether diseconomies or economies of scale exist in producing water under the first-best situation. The break-even constraint (15), which is deduced under the assumption that monetary transfers between the utility and disconnected customers are infeasible, plays a key role in deriving this result: If diseconomies of scale exist in producing water under the first-best situation, then $\hat{\Pi}(\tau^{fb}) = C'(Y^{fb})Y^{fb} - C(Y^{fb}) > 0$, i.e. the utility obtains excess profits when the water allocation is determined by solving the system of Eqs. (22) – (24) with τ set equal to τ^{fb}. By the increasing property of $\hat{\Pi}(\cdot)$ described in Theorem 1, the optimal marginal customer type τ^* must be less than τ^{fb} for the utility to avoid excess profits and fulfill the break-even constraint. In this case, because $\alpha^* = \hat{\alpha}(\tau^*) < \hat{\alpha}(\tau^{fb}) = 0$, Eqs. (23) and (24) prescribe that downward distortions be introduced into the marginal price and minimum charge. These adjustments encourage the purchase of water and connection to the system, thereby raising information rents for customers. The adjustments raise the total cost of water services and consequently enable the utility to satisfy the break-even constraint with restriction of profit. Conversely, if scale economies exist in producing water under the first-best situation, the utility incurs a loss when the water allocation is determined by solving the system of Eqs. (22) – (24) with τ set as equal to τ^{fb}. In such cases, the optimal marginal customer type must be greater than τ^{fb} for the utility to satisfy the break-even constraint (15) along with promotion of profit. The optimal water tariff then introduces upward distortions in the marginal price and minimum charge because $\alpha^* = \hat{\alpha}(\tau^*) > \hat{\alpha}(\tau^{fb}) = 0$ and because of Eqs. (23) and (24). Curtailing information rents to customers and the total cost of water services, those price distortions allow the utility to increase its profit to satisfy the break-even constraint. Thus, if diseconomies (resp. economies) of scale exist in producing water under the first-best case, then the optimal marginal price is distorted below (resp. above) the marginal production cost and increases (resp. decreases) with the quantity of water purchased because of the increasing property of the elasticity η.

We compare this result with two representative studies of nonlinear pricing of public utilities:

Goldman, Leland, and Sibley [9] studied nonlinear pricing of a public utility that maximizes the weighted sum of the utility's profit and consumer surplus, where the weight on the former is greater than that on the latter. Wilson [26, chapters 5, 6, and 8], on the other hand, examined nonlinear pricing of a public utility that maximizes the unweighted sum of the utility's profit and consumer surplus subject to the nonnegativity of the profit. In these studies, because of the specific settings, the optimal marginal price is necessarily greater than the marginal (production) cost except at the maximum consumption rate; the percentage profit margin of the optimal tariff is inversely proportional to the price elasticity of the demand for an increment of consumption. Therefore, quantity discounts are optimal if and only if the price elasticity of the demand for an increment of consumption increases with consumption. In our analysis, by contrast, whether the optimal marginal price of water is greater than the marginal production cost depends on the presence or absence of scale economies in producing water at the first-best solution because the water utility faces the break-even constraint (15), together with the infeasibility of monetary transfers with customers disconnected. As a result, while the price elasticity of the water demand for an increment of consumption increases with water consumption, both quantity premiums and discounts can be optimal in this study, depending on whether diseconomies or economies of scale exist in producing water under the first-best solution.

5. Concluding Remarks

In this paper, we have modeled a water market in which a monopolistic municipal water utility provides water services while incurring customer costs. We have investigated an optimal water tariff that maximizes consumer surplus in the water market under the constraint that the utility's tariff revenue collected from the customers connecting to the water service system match the total cost of water services. Under certain conditions, the analysis presented in this paper demonstrates that if diseconomies (resp. economies) of scale exist in producing water under the first-best situation, then (i) the marginal price in the optimal water tariff increases (resp. decreases) monotonically with the quantity of water purchased, and (ii) both the water supply and the customers connected to the system are more (resp. less) numerous under the optimal water tariff than under the first-best situation. It is thus demonstrated that the presence or absence of those diseconomies of scale can affect the rate structure and efficiency of the optimal water tariff.

This study has paid scant attention to cases in which the shadow price of the break-even constraint in problem (WP’) is so low that condition (25) fails. In such situations, generally, bunching of taste types is necessary to solve the optimal water consumption in problem (WP), and the necessary conditions for the optimality of (WP) become more complex than those described above. Even in such cases, however, it will be possible to define functions corresponding to $\hat{\alpha}$, $\hat{Y}$, and $\hat{\Pi}$ from the necessary conditions through a procedure similar to that described above. Examination of the properties of those defined functions will clarify how the results in Propositions 2 and 3 can be extended to those cases. Exploration of this issue is left as a focus for additional research.

Appendix: Proof of Theorem 1

First, to examine the signs of $\hat{Y}'(\tau)$ and $\hat{\alpha}'(\tau)$, we introduce a function ζ defined as

$$\zeta(\tau, Y, \alpha) \equiv Y - N\int_{\tau}^{M} \bar{x}[t, Y, \alpha] f(t)dt. \tag{35}$$

Using Eq. (26), the partial derivatives of ζ are given as follows:

$$\begin{aligned}
\zeta_\tau(\tau, Y, \alpha) &= N\bar{x}[\tau, Y, \alpha] f(\tau) > 0, && (36)\\
\zeta_Y(\tau, Y, \alpha) &= 1 + N\gamma C''(Y)\int_{\tau}^{M} \bar{x}[t, Y, \alpha] f(t)dt > 0, && (37)\\
\zeta_\alpha(\tau, Y, \alpha) &= N\int_{\tau}^{M} \bar{x}[t, Y, \alpha] H(t) f(t)dt > 0. && (38)
\end{aligned}$$

By definition, $\bar{Z}(\tau, Y) = \zeta(\tau, Y, \bar{\alpha}(\tau, Y))$. Differentiating this equality with respect to τ, and incorporating $Y = \hat{Y}(\tau)$, we deduce the following:

$$\bar{Z}_\tau(\tau, \hat{Y}) = \zeta_\tau(\tau, \hat{Y}, \bar{\alpha}(\tau, \hat{Y})) + \zeta_\alpha(\tau, \hat{Y}, \bar{\alpha}(\tau, \hat{Y}))\bar{\alpha}_\tau(\tau, \hat{Y}) = \zeta_\tau(\tau, \hat{Y}, \hat{\alpha}) + \zeta_\alpha(\tau, \hat{Y}, \hat{\alpha})\bar{\alpha}_\tau(\tau, \hat{Y}), \tag{39}$$

where $\hat{Y} = \hat{Y}(\tau)$ and $\hat{\alpha} = \hat{\alpha}(\tau) = \bar{\alpha}(\tau, \hat{Y}(\tau))$. From the above, we have $\bar{Z}_\tau(\tau, \hat{Y}) > 0$ because of (36) and (38) and because inequality (32) is satisfied under condition (33). We also have $\bar{Z}_Y(\tau, \hat{Y}) > 0$ by virtue of Eq. (29). We thus show that $\hat{Y}'(\tau) = -\bar{Z}_\tau(\tau, \hat{Y})/\bar{Z}_Y(\tau, \hat{Y}) < 0$.

Because of inequality (32), and because $\bar{\alpha}_Y(\tau, Y) \leq 0$, we then establish that $\hat{\alpha}'(\tau) = \bar{\alpha}_\tau(\tau, \hat{Y}) + \bar{\alpha}_Y(\tau, \hat{Y})\hat{Y}'(\tau) > 0$.

Differentiating $\bar{Z}(\tau, Y) = \zeta(\tau, Y, \bar{\alpha}(\tau, Y))$ with regard to Y, and incorporating $Y = \hat{Y}(\tau)$, we get

$$\bar{Z}_Y(\tau, \hat{Y}) = \zeta_Y(\tau, \hat{Y}, \bar{\alpha}(\tau, \hat{Y})) + \zeta_\alpha(\tau, \hat{Y}, \bar{\alpha}(\tau, \hat{Y}))\bar{\alpha}_Y(\tau, \hat{Y}) = \zeta_Y(\tau, \hat{Y}, \hat{\alpha}) + \zeta_\alpha(\tau, \hat{Y}, \hat{\alpha})\bar{\alpha}_Y(\tau, \hat{Y}), \quad (40)$$

where $\hat{Y} = \hat{Y}(\tau)$ and $\hat{\alpha} = \hat{\alpha}(\tau) = \bar{\alpha}(\tau, \hat{Y}(\tau))$. Using (39) and (40), $\hat{Y}'(\tau)$ is expressible as

$$\hat{Y}'(\tau) = -\frac{\zeta_\tau(\tau, \hat{Y}, \hat{\alpha}) + \zeta_\alpha(\tau, \hat{Y}, \hat{\alpha})\bar{\alpha}_\tau(\tau, \hat{Y})}{\zeta_Y(\tau, \hat{Y}, \hat{\alpha}) + \zeta_\alpha(\tau, \hat{Y}, \hat{\alpha})\bar{\alpha}_Y(\tau, \hat{Y})}. \quad (41)$$

If Eq. (41) is substituted into $\hat{\alpha}'(\tau) = \bar{\alpha}_\tau(\tau, \hat{Y}) + \bar{\alpha}_Y(\tau, \hat{Y})\hat{Y}'(\tau)$, then $\hat{\alpha}'(\tau)$ is transformed as

$$\hat{\alpha}'(\tau) = \frac{\zeta_Y(\tau, \hat{Y}, \hat{\alpha})\bar{\alpha}_\tau(\tau, \hat{Y}) - \zeta_\tau(\tau, \hat{Y}, \hat{\alpha})\bar{\alpha}_Y(\tau, \hat{Y})}{\zeta_Y(\tau, \hat{Y}, \hat{\alpha}) + \zeta_\alpha(\tau, \hat{Y}, \hat{\alpha})\bar{\alpha}_Y(\tau, \hat{Y})}. \quad (42)$$

Next, substituting $x(t) = \bar{x}[t, Y, \alpha]$ into Eq. (11), we define a function π as

$$\begin{aligned} \pi(\tau, Y, \alpha) &\equiv N \int_\tau^M \left\{ b(\bar{x}[t, Y, \alpha], t) - \int_\tau^t b_t(\bar{x}[s, Y, \alpha], s) ds \right\} f(t) dt - vN\bar{F}(\tau) - C(Y) \\ &= N \int_\tau^M \left[\{ b(\bar{x}[t, Y, \alpha], t) - v \} f(t) - b_t(\bar{x}[t, Y, \alpha], t)\bar{F}(t) \right] dt - C(Y). \quad (43) \end{aligned}$$

Furthermore, we define functions A and B as

$$A(\tau, Y, \alpha) \equiv \pi_\alpha(\tau, Y, \alpha)\zeta_Y(\tau, Y, \alpha) - \pi_Y(\tau, Y, \alpha)\zeta_\alpha(\tau, Y, \alpha), \quad (44)$$

$$\begin{aligned} B(\tau, Y) \equiv\ & \pi_\tau(\tau, Y, \bar{\alpha}(\tau, Y))\zeta_Y(\tau, Y, \bar{\alpha}(\tau, Y)) - \pi_Y(\tau, Y, \bar{\alpha}(\tau, Y))\zeta_\tau(\tau, Y, \bar{\alpha}(\tau, Y)) \\ & + \{ \pi_\tau(\tau, Y, \bar{\alpha}(\tau, Y))\zeta_\alpha(\tau, Y, \bar{\alpha}(\tau, Y)) - \pi_\alpha(\tau, Y, \bar{\alpha}(\tau, Y))\zeta_\tau(\tau, Y, \bar{\alpha}(\tau, Y)) \}\bar{\alpha}_Y(\tau, Y). \end{aligned} \quad (45)$$

By definition, $\hat{\Pi}(\tau) = \pi(\tau, \hat{Y}(\tau), \hat{\alpha}(\tau))$. Incorporation of Eqs. (41) and (42) enables representation of the derivative $\hat{\Pi}'(\tau)$ as

$$\begin{aligned} \hat{\Pi}'(\tau) &= \pi_\tau(\tau, \hat{Y}, \hat{\alpha}) + \pi_Y(\tau, \hat{Y}, \hat{\alpha})\hat{Y}'(\tau) + \pi_\alpha(\tau, \hat{Y}, \hat{\alpha})\hat{\alpha}'(\tau) \\ &= \frac{A(\tau, \hat{Y}, \hat{\alpha})\bar{\alpha}_\tau(\tau, \hat{Y}) + B(\tau, \hat{Y})}{\zeta_Y(\tau, \hat{Y}, \hat{\alpha}) + \zeta_\alpha(\tau, \hat{Y}, \hat{\alpha})\bar{\alpha}_Y(\tau, \hat{Y})}, \quad (46) \end{aligned}$$

where $\hat{Y} = \hat{Y}(\tau)$ and $\hat{\alpha} = \hat{\alpha}(\tau) = \overline{\alpha}(\tau, \hat{Y}(\tau))$. We have $\zeta_Y(\tau, \hat{Y}, \hat{\alpha}) + \zeta_\alpha(\tau, \hat{Y}, \hat{\alpha})\overline{\alpha}_Y(\tau, \hat{Y}) = \overline{Z}_Y(\tau, \hat{Y}) > 0$, as shown before. Thus, in order to complete the proof, it suffices to verify that $A(\tau, \hat{Y}, \hat{\alpha})\overline{\alpha}_\tau(\tau, \hat{Y}) + B(\tau, \hat{Y}) > 0$.

Differentiating Eq. (43) with respect to Y, and using the fact that Eq. (23) holds true for $x(t) = \overline{x}[t, Y, \alpha]$, we obtain the following expression for π_Y:

$$
\begin{aligned}
\pi_Y(\tau, Y, \alpha) &= N \int_\tau^M \left\{ b_x(\overline{x}[t, Y, \alpha], t) f(t) - b_{xt}(\overline{x}[t, Y, \alpha], t)\overline{F}(t) \right\} \overline{x}_Y[t, Y, \alpha] dt - C'(Y) \\
&= N \int_\tau^M \left\{ (\alpha - 1) b_{xt}(\overline{x}[t, Y, \alpha], t)\overline{F}(t) + C'(Y) f(t) \right\} \overline{x}_Y[t, Y, \alpha] dt - C'(Y) \\
&= NC''(Y)(1 - \alpha) \int_\tau^M \overline{x}[t, Y, \alpha] H(t) f(t) dt - C'(Y)\zeta_Y(\tau, Y, \alpha),
\end{aligned} \tag{47}
$$

where Eq. (37) is used to deduce the last equality. Similarly, the partial derivative π_α is expressible as

$$
\begin{aligned}
\pi_\alpha(\tau, Y, \alpha) &= N \int_\tau^M \left\{ b_x(\overline{x}[t, Y, \alpha], t) f(t) - b_{xt}(\overline{x}[t, Y, \alpha], t)\overline{F}(t) \right\} \overline{x}_\alpha[t, Y, \alpha] dt \\
&= N \int_\tau^M \left\{ (\alpha - 1) b_{xt}(\overline{x}[t, Y, \alpha], t)\overline{F}(t) + C'(Y) f(t) \right\} \overline{x}_\alpha[t, Y, \alpha] dt \\
&= \frac{N(1 - \alpha)}{\gamma} \int_\tau^M \overline{x}[t, Y, \alpha] H^2(t) f(t) dt - C'(Y)\zeta_\alpha(\tau, Y, \alpha),
\end{aligned} \tag{48}
$$

where Eq. (38) is used to obtain the last equality. Substituting Eqs. (37), (38), (47), and (48) into Eq. (44), and rearranging the terms, we can rewrite $A(\tau, Y, \alpha)$ as

$$
\begin{aligned}
A(\tau, Y, \alpha) = N^2(1-\alpha)C''(Y)\Bigg[&\int_\tau^M \overline{x}[t, Y, \alpha] H^2(t) f(t) dt \int_\tau^M \overline{x}[t, Y, \alpha] f(t) dt \\
&- \left\{ \int_\tau^M \overline{x}[t, Y, \alpha] H(t) f(t) dt \right\}^2 \Bigg] + \frac{N(1-\alpha)}{\gamma} \int_\tau^M \overline{x}[t, Y, \alpha] H^2(t) f(t) dt.
\end{aligned} \tag{49}
$$

Application of Schwarz's inequality yields the following because $H(t)$ is a decreasing function:

$$
\begin{aligned}
\left\{ \int_\tau^M \overline{x}[t, Y, \alpha] H(t) f(t) dt \right\}^2 &= \left\{ \int_\tau^M H(t) \sqrt{\overline{x}[t, Y, \alpha] f(t)} \sqrt{\overline{x}[t, Y, \alpha] f(t)} dt \right\}^2 \\
&< \int_\tau^M \overline{x}[t, Y, \alpha] H^2(t) f(t) dt \int_\tau^M \overline{x}[t, Y, \alpha] f(t) dt.
\end{aligned}
$$

Consequently, if $\alpha < 1$, then Eq. (49) implies that $A(\tau, Y, \alpha) > 0$. Given condition (33), we thus obtain $A(\tau, \hat{Y}(\tau), \hat{\alpha}(\tau))\overline{\alpha}_\tau(\tau, \hat{Y}(\tau)) > 0$ using inequality (32).

On the other hand, by the definition of $\bar{\alpha}$, the following equality pertains [see Eq. (24)]:

$$b(\bar{x}[\tau, Y, \bar{\alpha}], \tau) = \bar{\alpha} b_t(\bar{x}[\tau, Y, \bar{\alpha}], \tau) I(\tau) + v + \bar{x}[\tau, Y, \bar{\alpha}] C'(Y),$$

where $\bar{\alpha} = \bar{\alpha}(\tau, Y)$. Differentiating Eq. (43) with respect to τ, and incorporating the above, we obtain

$$\begin{aligned} \pi_\tau(\tau, Y, \alpha) &= -N\left[\{b(\bar{x}[\tau, Y, \alpha], \tau) - v\} f(\tau) - b_t(\bar{x}[\tau, Y, \alpha], \tau)\bar{F}(\tau)\right] \\ &= -N\left\{(\bar{\alpha} - 1) b_t(\bar{x}[\tau, Y, \bar{\alpha}], \tau)\bar{F}(\tau) + \bar{x}[\tau, Y, \bar{\alpha}] C'(Y) f(\tau)\right\} \\ &= N(1 - \bar{\alpha})\bar{x}[\tau, Y, \bar{\alpha}] H(\tau) f(\tau)/\gamma - C'(Y)\zeta_\tau(\tau, Y, \bar{\alpha}), \end{aligned} \tag{50}$$

where Eq. (36) is used to derive the last equality. Inserting Eqs. (36) – (38), (47), (48), and (50) into Eq. (45), and rearranging the terms, we can transform $B(\tau, Y)$ as follows:

$$B(\tau, Y) = \frac{N(1 - \bar{\alpha})\bar{x}[\tau, Y, \bar{\alpha}] f(\tau)}{\gamma H(\tau)} \left[H^2(\tau) + N\gamma C''(Y) \int_\tau^M \bar{x}[t, Y, \bar{\alpha}]\{H(t) - H(\tau)\}^2 f(t) dt\right],$$

where $\bar{\alpha} = \bar{\alpha}(\tau, Y)$. Hence, $B(\tau, Y) > 0$ if $\bar{\alpha}(\tau, Y) < 1$. Accordingly, $B(\tau, \hat{Y}(\tau)) > 0$ if $\bar{\alpha}(\tau, \hat{Y}(\tau)) = \hat{\alpha}(\tau) < 1$. Because of these results and Eq. (46), we have $\hat{\Pi}'(\tau) > 0$ under condition (33). Q.E.D.

NOTES

[1] Even in developed countries, some households living in suburbs and rural areas where the availability and quality of groundwater are favorable choose to use private wells for a potable water supply rather than connect to water service systems. See Stone [23] for information on residential groundwater use in the US.

[2] External diseconomies of scale refer to diseconomies of scale that are external to any one firm in a given industry. More specifically, an industry is subject to external diseconomies of scale if the long-run average cost of the industry rises concomitant with the supply, whether the industry comprises one firm or many (Bonbright [2, p. 16]; Kahn [14, Vol. II, p. 124]).

[3] Bonbright [2, p. 16] characterized internal economies of scale as: “the economies enjoyed by a monopolistic utility company through its ability to make use of larger generating equip-

ment and of a more capacious distribution network are referred to as internal economies — economies internal to a given firm or company.”

4) Kim [15] confirmed empirically that large water utilities in the US suffer from diseconomies of scale, whereas small ones enjoy substantial economies of scale. That study suggested that in water utilities in the US, scale economies tend to be exhausted as the size of water services grows. Kahn [14, Vol. II, p. 124] illustrated the fact that an industry subject to external diseconomies of scale can be a natural monopoly if internal economies of scale exist in the industry. Kahn cited municipal water supply as an example of that phenomenon.

5) Schmalensee [21] and Sherman and Visscher [22] have shown similar results.

6) Taxation of the excessive profit accrued by the first-best two-part tariff can be a solution to this revenue-and-cost mismatch problem. However, the American Water Works Association [1, p. 25] notes that “municipally owned utilities are not normally subject to taxation by local, state, or federal authorities.” As confirmed empirically by Dalhuisen et al. [5], residential water demand tends to be income-inelastic, which implies that the payments by lower-income households to water utilities are higher in relation to their income levels. The taxation therefore has a regressive impact on lower-income households, which will make it impractical and difficult for authorities to introduce the taxation.

7) Similar concerns are echoed in the manual of water rate determination, published by the American Water Works Association:

> A tenable solution to the revenue and cost mismatching is critical to both managerial and public acceptance of the concept of marginal cost as a basis for rates. Mismatching arises because pure marginal cost rates generate greater revenues than the utility’s rate revenue requirements, demanding utility decision-makers to balance potential efficiency benefits of marginal cost rates with the difficulties of excess revenue generation (American Water Works Association [1, p. 121]).

8) Allowing for the influences of customer costs and exclusions, Wilson [26, chapters 6 and 8] examined nonlinear pricing of a public utility that maximizes the total surplus under the constraint that the utility should obtain a nonnegative profit. That study placed no restriction

on the upper bound of the utility's profit, nor did it address the possibility that the supply conditions of the utility exhibit external diseconomies of scale.

9) See Dinar and Subramanian [6] and McIntosh and Yniguez [17].

10) Hall and Hanemann [12] argued that if marginal cost pricing generates too much revenue for a water utility, IBTs can be an efficient way for the water utility to equate the actual revenue to the required revenue (see also Hall [11]). Boland and Whittington [2] challenged this view, claiming that when marginal cost pricing brings excess revenues to a water utility because of diseconomies of scale in water services, a pricing policy exists that achieves a higher economic efficiency than IBTs without collecting too much revenue. Both of these arguments are not based on a formal model analysis. Their validity has never been theoretically tested.

11) For more detailed classifications of water-supply cost components, see American Water Works Association [1] and Elnaboulsi [7].

12) In the model described herein, to preserve analytical simplicity, seasonal variations and uncertainty in the demands for water and in the costs of water services are ignored. Furthermore, differences in characteristics between the residential, commercial, and industrial demand for water are ignored.

13) When analyzing the efficiency of the water pricing policy in the city of Vigo in Spain, Castro-Rodriguez, Da-Rocha, and Delicado [4] assumed that the marginal willingness to pay for an additional unit of water is linear in water consumption, and that differences among customers are attributable to the different levels of satiation in water consumption.

14) In this paper, subscripted variables denote partial derivatives with respect to the subscripted variable.

15) In contrast to the assumptions used for the present study, Wilson [26, p. 126] assumed that the total cost for a regulated firm is expressible as the sum of the costs of supplying the product or service to each customer. Consequently, the conditions for the optimality of the marginal customer type derived in Wilson [26, p. 160 and p. 187] take slightly different forms from Eq. (24).

16) Alternatively, as described in Wilson [26, p. 159 and p. 187], the water utility can be

assumed to extend the tariff schedule to purchase levels below the consumption of a marginal customer, $x^*(\tau^*)$. However, we avoid making such an assumption because it makes the study of the optimal tariff schedule more complicated.

17) See Goldman, Leland, and Sibley [9] for explanations on the price elasticity of the demand for an increment of consumption.

18) Inserting $x(t) = x^*(t)$, $Y = Y^*$, and $\alpha = \alpha^*$ into Eq. (23), and dividing it by $b_{xt}(x^*(t), t)I(t) = H(t)/\gamma$, we obtain $b_x(x^*(t), t)/b_{xt}(x^*(t), t)I(t) = \alpha^* + \gamma C'(Y^*)/H(t)$. Incorporation of $t = t^*(x)$ into this equation produces $\eta(x) = \alpha^* + \gamma C'(Y^*)/H(t^*(x))$.

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