Individual Memory in Extensive Games and Info-memory Protocols

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ゲーム理論は、複数の意思決定主体（プレーヤー）の相互作用に関する理論である。そこでは、推論・知識・信念・規範・情報・記憶などのプレーヤーの心的内部構造が大きな役割を果たすはずだが、既存の理論はこれらの構造を十分に議論していない。本稿では、展開形ゲームに代わるフレームワーク「Info-memory protocol」を提唱する。そして、ゲームにおける情報と記憶について議論し、「経験に基づいたプレーヤーの社会観形成」に関する研究の足がかりとする。

「展開形ゲーム」はゲーム論の最も基礎的な理論の一つである。その理論は、プレーヤーが利用できる情報や、各行動のタイミングや因果関係、戦略的可能性、プレーヤーの目的、記憶などを表現している。

しかし、展開形ゲームの理論は、その定式化にいくつかの困難を抱えている。展開形ゲームは、プレーヤーが「学んだこと」と「行なったこと」に関する記憶を表現することができる。ただ、「忘却」を「可能性を区別できない」という形で表現する。問題点は、「忘却」を「可能性の集合」として記述し、それらの可能性そのものは分かっていると仮定することにある。これでは、プレイヤーの記憶を有意味に論ずることはできない。また、展開形ゲームは「経験できること」と「経験できないこと」の区別がない。

このような展開形ゲームの問題点を克服するため、Kaneko & Kline は「Info-memory protocol」を導入した。ここでは、記憶は、Information protocol における記憶関数として記述される。記憶関数によって、プレイヤーの忘却を「そもそも記憶がない」という状態として記述する。また、理論の原子的要素として、プレイヤーが経験できるものから出発する。このように上記の展開形ゲームの困難を克服し、プレイヤーの記憶と忘却を有意味にする。それによって、「各プレーヤーが自分の世界観を過去の記憶から構成する」ことの研究が可能になる。

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1. Individual Memory in Extensive Games

John von Neumann and Oskar Morgenstern formulated extensive games in their book: "Theory of Games and Economic Behavior" (1944). A few years later, Kuhn (1953) gave a simpler description of an extensive game, which we will follow here. The description has proved very beneficial for analysing strategic situations with several players. It is general enough to incorporate differences in: the information available to the various players, the timing and sequence of moves, strategic possibilities, objectives of the players, and even memory.

Since our focus is on individual memory here, we use examples involving only one player. Let's see how the memory of a player can be described by an extensive game. Individual forgetfulness can be broken into a forgetting *what one did*, and forgetting *what one learned.*

To show the first type of forgetfulness, consider a professor who drives to work each morning and parks his car in Lot A or Lot B. The professor is typically pre-occupied with deep thoughts, and by the end of the day he has forgotten where he parked that morning. Lot A is located to the East of the professor's office, and Lot B is located to the West. This situation is described in Figure 1.

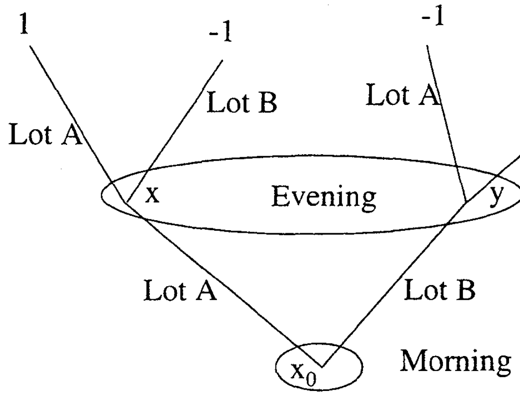

Figure 1

The bottom bubble in Figure 1 labelled "Morning" describes the situation the professor faces in the morning when he chooses either "Lot A" or "Lot B". The top bubble labelled "Evening" describes the situation he faces in the evening when he must choose to walk to "Lot A" or "Lot B". In order to capture his inability to recall what he did in the "Morning", we have put the two possibilities "x" and "y" in the same bubble. Possibility "x" occurs after "Lot A" was chosen in the morning and possibility "y" occurs after "Lot B" was chosen.

A basic assumption in game theory is that when a player plays the game, he will know when it is his move, i.e., which bubble he is at, but he will not be able to distinguish between the different possibilities in that bubble. If we wanted to capture a player who does not forget what he does in the morning, then "x" and "y" should appear in separate bubbles.

The above basic assumption in game theory is also typically accompanied by the presumption that each player knows the game, or essentially has a copy of the game tree in his possession. This assumption gives the player knowledge not necessarily of what he forgot, but at least of the set of possibilities, i.e., in the "Evening" either "he chose Lot A in the morning" or "he chose Lot B in the morning". We will come back to this assumption later in both this section and the next section where we argue that such knowledge on the set of possibilities would be too much in some contexts.

Now, we return to the game tree in question. The numbers at the top of the figure represent the payoffs of the professor. If he chooses the same Lot in both the "Morning" and the "Evening", then he gets a payoff of 1. If, however, his choice in the "Morning" differs from his choice in the "Evening", he winds up in the wrong parking lot and his payoff is -1. The extensive game tree of Figure 1 captures the forgetfulness of the professor about what he did. The negative payoffs capture the potential cost of his forgetfulness.

Extensive Games can also be used to capture forgetfulness of what a person learned. For example, a person might have found sometime in the past that the film "Amadeus" won the academy award for best picture in 1984. When later playing a trivia game, he recalls that it was either 1984 or 1985. His forgetfulness can be described in the following extensive game, where he wins a payoff of 1 if he chooses the correct year and zero if he chooses incorrectly.

To model forgetfulness here, we introduce a fictitious player called "Nature". The role of "Nature" is to describe multiple possibilities. The player's inability to recall what he actually learned may be interpreted as a belief that he might have learned something else. To incorporate this, we have extended the game to include the right path as another possibility. This path means that Amadeus won in 1985 and the player learned this. While this possibility never actually happened, it appears in the mind of the player when he is playing the "Trivia Game". In this way, we can describe a player who forgets what he learned.

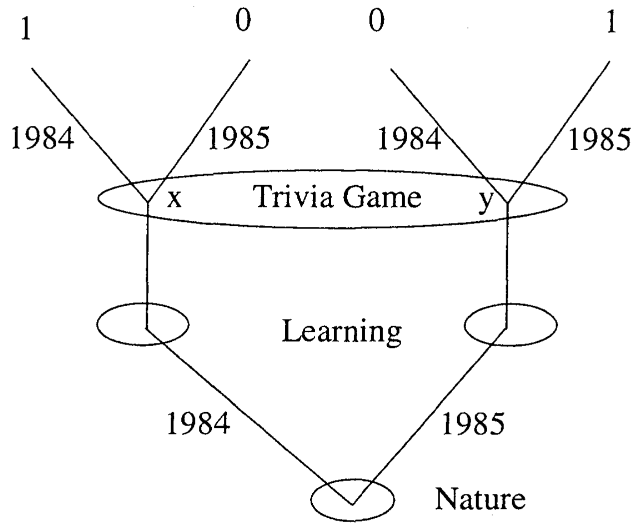

Figure 2

By these two examples, we see that extensive games can be used to describe forgetfulness about both what a player did and what he learned.[3] In each example, the forgetfulness was described using multiple possibilities that the player could not distinguish between.

In extensive games, forgetfulness is always represented by multiple possibilities. This is the same way that extensive games treat other types of incomplete or imperfect information. A reduction in information is translated into more possibilities. The most extreme case of no information may then be interpreted as the largest, possibly an infinite, set of possibilities. No memory could be treated in much the same way, i.e., as thinking there are a large set of possible things I might have done and/or learned.

Recall that with the standard assumption that the player has the game tree, he would always be able to determine the set of possible things he might have forgotten.

On the other hand, having limited, or no memory might not quite be the same as thinking a large number of things could have happened. In fact, a player with very limited memory might not think anything happened in the past, or might not be aware of anything having happened. In this case, we would not be able to accurately describe the player's lack of memory by a game theoretic model, or at least not by one that the player is aware of. Recall that a player with the game tree would be given the complete set of possible histories at each of his moves.

[3] In the literature of extensive games, several formal conditions on a player's information partition have been used to describe various types of memory starting with Kuhn's (1953) condition of perfect recall. Other conditions involving less than perfect recall are given and discussed by Okada (1987), Kaneko-Kline (1995), Piccione and Rubinstein (1997) and Bonanno (2002).

2. Individual Memory in Info-memory Protocols

Kaneko and Kline (2003) developed Info-memory protocols to describe individuals who might have no good idea about the complete set of possible histories. Rather than carving the set of possibilities using their experiences, these individuals might build up their view of the world from their memories of those experiences. To describe such players, it is important to allow for limited memory that is not described simply by sets of possibilities.

In order to do this, memory is described in an info-memory protocol by a *memory function*. While an extensive game can be used to describe individual memory of a player as we saw in the previous section, Kaneko and Kline take the position that the memory should be described separately. They still allow the extensive game to describe the *transmission of information* to players. For example, the game of Figure 1 describes the following transmission of information:

(a) In the Morning, information is transmitted to the player that he can park at Lot A or Lot B, and nothing else.

(b) In the Evening, information is transmitted to the player that he can walk to Lot A or Lot B, and nothing else.

In this way the extensive game is used to show that the information about the set of possible actions is transmitted to the player at each of his moves.

This says nothing about his memory and is consistent with players having various types of memory ranging from complete forgetfulness to perfect memory.

Continuing with this example, if we want to describe a player who forgets what he did, we could use the following memory function.

(1) $m(x) = m(y) = m(x_0) = \{ \text{⬣} \}$.

No matter what the situation is, the player recalls nothing about the past. The symbol ⬣ is used here to describe a memory of nothing.

To describe a player who recalls what he did, we might use the following memory function:

(2) $m(x_0) = \{ \text{⬣} \}$, $m(x)$={Lot A in morning} $m(y)$ ={Lot B in morning}.

Notice that the memory function assigns a memory to the player at each of the objective possibilities x_0, x, and y. The player need not be aware of this set; he simply has the memory $m(x_0)$ in the morning. Then in the evening, he has either the memory $m(x)$ or $m(y)$. We use set theoretic brackets in describing the value of a memory to allow the player's forgetfulness to involve a set of possibilities. For example, consider the following memory function:

(3) $m(x_0) = \{ \bullet \}$, $m(x)=m(y)=\{$"Lot A in morning", "Lot B in morning"$\}$.

The memory function (3) describes a player who remembers in the Evening that either he parked in Lot A in the morning or he parked in Lot B in the morning. This is different from the memory function (1) which describes a player who has the memory of nothing each time he moves.

We return now to the standard assumption in extensive game theory that the player knows the game tree. If the player is fully aware of the game tree described in Figure 1, then the memory functions (1) and (3) would leave him with equivalent information.

However, if we want to describe players who are not aware of the game tree, the distinction between (1) and (3) could be relevant. Suppose the player wanted to construct a view of the game he is playing. Since (3) has more ingredients than what is in (1), we might expect a better construction from a player with memory described by (3). The description of (1) gives only memories of nothing. The description of (3) gives memories of "Lot A in the morning" or "Lot B in the morning".

The additional structure of a memory function enhances our description of a game situation. Kaneko and Kline (2004) are using the memory function and its contents to describe how a player might construct his view of the game situation from his memories of past experiences. Because his experiences and memories are limited, so might be his view of the game situation.

In addition to this, we believe the description of memory by memory functions could have other applications in describing different types of memory that have escaped game theoretic analysis.

We end with one such example. Lisa has just been informed that a movie will be made of a book she read in the past. The book and movie are called: "Divine Secrets of the Ya Ya Sisterhood." Jeff asks Lisa what the meaning of the Ya Ya Sisterhood is. Lisa responds that she cannot recall the meaning, but that if she saw it in a list, she would know it.

We might simply describe Lisa's memory at the current true situation "x" by the memory function:

(4) $m(x)=\{$"Ya Ya Sisterhood means something I knew before and will recall if I see it in a list"$\}$

This, more or less, captures the situation at hand. Suppose, however, that we try to use an extensive game to describe this situation, and let Lisa know the game. The game tree will be constructed from a set of possible meanings for Ya Ya Sisterhood. If this set includes the truth, then Lisa will find the true meaning when she is made aware of the tree. If, on the contrary, the set does not include the truth, then while one could argue that this extensive game describes a situation like Lisa's, there are two difficulties.

First, we would now need a separate extensive game to describe the objective truth, by which I mean the true meaning of Ya Ya Sisterhood. A single extensive game could no longer simultaneously describe Lisa's memory and the objective truth. In contrast, a single info-memory protocol, by separating memory from the objective truth, allows for such a description, and involves no false memories.

Secondly, I don't believe that an incomplete list of possibilities would accurately describe Lisa's situation. In contrast, it would artificially introduce false memories that Lisa does not have in the form of false possibilities. The same criticism can be made of the description of forgetfulness in Figure 2 where a false memory was introduced to describe forgetfulness of the year Amadeus won the Oscar.

3. References

Bonanno, G., (2002), Memory of Past Beliefs and Actions, University of California, Davis, Working Paper.

Kaneko, M., and J. Kline (1995), Behavior Strategies, Mixed Strategies and Perfect Recall, *International Journal of Game Theory* 24, 127-145.

Kaneko, M. and J. Kline (2003), Modeling a Player's Perspective I: Info-memory protocols, Working Paper, Tsukuba University.

Kaneko, M. and J. Kline (2004), Modeling a Player's Perspective II: Inductive Derivations of an Individual View, In Progress.

Kuhn, H.W., (1953), Extensive Games and the Problem of Information, *Contributions to the Theory of Games II*, Kuhn, H.W., and A.W. Tucker, eds., 193-216. Princeton University Press.

Okada, A. (1987), Complete Inflation and Perfect Recall in Extensive Games, *International Journal of Game Theory* 16, 85-91.

Piccione, M., and Rubinstein, A. (1997), On the Interpretation of Decision Problems with Imperfect Recall, *Games and Economic Behavior* 20, 3-24.