Dynamical versus static imperfections in quantum computers

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Abstract

We study the effects of imperfections in a spin model of a quantum computer. We identify different regimes, ranging from low-frequency fluctuations, where the imperfections can be considered static, to the high-frequency case, where the imperfections are purely dynamical and their effects are shown to be completely wiped out.

Dynamical errors, yielding decoherence, can be extremely detrimental in quantum computation [1]. On the other hand, the role of static imperfections, such as small inaccuracies in the coupling constants, is often considered on a different footing, as these do not induce, strictly speaking, any decoherence, but rather errors that can be tolerated up to a certain threshold [2]. Also, the role of static imperfections is regime dependent, and can be utilized as an indicator of an underlying chaotic dynamics [2].

However, *strictu senso*, a discrimination between "static" imperfections and "dynamical" noise is given by the physics and depends on the speed of the quantum computer: dynamical noise plays the role of static imperfections, if its timescale is much larger than the computational time. We intend to explore this problem in more details and discuss the suggestion [3] that static imperfections can be more disruptive than noise for quantum computation.

We model a quantum computer as a lattice of interacting spins (qubits). Due to the imperfections, the couplings between the qubits and with an external field are both random and fluctuate in time. We consider n qubits on a d-dimensional lattice, described by the Hamiltonian

$$H_\tau(t) = \sum_{j=1}^{n} [\Delta_0 + \delta_j(t)] \sigma_z^{(j)} + \sum_{\langle i,j \rangle} J_{ij}(t) \sigma_x^{(i)} \sigma_x^{(j)}, \tag{1}$$

where the $\sigma_\alpha^{(i)}$'s ($\alpha = x, y, z$) are the Pauli matrices for qubit i and the second sum runs over nearest-neighbor pairs. The number n_c of terms in the second sum depends both on the arrangement and dimensionality and is proportional to nd. The energy spacing between the up and down states of a qubit is $\Delta_0 + \delta_i(t)$, where the $\delta_i(t)$'s are uniformly distributed in the interval $[-\delta/2, \delta/2]$ and the $J_{ij}(t)$'s in the interval $[-J, J]$ (zero means and variances $\delta^2\sigma^2$ and $4J^2\sigma^2$, respectively, with $\sigma^2 = 1/12$).

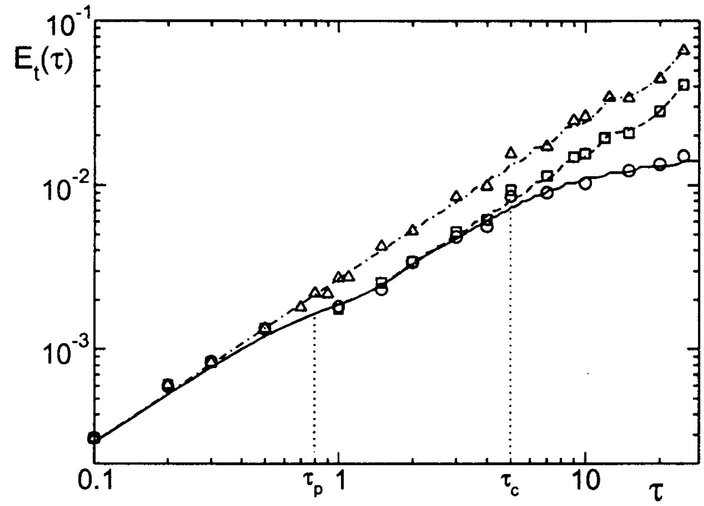

FIG. 1: Error E as a function of τ for $t = 25, n = 10, J = 5 \times 10^{-3}$, in the ergodic regime $\delta = 5 \times 10^{-3} = J$, with $n_{\uparrow\downarrow} = 8, n_{\uparrow\uparrow} = 5$ (squares), $n_{\uparrow\downarrow} = 13, n_{\uparrow\uparrow} = 0$ (triangles), and in the FGR regime $\delta = 3 \times 10^{-1} \ll J$, with $n_{\uparrow\downarrow} = 8, n_{\uparrow\uparrow} = 5$ (circles). We set $\sigma^2 = 1/12$, $n_c = 13$, $\Delta_0 = 1$. The fits are given by Eq. (3) with $n_{\uparrow\downarrow} = 8, n_{\uparrow\uparrow} = 5$ (dashed), $n_{\uparrow\downarrow} = 13, n_{\uparrow\uparrow} = 0$ (dot-dashed). The transition at τ_c is shown only in the former case.

We model the dynamical noise by supposing that both $\delta_i(t)$ and $J_{ij}(t)$ randomly change after each time interval τ and are constant otherwise.

For $J = \delta = 0$ the spectrum of the Hamiltonian is composed of $n + 1$ degenerate levels, with interlevel spacing $2\Delta_0$, corresponding to the energy required to flip a single qubit. We study the case $0 < \delta, J \ll \Delta_0$, in which the degeneracies are resolved and the spectrum is composed by $n + 1$ bands. In this limit the coupling between different bands is very weak and each state is effectively coupled to $O(n)$ other states inside the band. We assume free boundary conditions and express all energies in units Δ_0 ($\hbar = 1$). We study the behavior of the error (that is the logarithm of the fidelity [4])

$$E_t(\tau) = -\ln F_t(\tau) = -\ln \left| \langle \Psi | \mathcal{T} \exp \left(-i \int_0^t H_\tau(s) ds \right) | \Psi \rangle \right|^2, \qquad (2)$$

where $\mathcal{T}$ denotes time ordering. The behavior of E will be studied at fixed t as a function of τ, the inverse frequency of the noise characterizing the fluctuations of δ and J. The initial state $|\Psi\rangle$ is taken to be an eigenstate of $\sigma_z^{(j)}$ ($j = 1, \ldots, n$) and we concentrate on the central band of zero total magnetization, characterized by the highest density of states, and for which one expects the effect of noise to be most pronounced.

An exact calculation of the error to order J^2 can be carried out explicitly [5]. We only give here the approximate expression at a fixed time $t \geq \tau$:

$$E_t(\tau) \simeq 4J^2\sigma^2 t \begin{cases} n_c\tau & \tau < \tau_p & \text{(all regimes)} \\ n_{\uparrow\downarrow}\tau & \tau_p < \tau < \tau_c & \text{(all regimes)} \\ n_{\uparrow\downarrow}\tau & \tau > \tau_c,\ J \simeq \delta & \text{(ergodic regime)} \\ n_{\uparrow\downarrow}\pi/\delta & \tau > \tau_c,\ J < \delta/n & \text{(FGR regime)} \end{cases} \tag{3}$$

where $n_c = n_{\uparrow\downarrow} + n_{\uparrow\uparrow}$ is the total number of links, $n_{\uparrow\uparrow}$ ($n_{\uparrow\downarrow}$) being the number of nearest-neighbor parallel (antiparallel) pairs in the initial state. Notice that, unlike $n_{\uparrow\downarrow}$ and $n_{\uparrow\uparrow}$, n_c does not depend on the initial state $|\Psi\rangle$ but only on the geometry of the spin lattice.

In Fig. 1 we show the behavior of $E_t(\tau)$ for different values of δ. In the *static* situation (large τ, so that δ's and J's can be considered constant) system (1) is characterized by two distinct dynamical regimes: the Fermi Golden Rule (FGR) ($J < J_c$) and the ergodic regime ($J > J_c$), where $J_c \sim \delta/n$ [2, 3]. The FGR holds below threshold (weak coupling $J \ll \delta$) and is characterized by a Lorentzian local density of states. The ergodic regime takes place in the strong-coupling regime $\delta \simeq J$, when all the levels inside the band participate to the dynamics [6] and the local density of states coincides with the (Gaussian) density of states. The fidelity $F_t(\tau)$ [from which the error (2) is computed] is always the Fourier transform of the local density of states [6].

When τ becomes smaller, the imperfections become *dynamical* and different regimes emerge as a function of the frequency τ^{-1}. The transition at $\tau = \tau_c$ occurs when the error starts deviating from the linear behavior given by Eq. (3). As $\tau < \tau_c$ the two distinct (ergodic and FGR) behaviors characterizing the static case (compared in Fig. 1 only for the sets with $n_{\uparrow\downarrow} = 8$) cannot be resolved anymore. The additional kink at $\tau \simeq \tau_p = \pi/4\Delta_0$ sets in when the single spin dynamics starts to play a role. As a global feature, the error tends to vanish linearly with τ.

In conclusion, below a given (frequency) threshold, the errors can be considered static, and thus can be corrected by using any of the known methods. One observes in this case two different dynamical regimes. Above this threshold these regimes become unresolved. The difference between these regimes, found for static imperfections, holds also in the quasi-static case. On the other hand, unitary dynamical errors average to zero in the high frequency case. Our results can be relevant in the context of the strategies that have been proposed during the last few years in order to suppress decoherence [7].

These results are independent of the form and the size of the quantum computer. They remain valid under quite general conditions on the system Hamiltonian [5], allowing a more general application of these findings. Our results show that it is crucial to optimize the computing timescale, by choosing it between the two competing types of noise (static and dynamic). In turn, this suggests new strategies to develop general error correcting techniques.

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