Reconnection of Stable/Unstable Manifolds of the Harper Map

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Harper 方程式は、パラメーターの変化によりセパラトリックスのつなぎ変えを起こす最も簡単なモデルのひとつである(リコネクション)。Harper 方程式の時間を離散化した Harper 写像において安定/不安定多様体の公式を解析的に導いた。これにより、両多様体はリコネクションが起こるパラメーター付近でリコネクションの前駆となる振動を始めることを示した。

The Harper map is one of the simplest chaotic system exibiting reconnection of invariant manifold. The map depends on a real parameter k and is defined on $(v,u) \in [-\pi,\pi]^2$:

$$\begin{aligned} v(t+\sigma)-v(t) &= -\sigma\sin u(t) \\ u(t+\sigma)-u(t) &= k\sigma\sin v(t+\sigma) \end{aligned} \tag{1}$$

where $\sigma(>0)$ is the time step and plays a role of the small parameter. In the continuous limit, $\sigma\to 0$, the map reduces to a set of differential equations which admit topologically different separatrices depending on the parameter k (Fig. 1).

In this paper, we consider the change of the unstable manifolds for $k\to 1-0$. The Laurent expansions of perturbative solution near its singularity points is expressed as a Borel summable asymptotic expansion in a sector including $t=-\infty$ and it is analytically continued to the other sector, where the solution acquires new terms (ABAO terms) describing heteroclinic tangles. ABAO term corresponds to the difference between stable and unstable manifolds which is exponentially small ($\epsilon \equiv e^{-\frac{A}{\sigma}}$ (Re$[A]>0$)) with respect to σ (See more details in [2]). Therefore

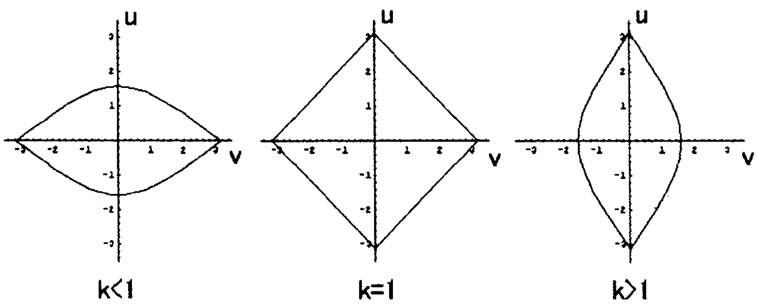

図 1: Separatrix of the Harper map in the continuous limit

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in the sector which including $t=\infty$, the unstable manifold (v_u, u_u) is expressed as the doulbe expansions in σ and ϵ:

$$v_u(t) = \sum_{n=1}^{\infty}\sum_{l=0}^{\infty} \sigma^{j_n}\epsilon^{2n-1} v_{nl}(t)\sigma^l, \quad u_u(t) = \sum_{n=1}^{\infty}\sum_{l=0}^{\infty} \sigma^{j_n}\epsilon^{2n-1} u_{nl}(t)\sigma^l \tag{2}$$

where j_n is determined by matching v_{nl}, u_{nl} with ABAO term near singular points of perturbative solution. Note that (2) is the only form which admits matching [2]. We get the formula of the stable/unstable manifolds of the Harper map. This formula indicates taht stable/unstable manifolds acquire new oscillatory portion corresponding to the heteroclinic tangle after the reconnection. Fig. 2 shows the unstable manifold for $k = 1.0 - 10^{-9}$. One sees the oscillation near $(0, \pi)$.

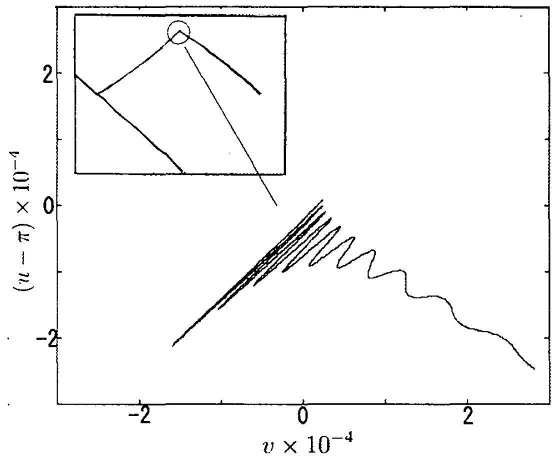

図 2: The analytically constructed unstable manifold near $(0,\pi)$. $\sigma = 0.35$, $k = 1.0 - 10^{-9}$

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